Global Asset Allocation with Time-varying Risk.

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Abstract

We extend the number of assets available to a UK investor who wishes to select a portfolio of international financial assets. A two-stage allocation strategy is adopted by first forming time-varying portfolios of international government bonds and European equity, both of which constitute a single asset in the final asset allocation procedure. We find that extending the investment opportunity set presents substantial risk-returns advantages to the investor, together with better performing portfolios. Finally, we show that the level of home country bias prevalent in the UK is quite large. Our results show that, on average, home assets constitute only 57% of the optimal portfolio, while survey results suggest the actual proportion of home assets held by UK investors is 82%. We find that on average six foreign assets should be held in the optimal portfolio with US, French and German equity all having a major role to play.

Keywords: Asset allocation, international diversification, M-GARCH
JEL Classification: G11, G15.

1 Introduction

The potential benefits of international portfolio diversification are well documented, from the early literature of Grubel (1968) and Levy and Sarnat (1970) to the present day (e.g. De Santis and Gerard (1997)). Flavin and Wickens (1998a,b) show that asset allocation can be greatly improved by re-balancing the portfolio in response to time variation in the conditional covariance matrix of asset excess returns compared with using an allocation based on a constant estimate of the matrix. This stems from the fact that variances and covariances exhibit a great deal of persistence unlike the level of returns which are virtually serially uncorrelated. These papers employ a multivariate GARCH model to capture this time variation and use the estimated variances and covariances as inputs into the portfolio selection problem. One potential criticism of these optimal allocation models is that, due to the well documented problems of dimensionality in estimating M-GARCH models, the number of potential assets that an investor is allowed to hold is small - three in each of the aforementioned papers. In this paper we analyse the effects of increasing the number of assets in which a financial market participant may invest. In particular, we address the question from the perspective of a UK investor who is willing to internationally diversify his portfolio of risky financial assets.

To circumvent the dimensionality problem, we adopt a two stage allocation. This is not unlike how many investment companies operate, with teams of experts specialising in selecting portfolios of a given class of assets or portfolios from a specific geographical region. The first step in our analysis allows the UK investor to form an optimal portfolio of international government bonds. Levy and Lerman (1988) find that a US investor who diversified across world bond markets could have realised returns more than twice the mean rate of return on a domestic US bond portfolio at the same risk level. Having found the optimal time-varying asset proportions that form these portfolios in each period of the analysis, we calculate the realised returns on this optimal portfolio and use this series of returns as the returns on a single asset in the final allocation decision. A similar process is followed to form an optimal time-varying portfolio of European equities. Therefore the final portfolio selection decision focuses on five assets, namely UK equity, US equity, Japanese equity, European equity and a global bond.

We find that the optimal portfolio of bonds is dominated by the home bond with the Japanese bond also playing a significant role in its composition. There is also evidence that the excess returns on national government bonds may be more predictable than equity returns. The conditional second order moments are well characterised by an M-GARCH(1,1) process. Meanwhile the most important component of the European equity portfolio is French equity, followed by German equity with the Italian asset making a very limited contribution. In the time-varying optimal global portfolios we discover that UK equity dominates the portfolio, that US and French equity are major components, and that German equity and the UK bond also have a significant role.

Our results suggest that in the long run, volatility is transmitted between stock markets with the exception of Japanese and European markets but there is no evidence of any spillover effects between stock and bond markets. These markets appear to be segmented in the long run. In the short run, volatility spillovers again occur between the stock markets with Japan-Europe remaining the exception. We find limited evidence of volatility spillovers from the UK and Japan stock markets to bond markets but this is probably due to the fact that the bond portfolio is dominated by the government bonds of these countries.

Expanding the investment opportunity set of the UK investor beyond just UK and US equity and bonds as considered in Flavin and Wickens (1998b) by including additional international assets is found to offer the investor a significant risk-return advantage and to substantially improve the performance of the optimal portfolio. However, the home bias problem discussed in Flavin and Wickens (1998b) is further accentuated by the introduction of these extra assets. Our results show that on average, 57% of investable funds should be held in home assets with the remaining 43% invested in foreign assets. This compares with the survey findings of French and Poterba (1991) who report that 82% of the portfolios of UK investors are home assets. These new results suggests that the problem of home bias is more acute than previously thought.

The remainder of the paper is structured as follows; Section 2 presents the M-GARCH model that we use in the paper and discusses the data. In section 3, we present the results of the analysis. Finally, section 4 contains our concluding remarks.

2 The Model and Data

2.1 The Model

The aim of this paper is to identify the optimal global portfolio of risky assets in each period for a UK investor. We adopt a two-stage asset allocation strategy in which it is assumed that investors use Markowitz's mean-variance portfolio theory to determine the optimal allocation of assets at each stage. In stage one, investors form an optimal portfolio for each class of asset. These portfolios are then treated as individual assets in stage two and formed into a single global portfolio. At each stage, investors therefore select the optimal portfolio identified by the point of tangency between a ray from the origin and the portfolio frontier of excess returns on the risky assets. This is fully described in Flavin and Wickens (1998a). The risk-free asset in each case is the UK domestic risk-free bond. All excess returns are expressed in sterling terms.

The vector of excess returns is assumed to be Normally distributed with a time-varying conditional mean and conditional variance generated by an M-GARCH(1,1) process. It is widely recognised that there is a major computational problem in the full information estimation of such models due to the large number of parameters they possess. We therefore adopt the M-GARCH representation used in Flavin and Wickens (1998a).[1] This model is a variant of the Berndt, Engle, Kraft & Kroner (BEKK) representation.[2] If n is the number of risky assets, then compared with the most general formulation of the model, this representation results in the number of parameters to be estimated increasing at the rate n^2 instead of n^4. When $n = 3$, the number of parameters to be estimated is reduced from 78 using the most general model to 18, a substantial saving. This model has a VAR(1) structure in the conditional mean equations with the errors following a GARCH(1,1) process. When we deal with national stock market indices, we allow for the possibility of including a dummy variable to capture the influence of the October 1987 Stock market crash. Therefore the model which we estimate is as follows:

[1] This parameterisation is consistent with the covariance stationary model developed in Engle and Kroner (1995).

[2] For a full treatment of multivariate (G)ARCH models, the reader is referred to Bollerslev, Engle and Nelson(1994), Bera and Higgins(1993) or Bollerslev, Chou and Kroner(1992) for a survey of (G)ARCH models in finance

$$\begin{aligned}
\mathbf{r}_t &= \mathbf{c} + \boldsymbol{\beta}\mathbf{r}_{t-1} + \gamma dum87 + \boldsymbol{\xi}_t \\
\boldsymbol{\xi}_t &\mid \Psi_{t-1} \sim N(0, \mathbf{H}_t) \qquad (1) \\
\mathbf{H}_t &= \mathbf{V}'\mathbf{V} + \mathbf{A}'(\mathbf{H}_{t-1} - \mathbf{V}'\mathbf{V})\mathbf{A} + \mathbf{B}'(\boldsymbol{\xi}_{t-1}\boldsymbol{\xi}'_{t-1} - \mathbf{V}'\mathbf{V})\mathbf{B}
\end{aligned}$$

where $\mathbf{r}_t$ represents a $kx1$ vector of financial asset excess returns to be defined for each of the applications undertaken in this paper. $\boldsymbol{\beta}$ is a kxk matrix of regression parameters and $\boldsymbol{\gamma}$ is a $kx1$ vector of parameters. $\mathbf{V}$ is a lower triangular matrix, while $\mathbf{A}$ and $\mathbf{B}$ are full kxk symmetric matrices.[3] $\mathbf{V}'\mathbf{V}$ denotes the long-run conditional covariance matrix of asset excess returns. The parameters of the $\mathbf{A}$ and $\mathbf{B}$ matrices captures the importance of the short-run dynamics and identify the sources of these short-run deviations from the computed conditional long-run value.

2.2 The Data

This paper uses time series data on the annualised monthly returns on equity and bonds for a number of countries for the period January 1982 to May 1998. The equity returns are calculated from national stock market indices

[3] Defining $\mathbf{A}$ and $\mathbf{B}$ allows the interaction between assets to take place with minimum time delay. A simple example will illustrate the difference. If we assume that we have a two asset system with returns, $r_{1,t+1}$ and $r_{2,t+1}$, and conditional variances denoted by $H_{11}$and H_{22} and covariance by H_{12}.

Definition 1. $\mathbf{V}$, $\mathbf{A}$ and $\mathbf{B}$ are defined as symmetric, triangular matrices.

$$H_{11,t} = V_{11}^2 + A_{11}^2 H_{11,t-1} + B_{11}^2 \varepsilon_{1,t}^2$$

and

$$\varepsilon_{1,t+1} = r_{1t} - b_1 - \lambda_1 r_{1,t-1} - \lambda_2 r_{2,t-1}$$

It takes two periods for the second asset to influence the conditional variance of the first.

Definition 2. $\mathbf{A}$ and $\mathbf{B}$ are defined as full, symmetric matrices.

$$\begin{aligned}
H_{11,t} = V_{11}^2 + & \quad (A_{11}^2 H_{11,t-1} + 2A_{11}A_{12}H_{12,t-1} + A_{12}^2 H_{22,t-1}) \\
+ & \quad (B_{11}^2 \varepsilon_{1,t}^2 + 2B_{11}B_{12}\varepsilon_{1,t}\varepsilon_{2,t} + B_{12}^2 \varepsilon_{2,t}^2)
\end{aligned}$$

Now the conditional variance of the first asset is influenced by the second with only a one period time lag through the covariance term, $H_{12,t-1}$ and its own variance, $H_{22,t-1}$.

for the UK, the US, Japan, Germany, France and Italy. The UK equity market is represented by the Financial Times All share index, US equity by the Standard and Poors Composite index, while the new Tokyo stock exchange is used to represent Japanese equity. The other European indices are represented by Datastream calculated indices. The bond returns are calculated from Datastream's government bond indices for the UK, the US, Japan and Germany. The analysis is conducted from the perspective of a UK investor, and so each of the national indices is converted to sterling using end of month exchange rates. Dividend payments are taken into account in the calculation of the total returns to equity, and coupon payments are included in the holding period returns to government bond. The risk-free rate is represented by the return on the UK government 30 day Treasury Bill which is riskless in nominal terms.

As the analysis is conducted in terms of rates of return in excess of the risk-free rate, the volatility of the risk-free rate will have no effect on the conditional volatilies of the risky assets and will not contribute to the riskiness of the optimal portfolio.

3 Results and Analysis

3.1 Global Bond Portfolio

One of the assets appearing in stage two of the global asset allocation is a portfolio of international government bonds, a global bond portfolio. Equation (1) is estimated, therefore, for the monthly excess holding period returns on government bonds expressed in sterling terms. The vector of excess returns is defined as $\mathbf{r} = (ukb, usb, grb, jpb)'$, where ukb denotes the UK government bond and usb, grb and jpb represent the bonds of the US, Germany and Japan respectively. The dummy variable for the stock market crash was omitted from the global bond model.

Maximum likelihood estimates of equation (1) were obtained using the BHHH algorithm. As $\mathbf{V}$, $\mathbf{A}$ and $\mathbf{B}$ are symmetric, we report only their lower triangles. t-statistics are in parentheses.

3.1.1 Conditional Mean

$$\mathbf{c} = \begin{bmatrix} 2.94 \\ (1.73) \\ 0.30 \\ (0.10) \\ -0.54 \\ (0.22) \\ -0.001 \\ (0.04) \end{bmatrix}, \boldsymbol{\beta} = \begin{bmatrix} -0.01 & 0.02 & -0.20 & 0.14 \\ (0.15) & (0.36) & (2.71) & (2.22) \\ -0.17 & -0.04 & 0.37 & 0.04 \\ (1.19) & (0.42) & (2.86) & (0.38) \\ -0.09 & -0.04 & 0.32 & 0.02 \\ (0.75) & (0.53) & (2.83) & (0.24) \\ -0.13 & -0.16 & 0.48 & 0.15 \\ (0.76) & (1.62) & (3.75) & (1.69) \end{bmatrix}$$

3.1.2 Conditional Covariance

$$\mathbf{V} = \begin{bmatrix} 23.73 & & & \\ (4.79) & & & \\ 7.51 & 41.35 & & \\ (1.06) & (15.03) & & \\ 5.04 & 12.26 & 26.54 & \\ (1.16) & (3.79) & (10.43) & \\ 8.75 & 20.56 & 16.13 & 30.57 \\ (2.56) & (6.33) & (3.15) & (10.67) \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 0.37 & & & \\ (1.35) & & & \\ -0.23 & 0.50 & & \\ (1.13) & (1.48) & & \\ -0.56 & 0.07 & 0.29 & \\ (3.33) & (0.24) & (0.77) & \\ -0.63 & -0.47 & -0.43 & 0.09 \\ (4.14) & (2.18) & (1.80) & (0.35) \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} 0.18 & & & \\ (2.45) & & & \\ 0.21 & 0.12 & & \\ (4.52) & (1.27) & & \\ -0.03 & 0.65 & 0.02 & \\ (0.50) & (4.06) & (0.23) & \\ -0.01 & 0.13 & -0.12 & 0.12 \\ (0.12) & (1.99) & (1.63) & (0.13) \end{bmatrix}$$

The long-run covariance matrix of government bond excess returns is

$$\begin{bmatrix} 562.93 & & & \\ 178.16 & 1795.94 & & \\ 119.77 & 544.96 & 879.99 & \\ 207.68 & 915.83 & 724.21 & 1694.03 \end{bmatrix} \tag{2}$$

3.1.3 Interpretation of the Results

The most interesting feature of the estimates of the conditional mean of each of the excess bond returns is the significance of the first lag of the excess return on the German bond (the third column of the $\boldsymbol{\beta}$ matrix). Although only weakly significant, the results imply that the greater the excess return on the German bond, the higher the excess return in the next period of the other government bonds. An exception is the UK bond, which has a lower excess return next period. A possible explanation for this difference is that exchange rate effects are dominating the excess returns. The lagged Japanese bond is also significant for the UK and Japanese bonds

In the conditional covariance matrix, the elements of the long-run, unconditional, covariance matrix, $\mathbf{V}$, are predominantly statistically significant. All of the elements on the leading diagonal are significant as are the off-diagonal terms, with the exception of those relating the UK bond to the US and the German bonds. This suggests a considerable volatility contagion between markets in the long run. Again this could be due to exchange rate effects. The short-run deviations from the long-run covariance matrix are due mainly to the off-diagonal elements. Each $(i,j)th$ element of $\mathbf{H}_t$ has a corresponding statistically significant $(i,j)th$ element in either the $\mathbf{A}$ or $\mathbf{B}$ matrices. The Japanese bond seems to be a particularly important source of short-run volatility in the other bond markets.

The importance of this short-run volatility is depicted for each of the bonds in Figure 1. The total conditional variance of each bond and its long-run value, depicted as a heavy horizontal line is shown. For each bond, there is considerable short-run activity and this is likely to influence the optimal bond portfolio in each period. The US and Japanese bonds are the most volatile. The returns on the German bond are surprisingly stable especially given that they are expressed in sterling terms and hence contain exchange rate risk. In fact, its range of movement is less than that of the UK bond, which is the home asset. However, the unconditional variance of returns is smallest for the UK bond (see matrix 2).

3.1.4 Optimal Bond Portfolio

Given the limited predictability of the excess bond returns - a feature accentuated even more in the equity returns - in generating the time-varying portfolio shares, we assume a constant vector of expected bond returns. This has the added advantage that all of the variation in the estimated frontiers, and hence the portfolio shares, can be attributed to variation in the conditional covariance matrix of excess returns. This approach is also consistent with Cumby, Figlewski and Hasbrouck (1994) who use the historical mean of each asset as its expected value. Jobson & Korkie (1981) advocate the use of global shrinkage based on Stein estimators whereby all assets of the same class have the same expected excess return. This is an extreme case of Stein estimation with the individual asset being assigned a weight of zero and the global mean having a weight of one. Jobson & Korkie show that this approach significantly improved the practical application of the mean-variance framework. Since we are working with financial asset indices as opposed to individual securities, these appoaches reduce to the same thing. Another reason for making this assumption is that the sensitivity of the portfolio shares to small variations in the mean is far greater than that to variations in the covariance matrix, Kallberg and Ziemba (1984). Best and Grauer (1991) show that even small changes in the mean vector can result in dramatic variation in the composition of the estimated optimal portfolio of risky assets.

Continuous re-balancing of the portfolio to changes in the predicted excess return would not only be expensive due to transaction costs, it would also be counter-productive because of the lack of persistence of the deviations of excess returns from their unconditional means. This is not true of rebalancing due to changes in the conditional variance because of their much higher degree of persistence and their lower volatility.

We adopt two asset allocation strategies, both based on the seminal work of Markowitz (1952, 1959). First, we compute the optimal bond mix assuming there is no constraint on the sign of the assets in the portfolio, i.e. the investor is allowed to hold unlimited short positions in each of the assets. With this allocation strategy, we find that the UK investor should hold the individual bonds shares depicted in Figure 2. The UK bond is the dominant asset in the portfolio in each period, with a mean holding of 84%. This is not surprising as the UK bond has the least conditional variance due largely to being free of exchange rate risk. The next highest share is of Japanese bonds which account for 17% of the portfolio on average, and contributed greatly to the composition of the portfolio in the early period of the sample. After 1996, however, its importance is diminished and it is often held short. The US bond is the third most important accounting for 10% of the portfolio; in nearly every period it has a small, but positive, holding. In contrast, the German bond is usually held short by as much as 11% of investable funds. The proceeds from this position allow greater investment in the other bonds. The main features of this allocation strategy are summarised in Table 1.

	Mean	Minimum	Maximum
UK Bond	84	56	106
US Bond	10	-30	31
German Bond	-11	-62	28
Japanese Bond	17	-14	81

Table 1: Key features of the Unrestricted Bond allocation

In practice, investors are probably unlikely to follow this investment strategy for two reasons. First, such large volatilities in the optimal shares would give rise to high transactions costs. Second, many institutional investors are prohibited by law from holding a short position. We therefore construct optimal portfolios subject to the constraint that asset shares must be non-negative. For the unconstrained portfolio it was possible to obtain a closed-form expression for the portfolio shares and hence find the return on the portfolio in each period. For the constrained portfolio we use quadratic programming to minimise the variance of the portfolio subject to a target rate of return which is chosen as the mean return on the unconstrained optimal portfolio. Thus, in terms of the mean portfolio return, the investor is not penalised by adding this restriction. Also, it aids comparison with the unrestricted case.

Figure 3 shows the composition of the optimal constrained global bond portfolio over the entire sample. The UK bond continues to dominate the portfolio. On average, it accounts for 80% of the portfolio and its range of movement - 69% to 94% - is substantially dampened. The relative importance of each of the other assets remains unaltered, with the Japanese bond having a mean holding of 13%, followed by the US bond with 6.5% and the German bond contributing only 0.5% on average. Each of the foreign assets have a minimum holding of 0%, indicating that this added constraint is binding. Table 2 summarises the key features of this allocation procedure.

	Mean	Minimum	Maximum
UK Bond	80	69	94
US Bond	6.5	0	13
German Bond	0.5	0	5
Japanese Bond	13	0	31

Table 2: Key features of the Restricted Bond allocation

In constructing the global portfolio, we assume that there is a single international government bond formed from these individual government bonds using the time-varying restricted weights just computed. This global bond portfolio will constitute one asset in the final global portfolio. We now turn to the treatment of European equity in the global portfolio.

3.2 European Equity Portfolio

A second asset in stage two of the global asset allocation strategy is a portfolio of European equities. Once again, all returns are expressed in sterling as we are addressing the problem from the perspective of a UK investor. The vector of returns defined by equation (1) is $\mathbf{r} = (greq, freq, iteq)'$, where $greq$, $freq$ and $iteq$ denote total returns on German, French and Italian equity market indices respectively. The dummy variable for the 1987 stock market crash proved to be insignificant and is omitted from the model. The following estimates were obtained.

3.2.1 Conditional Mean

$$\mathbf{c} = \begin{bmatrix} 9.66 \\ (2.01) \\ 12.56 \\ (2.30) \\ 10.97 \\ (1.62) \end{bmatrix}, \boldsymbol{\beta} = \begin{bmatrix} -0.10 & 0.08 & 0.11 \\ (1.08) & (0.86) & (1.65) \\ -0.07 & 0.003 & 0.07 \\ (0.58) & (0.03) & (1.04) \\ -0.09 & 0.11 & 0.03 \\ (0.74) & (0.85) & (0.35) \end{bmatrix}$$

3.2.2 Conditional Covariance

V, A and **B** are all symmetric matrices.

$$\mathbf{V} = \begin{bmatrix} 59.24 & & \\ (9.76) & & \\ 54.82 & 39.46 & \\ (3.66) & (1.38) & \\ 41.14 & 26.31 & 62.11 \\ (6.62) & (1.66) & (4.12) \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 0.02 & & \\ (0.03) & & \\ 0.06 & 0.46 & \\ (0.23) & (1.51) & \\ 0.74 & -0.55 & 0.04 \\ (6.30) & (2.39) & (0.11) \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0.11 & & \\ (1.13) & & \\ 0.32 & 0.11 & \\ (3.64) & (1.11) & \\ 0.14 & 0.01 & 0.08 \\ (1.83) & (0.12) & (1.21) \end{bmatrix}$$

The long-run conditional covariance matrix of asset excess returns is given by:

$$\begin{bmatrix} 3509.65 & & \\ 3247.90 & 4562.95 & \\ 2437.30 & 3293.59 & 6242.76 \end{bmatrix} \tag{3}$$

3.2.3 Interpretation of the Results

The absence of any significant estimate in the $\boldsymbol{\beta}$ matrix of the conditional mean reflects the usual difficulty of predicting equity returns and is in contrast with the conditional variance. The coefficients of the long-run covariance matrix, which is captured by the $\mathbf{V}$ matrix, are significantly different from zero, and very strongly so in most cases. The significance of elements of the $\mathbf{A}$ and $\mathbf{B}$ matrices show that the conditional covariance matrix is time-varying in the short run. The estimates suggest short-run volatility transmission between Italian-German (A_{31}, B_{31}), Italian-French (A_{32}) and German-French (B_{21}) markets, and hence provides strong support for our approach to modelling time-varying volatility. Many other equity studies assume a diagonal covariance structure (e.g. Engle et al. (1990)), or adopt the more restrictive constant correlation approach of Bollerslev (1990). The more general approach should lead to improved asset allocation.

The estimates of the time-varying variances are depicted in Figure 4. Each of the assets exhibits a large degree of variability. All three markets seem to be affected by the US stockmarket crash, French equity more than either German or Italian. These markets were effected to a lesser degree but have other periods of turbulence also. In the long run, the German stockmarket seems to be the most stable and the Italian market the most volatile.

3.2.4 Optimal European Equity Portfolio

As before, first we consider the optimal unrestricted portfolio. This is depicted in Figure 5. French equity clearly dominates the portfolio. It has a mean holding of 66.5% of investable funds and is always held long. German equity, on the other hand, has a holding of 34.5% but this is quite volatile and in at least 10 periods is held short. The Italian equity holding is usually quite small and, on average, is held short, composing -1% of the portfolio. Table 3 summarises this asset allocation strategy.

	Mean	Minimum	Maximum
France	66.5	15	191
Germany	34.5	-71	90
Italy	-1	-19	29

Table 3: Key features of Unrestricted European Equity allocation

When short sales are prohibited the portfolio shares are much more stable, see Figure 6. The portfolio still consists mainly of French equity. The average share is 66% and the variation is between 65% and 75% of the portfolio. German equity comprises most of the remainder of the portfolio, having a mean holding of 31.5%. It never exceeds 34% and there are a number of occasions when the short sales restriction is binding and German equity is not held. Finally, Italian equity was not a very attractive asset from the perspective of a UK investor. It is very often omitted from the optimal portfolio of European equities and has a mean holding of 2.5%. The returns on Italian stock were obviously not sufficient to compensate the investor for holding such a volatile asset. This volatility was due in part to currency risk. Table 4 presents a summary of the constrained allocation strategy.

	Mean	Minimum	Maximum
France	66	65	75
Germany	31.5	0	34
Italy	2.5	0	25

Table 4: Key features of the Restricted European Equity allocation

In constructing the global portfolio, we assume that the constrained European equity portfolio constitutes another asset.

3.3 Global Asset Allocation

The global portfolio is assumed to consist of five assets: the global bond, European equity, and UK, US and Japanese equity. The vector of excess returns with respect to the UK risk-free rate is $\mathbf{r} = (ukeq, useq, jpeq, eueq, gblb)'$, where ukeq, useq and jpeq are the sterling excess returns of the national stockmarkets of the UK, the US and Japan respectively; eueq is the European equity portfolio and gblb is the global bond portfolio. The estimates of equation (1) are

3.3.1 Conditional Mean

$$\mathbf{c} = \begin{bmatrix} 11.74 \\ (2.78) \\ 11.77 \\ (1.96) \\ 4.98 \\ (0.61) \\ 14.25 \\ (2.44) \\ 2.88 \\ (1.12) \end{bmatrix}, \boldsymbol{\beta} = \begin{bmatrix} 0.03 & 0.07 & 0.07 & -0.18 & 0.22 \\ (0.31) & (0.64) & (1.11) & (2.12) & (0.96) \\ 0.03 & 0.04 & 0.03 & -0.12 & 0.05 \\ (0.19) & (0.30) & (0.41) & (1.04) & (0.17) \\ -0.002 & 0.01 & 0.11 & -0.10 & 0.31 \\ (0.02) & (0.05) & (1.11) & (0.65) & (0.80) \\ -0.11 & 0.13 & 0.05 & -0.06 & 0.17 \\ (0.88) & (1.02) & (0.67) & (0.57) & (0.56) \\ 0.03 & -0.01 & 0.03 & -0.09 & 0.10 \\ (0.41) & (0.30) & (0.91) & (2.00) & (0.95) \end{bmatrix}$$

$$\boldsymbol{\gamma} = \begin{bmatrix} -278.12 \\ (1.67) \\ -160.54 \\ (1.85) \\ 0 \\ \\ 0 \\ \\ 0 \end{bmatrix}$$

3.3.2 Conditional Covariance

$$\mathbf{V} = \begin{bmatrix} 51.72 & & & & \\ (7.36) & & & & \\ 31.78 & 53.46 & & & \\ (2.81) & (9.54) & & & \\ 24.66 & 25.54 & 74.83 & & \\ (1.95) & (1.93) & (5.66) & & \\ 30.37 & 13.12 & 11.99 & 46.96 & \\ (4.37) & (1.68) & (0.80) & (4.17) & \\ 13.51 & 3.88 & 4.84 & -2.31 & 0.07 \\ (0.94) & (0.16) & (0.09) & (0.03) & (1.50) \end{bmatrix}$$

$$
\mathbf{A} = \begin{bmatrix} -0.06 & & & & \\ (0.17) & & & & \\ -0.45 & -0.53 & & & \\ (1.77) & (2.17) & & & \\ -0.03 & -0.16 & -0.17 & & \\ (0.07) & (1.63) & (0.25) & & \\ -0.41 & -0.15 & 0.13 & 0.59 & \\ (1.90) & (0.52) & (0.33) & (2.39) & \\ 0.04 & -0.01 & 0.43 & -0.25 & -0.70 \\ (0.09) & (0.02) & (1.16) & (1.04) & (1.77) \end{bmatrix},
$$

$$
\mathbf{B} = \begin{bmatrix} 0.28 & & & & \\ (1.96) & & & & \\ -0.09 & 0.14 & & & \\ (0.84) & (1.08) & & & \\ 0.18 & 0.05 & 0.11 & & \\ (2.42) & (0.63) & (1.12) & & \\ 0.09 & 0.17 & -0.07 & 0.20 & \\ (0.95) & (1.76) & (0.94) & (1.76) & \\ 0.10 & -0.02 & 0.12 & -0.03 & -0.04 \\ (1.69) & (0.21) & (2.35) & (0.51) & (0.41) \end{bmatrix}
$$

The long-run variance-covariance matrix of asset excess returns is given by;

$$
\begin{bmatrix} 2674.76 & & & & \\ 1643.51 & 3867.95 & & & \\ 1275.12 & 2148.83 & 6859.04 & & \\ 1570.81 & 1666.50 & 1981.23 & 3443.66 & \\ 699.15 & 636.92 & 794.73 & 411.08 & 226.58 \end{bmatrix} \tag{4}
$$

3.3.3 Interpretation of the Results

The conditional mean has little predictive power once again. Most coefficients in $\boldsymbol{\beta}$ are not statistically different from zero. The exception is the first lag of the excess return on European equity which seems to have some predictive power for both UK equity returns and global bond returns. A dummy variable for the October 1987 US stockmarket crash was found to be significant in the equations for UK and US equities. The inclusion of the dummy in the estimation provides estimates of the conditional covariance structure that is corrected for the stockmarket crash. Subsequent portfolio allocation will assume that a crash will not re-occur.

The estimate of **V** shows that the long-run conditional covariance matrix exhibits contagion effects between national equity markets, but not between equity and bonds. The long-run covariance matrix is given by (4) and shows that Japanese equity is the most volatile asset, UK equity is the least volatile equity reflecting the absence of exchange risk, but the global bond is by far the least volatile asset of all.

The estimates of **A** and **B** suggest that there is also contagion in the short-run deviations between equity markets. Evidence of short-run volatility spillovers exists between UK and US equity (A_{21}), UK and European equity (A_{41}), UK and Japanese equity (B_{31}), US and Japanese equity (A_{32}) and US and European equity (B_{42}). Only European and Japanese equity markets seem to be segmented (as they are in the long run too). The size and significance of the diagonal elements of **A** and **B** suggest persistence in the short-run deviations in the UK (B_{11}), the US (A_{22}) and European equity markets (A_{44} and B_{44}).

Finally in the short-run, we find some evidence that stock and bond market volatility may be linked. There is a statistically significant relationship between our glo al bond portfolio and the UK and Japanese equity markets. This may be due to the fact that our global bond portfolio is dominated by UK and Japanese bonds (on average these two bonds comprise 93% of the portfolio). Short-run deviations in bond market volatility seem to be caused by its own variance effect and the aforementioned relationships with UK and Japanese equity markets. Figure 7 plots the conditional variance of each asset over time.

3.3.4 Portfolio Frontiers

The estimated time-varying conditional covariances can be used to determine mean-variance portfolio frontiers which will also vary over time. Figure 8 shows the mean, minimum and maximum frontiers over the entire sample. These indicate that there has been considerable variation in both the shape and location of the frontier. This alone suggests that asset allocation can be improved by taking account of time variation in the covariance matrix of asset returns, and justifies our modelling approach.

We can compare these frontiers with those obtained in previous studies we have undertaken on international portfolio diversification. In the first study (Flavin and Wickens (1998b)), we allowed the UK investor only one opportunity to diversify internationally by holding US equity. Figure 9 plots the mean frontiers generated by the two investment opportunity sets. It is clear from the fact that the frontier generated by the "global" investment set is closer to the origin at all points that the UK investor can reap substantial risk-return gains by considering the broader range of potential investment vehicles examined in this paper. Figure 10, which shows the minimum and maximum frontiers in addition, supports this. The global frontiers (shown by the heavy lines) always lie inside their counterparts generated by the more restricted investment set.

3.3.5 Optimal Global Portfolio Performance

The unconstrained optimal global portfolio is obtained as the point of tangency between the capital market line and the portfolio frontier. Figure 11 shows the behaviour of both the excess return on the global portfolio and its conditional standard deviation over time. As expected, they move together, showing that greater risk must be compensated by a higher return. Furthermore, we can see that the optimal portfolio always has a positive expected return, implying that the optimal portfolio is always located on the efficient segment of the portfolio frontier.

The Sharpe Performance Index ($SPI = \frac{return}{risk}$) for the global portfolio is plotted in Figure 12. The average value is 0.30. This represents a massive increase of 67% over the average SPI value achieved when the only foreign asset is US equity, and an even bigger increase of 87.5% over an investment set limited to domestic assets only. This provides a clear message that international diversification may lead to better portfolio performance, and the greater the number of assets considered, the larger the gains may be.

3.3.6 Optimal Global Portfolio Composition

The shares of each asset in the unconstrained optimal portfolio are shown in Figure 13. They are highly volatile. Domestic UK equity dominates the portfolio with an average holding of 52%, but there is a period when this asset is held short. In fact each of the assets in the analysis is sold short at some stage, leading to large and frequent changes in the composition of the portfolio. US equity is the next most important asset, accounting on average for 28% of investable funds. Both the European equity and the global bond have mean positive holdings of 20% and 11% of the portfolio respectively. Japanese equity is frequently held short and on average has a position of -11%. The portfolio shares are summarised in Table 5.

	Mean	Minimum	Maximum
UK Equity	52	-11	305
US Equity	28	-21	216
Japanese Equity	-11	-45	5
European Equity	20	-47	142
Global Bond	11	-353	62

Table 5: Key features of the Unrestricted global asset allocation

Even if there were no constraints on going short, this strategy is clearly not implementable in practice due to the large transaction costs incurred as a result of the massive portfolio re-balancing entailed. Introducing a constraint on short sales greatly reduces the need for re-balancing. Figure 14 shows the composition of the constrained optimal global portfolio over the entire sample. UK equity dominates again with a mean holding of 52%, but this still fluctuates widely with a range of between 0 and 90%. With the investor no longer able to generate extra funds through the short sale of Japanese equity, European equity replaces US equity as the second most important asset, with 21% of the portfolio as against 20% previously. (Of course, US equity still has a larger share than the equity of any other single country.) Most of the re-balancing of the portfolio seems to involve switching between European and US equity. Japanese equity is almost always omitted from the optimal portfolio and has a mean position of zero. Even when it makes a fleeting appearance, it never accounts for more than 4% of the portfolio. The share of the global bond fluctuates least in the portfolio, staying at around 7% of investable funds and rarely altering; throughout the 16 year sample, the range is between 5% and 8%. Table 6 summarises the asset holdings.

	Mean	Minimum	Maximum
UK Equity	51.8	0	90
US Equity	20	0	56
Japanese Equity	0	0	4
European Equity	21.2	0	72
Global Bond	7	5	8

Table 6: Key features of the Restricted global asset allocation

We now proceed to decompose the European equity and the global bond into their constituent parts (see Figures 3 and 6). Since the French equity dominated the European portfolio, it becomes a very important constituent of the global portfolio and contributes 14% on average. German equity also plays an important role in the global portfolio, with an average holding of almost 7%. The Italian equity, on the other hand, does not hold much appeal to the UK investor and has a mean holding of only 0.5%.

Previously, it was found that the global bond portfolio was mainly composed of UK and Japanese bonds. In the global portfolio, they account for almost 6% and 1% of the portfolio respectively. Reflecting its stability in the global bond portfolio, the UK bond is the only asset that is always held in the global portfolio. Neither US nor German bonds contribute significantly to the portfolio as they have mean holdings of 0.5% and 0%, respectively. In Table 7, we put all of the parts of this study together and show the mean and range of asset shares in the optimal global portfolio of assets of a UK investor.

These results reveal that the degree of home-country bias is greater than estimated in Flavin and Wickens (1998b). When we allowed the UK investor hold only one foreign asset, US equity, we found that 75% of the portfolio should on average be held in domestic assets. Survey results show that the actual holding was approximately 82% (French and Poterba (1991)), which suggested that the degree of home bias in the UK was not acute and certainly not as large as in the US. Expanding the number of foreign assets which the UK investor may potentially hold, results in a maximum of only 57% in domestic assets, a sizeable increase in the estimate of home bias. On average, the optimal global portfolio is comprised of 8 assets, six of which

	Mean	Minimum	Maximum
UK Equity	51.7	0	90.3
US Equity	20.0	0	56.5
Japanese Equity	0.0	0	4.1
French Equity	14.0	0	47.3
German Equity	6.7	0	25.3
Italian Equity	0.5	0	14.6
UK Bonds	5.6	4.1	7.1
US Bonds	0.5	0	1.0
German Bonds	0	0	0.3
Japanese Bonds	1.0	0	2.6

Table 7: Key features of final asset allocation

are foreign. This implies that increased international diversification by UK investors would yield portfolios with higher performance, having a higher return for each level of risk.

4 Conclusion

Using an optimal dynamic asset allocation strategy that takes account of time-varying volatility to construct an optimal global portfolio faces a major hurdle: the curse of dimensionality. Ideally, a global portfolio should be based on an extensive set of assets, domestic and international, equity and bonds. The problem is that the wider the set of assets, the more difficult it is to estimate the time-varying covariance structure. In this paper, we propose the use of a two-stage procedure in order to limit the size of the estimation problem at each stage. In the first stage two groups of assets are formed into optimal portfolios and are then treated in the second stage as two individual assets. In the second stage, these two assets are then combined with others to form the global portfolio. The time-varying weights in the global portfolio of the original assets can then be recovered.

We examine the optimal global portfolio of a representative UK investor and therefore express all returns in sterling terms and use a UK risk-free rate. The investment set for the global portfolio comprises 10 risky assets: UK, US, Japanese, German, French and Italian equity and UK, US, Japanese and German government bonds. Five assets are included in stage two: UK, US, Japanese equity and two portfolios of assets. One is a portfolio of European equity (German, French and Italian); the other is a portfolio of the four government bonds. At each stage the optimal portfolio is constrained to prohibit short sales. this has the effect of reducing the size of the re-balancing required each period and so reduces transactions costs, and it is consistent with the legal requirements imposed on the bulk of UK investment which is undertaken by pension funds.

The time-varying covariances of the excess returns is estimated using a multivariate GARCH model at each stage. The analysis uses the parsimonious parameterization of the M-GARCH(1,1) model presented in Flavin and Wickens (1998a,b).

We find that the most important asset in the constrained optimal global portfolio is UK equity; on average its share is 52% of the portfolio. UK bonds comprise nearly 6% of the portfolio on average. Taken together, they imply that domestic assets comprise only 57% of the portfolio. This contrasts sharply with the survey results of French and Poterba (1991), who found that UK investors hold 18% of their portfolios in international assets and reveals that the degree of home bias inherent in the UK market is very substantial, and larger than found in Flavin and Wickens (1998b) who restricted foreign asset holding to just US equity. On average, the UK investor should hold a portfolio of six foreign assets and two domestic. US, French and German equity together should make up 40% of the portfolio. Our results imply that increased levels of international diversification would benefit the UK investor.

Using the expanded set of investment opportunities also produces better overall portfolio performance. Our optimal portfolios have much larger SPI values than those where international diversification opportunities were limited to just US equity. On average, the SPI values were 67% higher (0.30 compared to 0.18). The gains in SPI values are even greater (87.5%) when compared to a purely domestic UK portfolio. The advantages of increasing the investment set can also be seen by comparing the mean portfolio frontiers generated by our two studies of international diversification. Introducing more assets moves the frontier significantly towards the origin and offers the investor the opportunity to enjoy higher levels of return for each risk level.

Our results show that in the long run, volatility is transmitted between individual country stock markets. But bond and stock markets appear to be segmented as there is no evidence of volatility spillover effects between the two types of market. Further, the Japanese and European stock markets appear to be unaffected by each others volatility. There is strong evidence of spillovers between all other pairs of equity markets. Similarly, we find evidence of short-run contagion between all stockmarkets, except for the Japanese-European markets.

In summary, it is beneficial to hold an internationally diversified portfolio as theory predicts even though domestic assets may dominate the portfolio. Our results suggest that the problem of home bias may be even larger than currently thought. Finally, although using a constrained optimization reduces the extent of re-balancing required each period, it is still considerable. If transactions costs were taken into account explicitly - which they aren't in this paper - then this would be expected to further reduce fluctuations in the shares.

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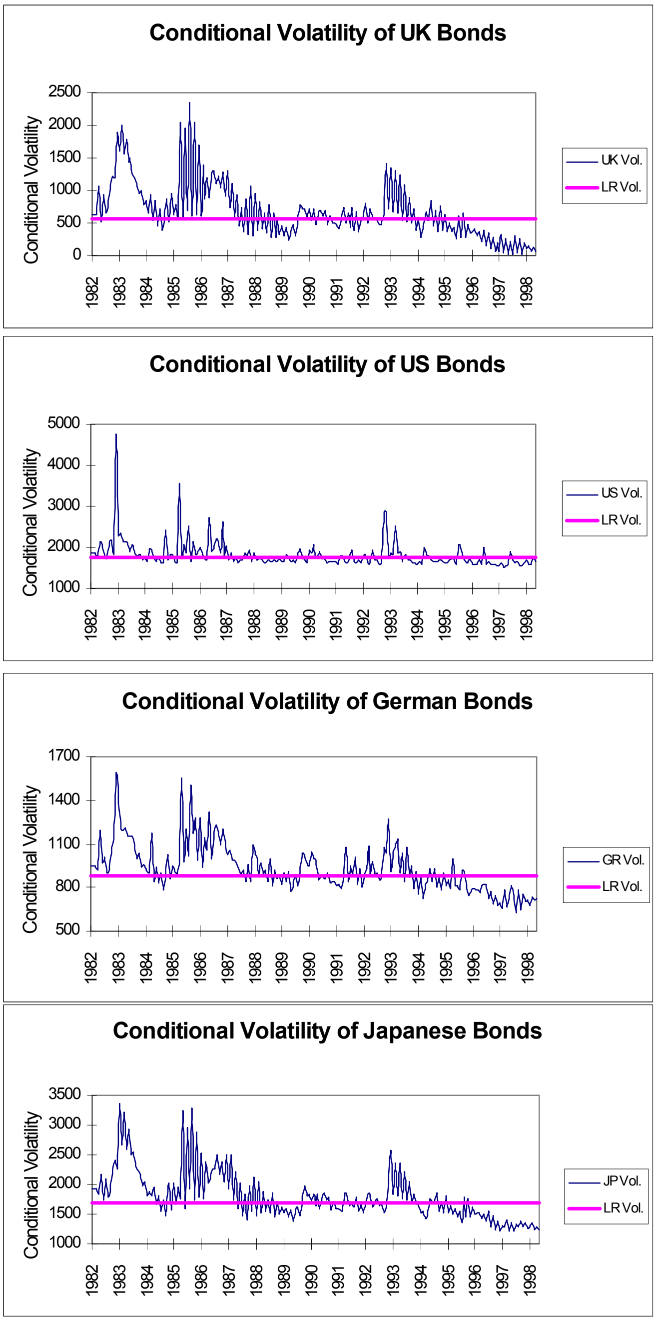
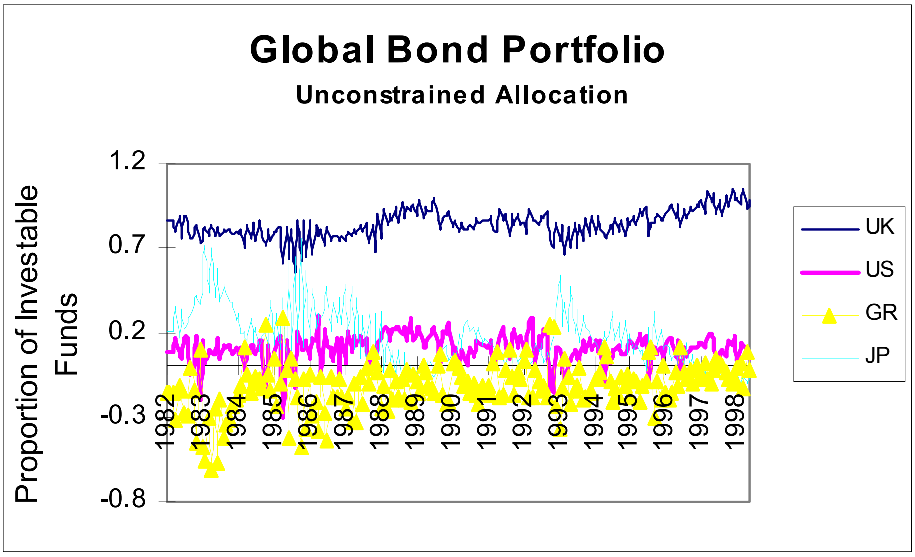

Conditional Volatility of UK Bonds
Conditional Volatility
2500
2000
1500
1000
500
0
1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998
UK Vol.
LR Vol.
Conditional Volatility of US Bonds
Conditional Volatility
5000
4000
3000
2000
1000
1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998
US Vol.
LR Vol.
Conditional Volatility of German Bonds
Conditional Volatility
1700
1400
1100
800
500
1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998
GR Vol.
LR Vol.
Conditional Volatility of Japanese Bonds
Conditional Volatility
3500
3000
2500
2000
1500
1000
1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998
JP Vol.
LR Vol.

Figure 1:



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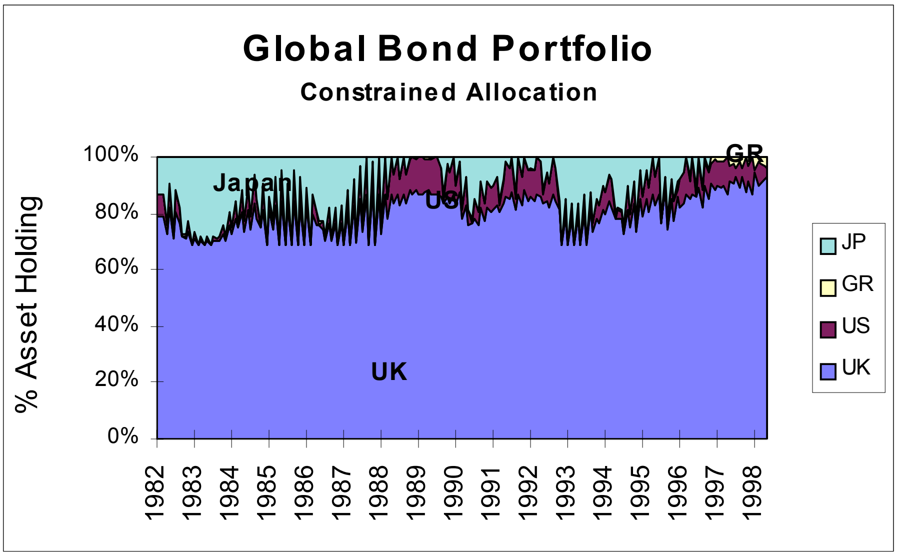

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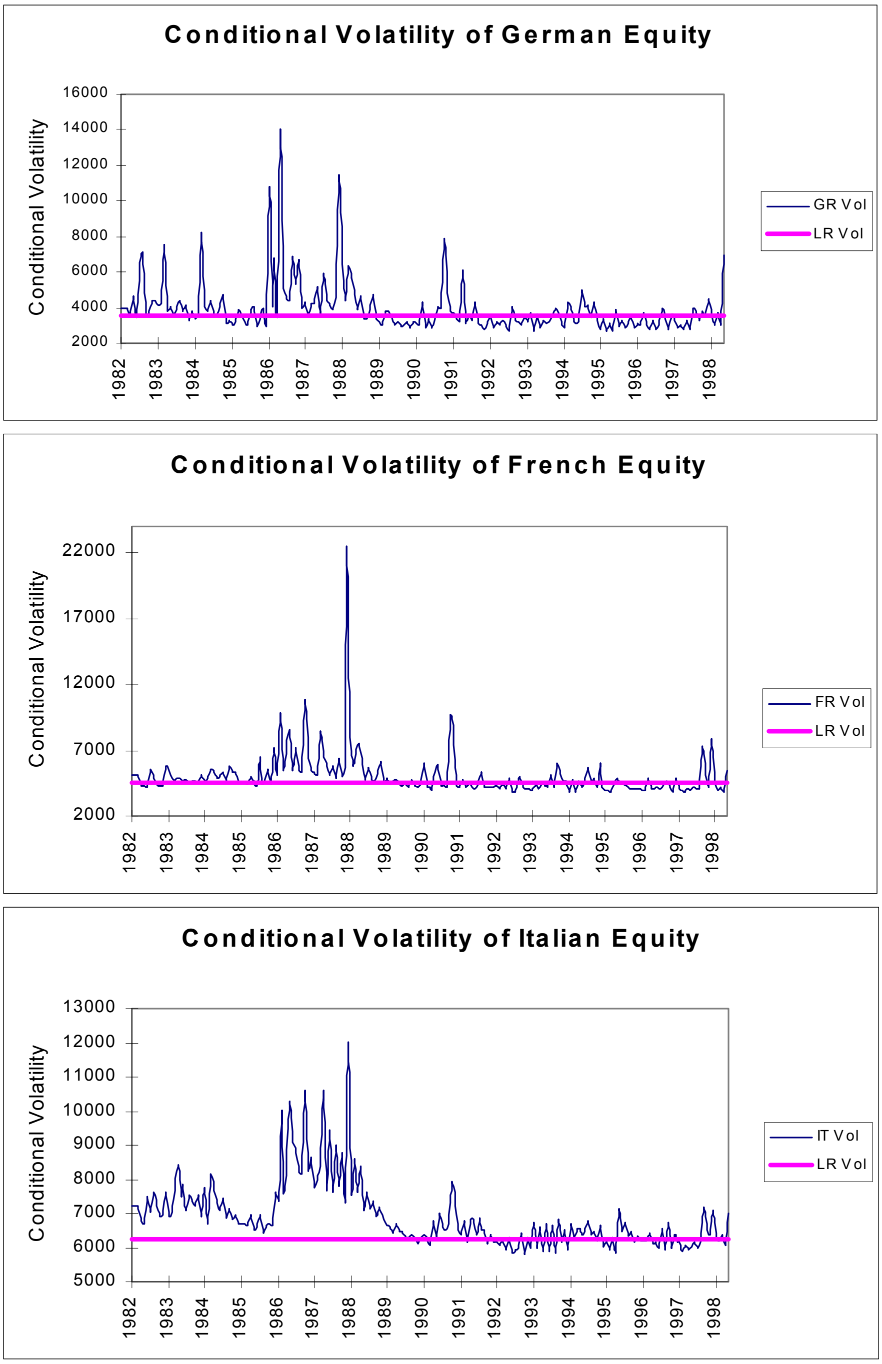
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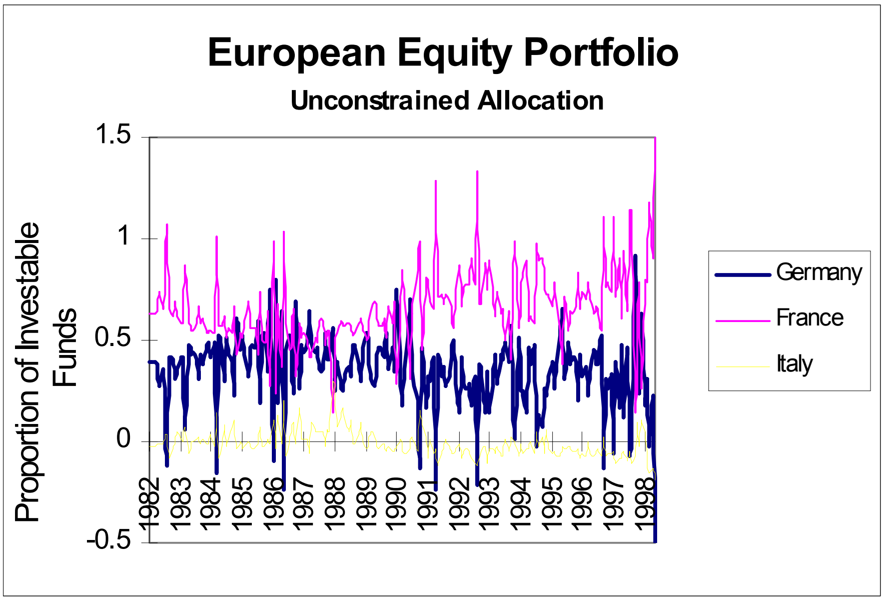
Conditional Volatility
16000
14000
12000
10000
8000
6000
4000
2000
1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998
GR Vol
LR Vol
Conditional Volatility of French Equity
Conditional Volatility
22000
17000
12000
7000
2000
1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998
FR Vol
LR Vol
Conditional Volatility of Italian Equity
Conditional Volatility
13000
12000
11000
10000
9000
8000
7000
6000
5000
1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998
IT Vol
LR Vol

Figure 4:



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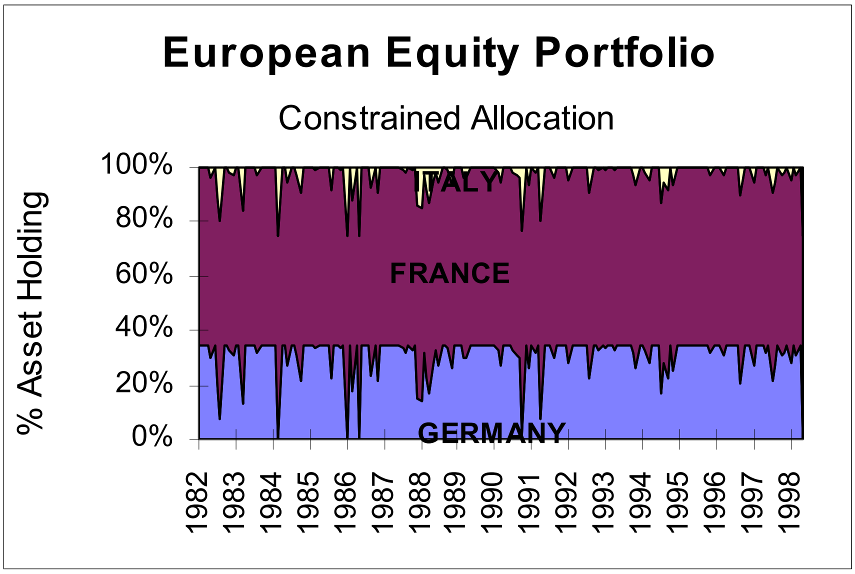

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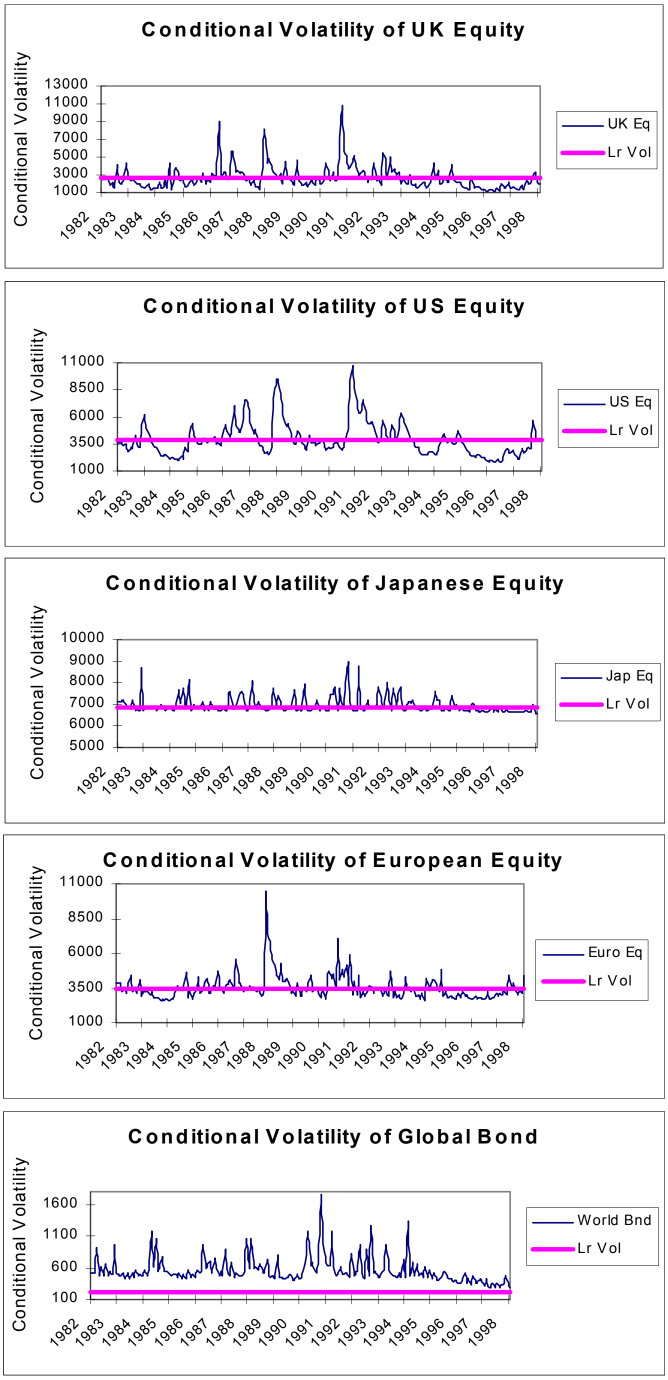
Conditional Volatility of UK Equity
Conditional Volatility
13000
11000
9000
7000
5000
3000
1000
1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998
UK Eq
Lr Vol
Conditional Volatility of US Equity
Conditional Volatility
11000
8500
6000
3500
1000
1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998
US Eq
Lr Vol
Conditional Volatility of Japanese Equity
Conditional Volatility
10000
9000
8000
7000
6000
5000
1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998
Jap Eq
Lr Vol
Conditional Volatility of European Equity
Conditional Volatility
11000
8500
6000
3500
1000
1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998
Euro Eq
Lr Vol
Conditional Volatility of Global Bond
Conditional Volatility
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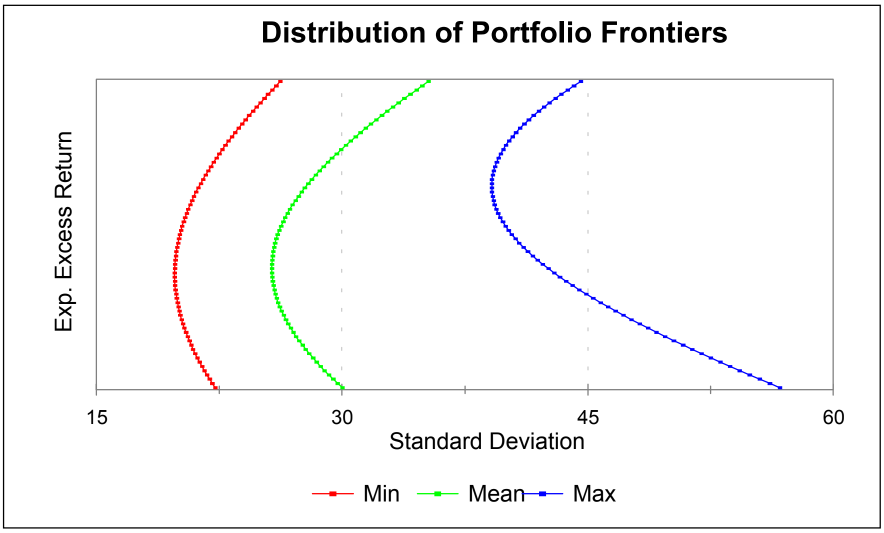
1100
600
100
1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998
World Bnd
Lr Vol

Figure 7:



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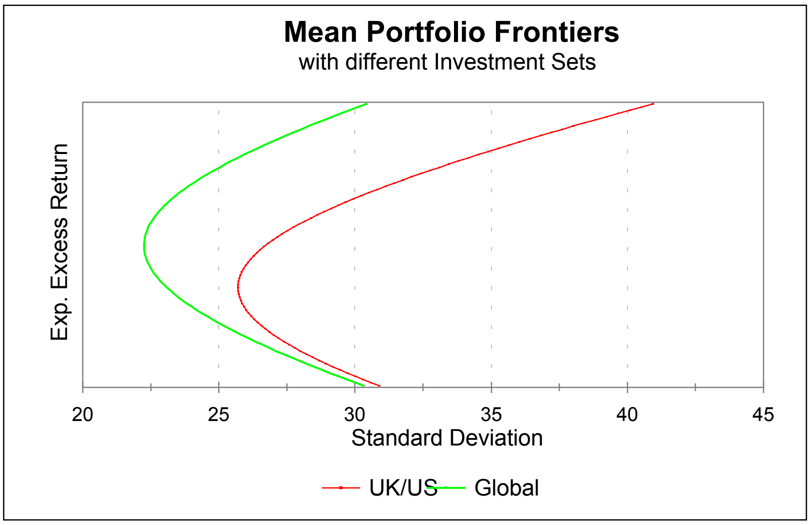

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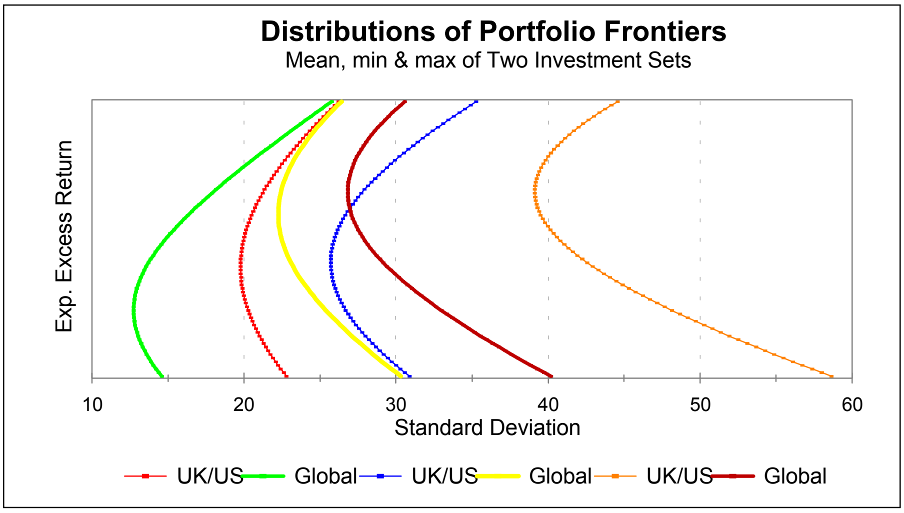
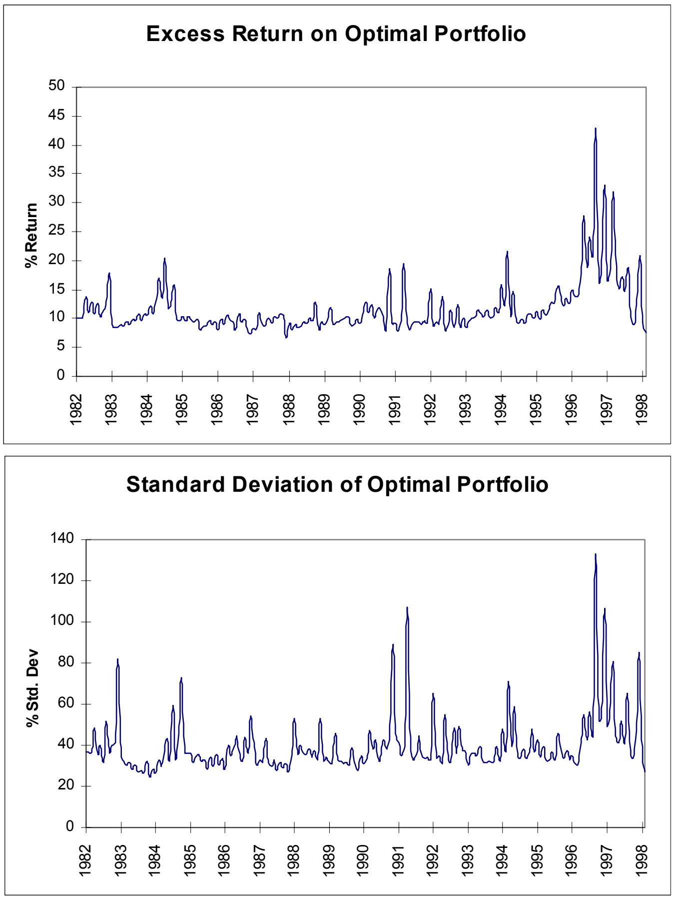

Distributions of Portfolio Frontiers
Mean, min & max of Two Investment Sets
Exp. Excess Return
10
20
30
40
50
60
Standard Deviation
UK/US
Global
UK/US
Global
UK/US
Global

Figure 10:


Excess Return on Optimal Portfolio
% Return
50
45
40
35
30
25
20
15
10
5
0
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
Standard Deviation of Optimal Portfolio
% Std. Dev
140
120
100
80
60
40
20
0
1982
1983
1984
1985
1986
1987
1988
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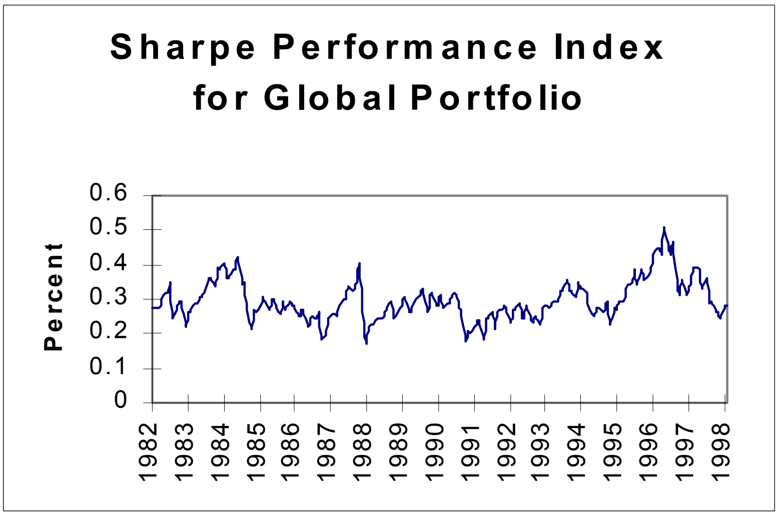
1990
1991
1992
1993
1994
1995
1996
1997
1998

Figure 11:



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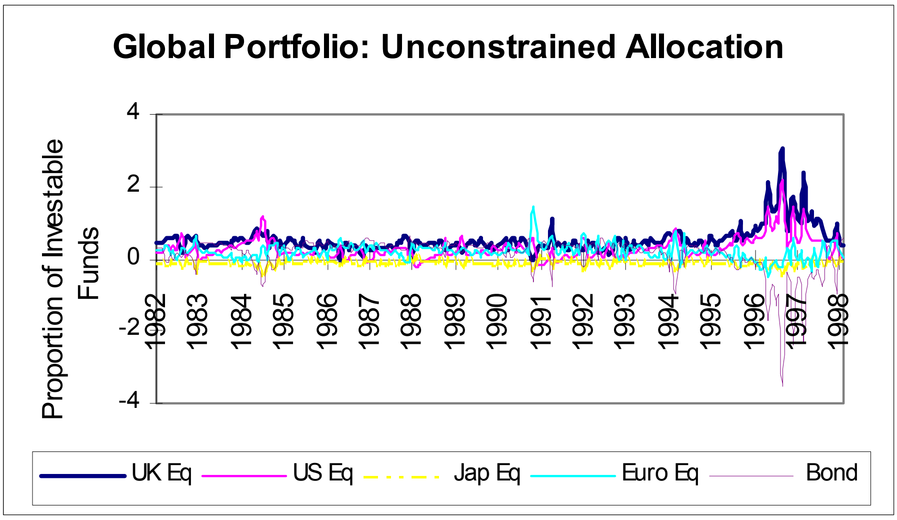

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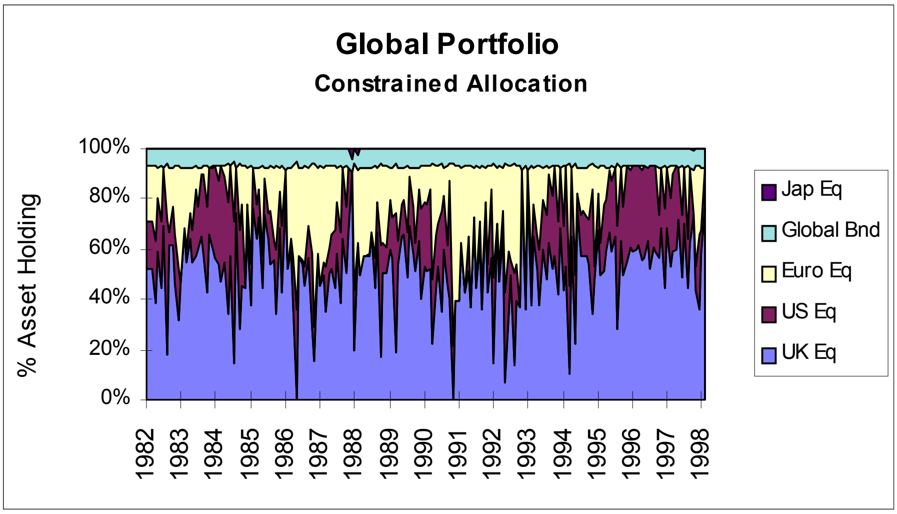
Global Portfolio
Constrained Allocation
% Asset Holding
100%
80%
60%
40%
20%
0%
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
Jap Eq
Global Bnd
Euro Eq
US Eq
UK Eq

Figure 14: