Limited Reserves and the Optimal Width of an Exchange Rate Target zone

by

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ABSTRACT

This paper analyses the stabilising properties of an exchange rate target zone when the stock of available reserves is limited. In these circumstances it is reasonable to suppose that the optimal bandwidth is affected by the expected lifetime of the zone. Our analysis uses Sutherland's (1995) target zone model to assess the importance of the expected lifetime in determining the optimal width of the zone. We find that the expected lifetime tends to widen the optimal bandwidth considerably but unless the stock of initial reserves is small and/or the fundamentals drift large, the extra lifetime bought is small in percentage terms.

Keywords: Stabilisation policy, exchange rate crises, optimal target zones

JEL Classification: F31, F33

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1. Introduction

A number of authors, Klein (1990), Miller & Weller (1991), Sutherland (1995) and Beetsma and Van der Ploeg (1998), have analysed the stabilising properties of target zones using versions of the original Krugman (1991) target zone model that incorporate more complex underlying models of exchange rate determination. In each case the target zone is treated as an infinitely lived structure so that the expected lifetime plays no part in the choice of bandwidth. In practice systems of managed exchange rates last for finite periods of time before being realigned or abandoned. This is especially true for countries with limited access to foreign borrowing. Therefore, if some form of managed regime is superior to a pure float, the optimal regime will be determined not just by how much stabilisation a particular regime provides, but also by how long the regime is expected to last. This argument is especially relevant to the target zone system that has the unique feature that foreign exchange interventions are required only at the edges of the zone.

Some support for omitting the lifetime from the determination of the optimal band is to be found in Dumas & Svensson (1994), who argue that for realistic parameters the expected lifetime of the Krugman model is long enough that it can be ignored when choosing the bandwidth. However, these results are based on the original Krugman target zone model in which the underlying structural form is a narrowly defined flexible price version of the monetary model. This paper examines the optimal width of a target zone when the stock of reserves is limited, the policymaker cares about stabilisation over the longer run, and when there is a richer model of exchange rate determination. The optimal band is chosen to create a zone of stability of targeted macroeconomic variables reflecting the trade-offs between the variability caused by different shocks, and the expected lifetime of the zone.

The remainder of the paper is organised as follows. Section 2 presents the exchange rate model that is used in the paper. Section 3 presents and discusses the objective function from which the optimal bandwidth is derived. This section also explains how the expected lifetime can be calculated using a fundamentals based approach to currency collapse. Section 4 contains the results.

2. A Target Zone Model

The Krugman (1991) target zone model is based upon a reduced form of the flexible price monetary model of exchange rate determination with random disturbances to money demand. The drawback of this structure is that because the real and monetary sectors are independent it is difficult to rationalise the existence of target zones or any other form of managed exchange rate regime. To overcome this Sutherland (1995) shows how the same basic equation used in Krugman (1987) can be derived from a underlying model in which the nominal and real sectors are not independent, and in which a variety of shocks cause variations in both real and nominal variables. The equations below represent a modified version of this model.

$$y_t^s = \alpha\left[p_t - E_{t-1}\left(p_t \middle| I_{t-1}\right)\right] + \varepsilon_t \qquad (1)$$

$$y_t^d = \eta\left(s_t - p_t + p_t^*\right) + \omega_t \qquad (2)$$

$$m_t - p_t = \phi y_t - \lambda i_t - \upsilon_t \qquad (3)$$

$$E\frac{(ds_t)}{dt} = i_t - i_t^* \qquad (4)$$

$$d\upsilon_t = u_\upsilon dt + \sigma_\upsilon dz_\upsilon \quad (5)$$

$$d\varepsilon_t = u_\varepsilon dt + \sigma_\varepsilon dz_\varepsilon \quad (6)$$

$$d\omega_t = u_\omega dt + \sigma_\omega dz_\omega \quad (7)$$

Where,

$y =$ domestic output

$p =$ price level

$m =$ nominal money supply

$i =$ nominal interest rate

$s =$ nominal exchange rate

$\varepsilon, \omega, \upsilon =$ goods supply, goods demand and velocity disturbances respectively.

All variables except i and i^* are in logs, and an asterix denotes a foreign variable. The foreign variables are assumed exogenous and omitted from the remainder of the paper.

Equation (1) is a surprise supply function where ε represents the disturbance term. Equation (2) defines aggregate demand to be a function of the real exchange rate and the demand disturbance ω. Equation (3) is the money market equilibrium condition, where real money demand is determined by output, the nominal interest rate and a monetary disturbance υ. Equation (4) is the Uncovered Interest Parity condition. The disturbance terms ε, ω and υ each follow independent Brownian motion processes with constant drifts.

In addition to the variety of disturbances the model differs from the flexible price monetary structure of the Krugman model by allowing the real exchange rate to vary if $\eta < \infty$, and for output to respond to surprise movements in the aggregate price level if

$\alpha > 0$. If $\alpha = 0$, $\eta = \infty$ and $\varepsilon = 0$ the real and monetary sectors are independent, disturbances to goods demand have no effects on prices or output and supply shocks are absent. The system of equations then simplifies to the reduced form used in the Krugman target zone model.

In the original Sutherland model the aggregate supply curve relates output to the price level rather than to surprise movements in the price level. This would be incompatible with our objective of examining optimal stabilisation policy under a finite lifetime and long run horizon. For the lifetime to matter it cannot be too long. As Dumas & Svensson (1994) show, a short lifetime very likely requires that the fundamentals driving the exchange rate exhibit substantial drifts. If the original supply function were retained in the presence of a drift, predictable movements in fundamentals including nominal variables affect output. To avoid this we use a surprise supply function. For simplicity we use continuous time implying that any deviations from equilibrium last only for an infinitesimal period of time. The impact effects of the shocks are the same as in the original Sutherland model and these determine the optimal instantaneous stabilisation policy. Our modification of the supply function ensures that predictable changes in the nominal variables do not affect output. Specifically solving (1) and (2) for the price level and output yields,

$$p = \frac{\eta s + \alpha E(s)}{\alpha + \eta} - \frac{\eta \varepsilon + \alpha E(\varepsilon)}{\eta(\alpha + \eta)} + \frac{\eta \omega + \alpha E(\omega)}{\eta(\alpha + \eta)} \qquad (8)$$

$$y = \frac{\alpha}{\alpha + \eta}\left[\eta(s - E(s)) + (\omega - E(\omega)) - (\varepsilon - E(\varepsilon))\right] + \varepsilon \qquad (9)$$

For simplicity the time subscripts are suppressed. Equations (8) and (9) show the effects of the various disturbances on prices and output. Intuitively both predictable and unpredictable movements in demand affect the aggregate price level whilst only surprise movements in demand affect output. The solution for the exchange can be found by rearranging (3) in terms of the interest rate, using (8), (9) and the UIP condition to obtain,

$$\frac{E(ds)}{dt} = \psi_1(m+v) + \psi_2\varepsilon + \psi_3\omega + \psi_4 s + \psi_5 E(\varepsilon) + \psi_6 E(\omega) + \psi_7 E(s) \tag{10}$$

Where,

$$\psi_1 = -\frac{1}{\lambda} \qquad \psi_2 = \frac{(\phi\eta - 1)}{\lambda(\alpha+\eta)} \qquad \psi_3 = \frac{(1+\alpha\phi)}{\lambda(\alpha+\eta)} \qquad \psi_4 = \frac{(1+\alpha\phi)\eta}{\lambda(\alpha+\eta)}$$

$$\psi_5 = \frac{\alpha(\phi\eta - 1)}{\lambda n(\alpha+\eta)} \qquad \psi_6 = \frac{\alpha(1-\phi\eta)}{\lambda n(\alpha+\eta)} \qquad \psi_7 = \frac{\alpha(1-\phi\eta)}{\lambda(\alpha+\eta)}$$

Next rewrite (10) in terms of the exchange rate and use that expression to eliminate the expected value of the exchange rate. Substituting back, noting that $\psi_4 + \psi_7 = 1/\lambda$ and letting f_t represent a composite fundamental yields,

$$s_t = f_t + \lambda E\frac{[ds]}{dt} \tag{11}$$

$$f_t = -\frac{1}{\psi_4}[\psi_1(m+v) + \psi_2\varepsilon + \psi_3\omega] + \frac{\lambda}{\psi_4}[\psi_7\psi_1 E(m+v) - (\psi_5\psi_4 - \psi_7\psi_2)E(\varepsilon) - (\psi_6\psi_4 - \psi_7\psi_3)E(\omega)] \tag{12}$$

By differentiating (12), simplifying, and using that for each of the shocks $dE(x) = \mu_x dt$, the dynamics of the unregulated part of f_t can be expressed as the following composite Brownian motion process,

$$df = \mu_f dt + \sigma_f dz \qquad (13)$$

The drift is equal to $\mu_f = \mu_v + ((1-\phi\eta)/\eta)\mu_\varepsilon - (1/\eta)\mu_\omega$ and the variance $\sigma_f^2 = (1/\psi_4)^2(\psi_1^2\sigma_v^2 + \psi_2^2\sigma_\varepsilon^2 + \psi_3^2\sigma_\omega^2)$. The solution under a target zone can now be obtained in the standard way. This is,

$$s = \lambda\mu_f + f_t + A_1 e^{\rho_1 f_t} + A_2 e^{\rho_2 f_t} \qquad (14)$$

Where $\rho_1, \rho_2 = \left(-\mu_f \pm \sqrt{\mu_f^2 + 2\sigma_f^2/\lambda}\right)\Big/\sigma_f^2$. The arbitrary constants are defined by the smooth pasting conditions. The solution is a characteristic S-curve such as depicted in figure 1.

Figure 1 here

3. The Optimal Band and the Lifetime of the Target Zone

The optimal band is chosen to minimise the variances of the potential target variables, prices, output and the nominal or real exchange rate over an infinite but discounted horizon. The long run horizon means the optimal band is influenced by the lifetime. Following Dumas & Svensson (1994) the lifetime is modelled using a 'fundamentals' based approach to currency crises. It is assumed that access to foreign borrowing is limited and implicit constraints, such as a government budget deficit, prevent the exclusive focus of monetary policy on the defence of the zone. This means the expected lifetime is finite, and since narrowing the zone increases the frequency of interventions, the lifetime is shorter the narrower the zone. Although there are a variety of possibilities we assume the post abandonment regime is a pure float that is expected to be permanent[1]. In deciding whether or not to narrow the zone any reduction in the variances of the target variables over the expected lifetime, are measured against the expected discounted increase in the variances caused by the earlier transition to the floating regime. The problem of choosing the optimal band is equivalent to choosing the optimal expected abandonment time. The objective function is,

$$\underset{\bar{s}}{Min}\, V = E_t \left\{ \pi_s \left(\int_0^T \sigma_s^2 e^{-\delta\tau} d\tau + \int_T^\infty \sigma_{sf}^2 e^{-\delta\tau} d\tau \right) + \pi_p \left(\int_0^T \sigma_p^2 e^{-\delta\tau} d\tau + \int_T^\infty \sigma_{pf}^2 e^{-\delta\tau} d\tau \right) \right. \quad (15)$$

$$\left. + \pi_y \left(\int_0^T \sigma_y^2 e^{-\delta\tau} d\tau + \int_T^\infty \sigma_{yf}^2 e^{-\delta\tau} d\tau \right) \right\}$$

Where π_s, π_p, π_y are the preferences of the policy maker towards exchange rate, price and output stability. The variances of the exchange rate, price level and output under the target zone regime are represented by $\sigma_s^2, \sigma_p^2, \sigma_y^2$, whilst $\sigma_{sf}^2, \sigma_{pf}^2, \sigma_{yf}^2$ are the corresponding variances under a free float regime. The rate of time preference is δ and T represents the date of abandonment of the target zone.

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[1] A more stable post abandonment regime reduces the gains from having a target zone and so reduces the importance of the lifetime. However, as in Krugman & Miller (1993), one could equally argue that the post abandonment regime exhibits excess volatility due to speculative behaviour not present in the target zone regime.

The non-linearity of the S-curve complicates the calculation of the variances in the target zone regime. For a fixed bandwidth the distribution of the fundamental and the exchange rate are determined by the relative sizes of the drift and variance of the composite fundamental, and by the length of time for which the zone is expected to last[2]. Instead we use the following simple approximation to the expected average relationship between the exchange rate and fundamental using the slope of the secant line between the limits on the exchange rate and the limits on the fundamental. Since $b-a$ increases as μ_f increases this approximation has the correct feature that a larger drift reduces the expected variance of the exchange rate for a given lifetime. If the drift is small and the lifetime short the approximation understates the average within band variance of the exchange rate. The estimated gap between within the free float and within band variability is overestimated and our results overstate the importance of the zone and the lifetime. Conversely if μ_f is not small and/or the lifetime not short our results understate the importance of the lifetime. For the short lifetimes in which we are interested this the essential features of the density function.

The lifetime is calculated by assuming domestic credit is fixed by an unspecified internal objective and there exists a known critical lower level of the money supply and reserves at which a speculative attack is triggered. In logs the critical lower level of reserves is $r = p$. If the zone is defended using only marginal interventions the limits on the exchange rate coincide with the limits on the fundamental. It follows

[2] If the lifetime is infinite, the asymptotic density is independent of time and relatively easy to calculate. It is not appropriate here because we are interested in cases where the lifetime is far from infinite. Although it is possible to numerically calculate the time contingent density function this is beyond the scope of the paper.

that since reserves only fall at the upper limit of the exchange rate $\bar{s}$, or equivalently at the upper limit of the fundamental $g = b$, the target zone collapses on the first occasion that $g = b$ and $r = p$. Similarly letting $r = q$ denote a critically high stock of reserves means the date of abandonment is the first occasion that reserves equal $r = q$ and the fundamental reaches $g = a$. The regulated dynamics of the fundamental are,

$$df = \mu_f dt + \sigma_f dz + dE(m) - (\psi_1/\psi_4)(dm_t - dEm_t) \tag{16}$$

The last term in (16) represents surprise movements in the money supply and these do not influence the expected lifetime, which depends only on the initial reserve stock and expected changes in money supply. The latter are determined by the parameters of the composite disturbance terms and the S - curve. This means the expected lifetime is equivalent to the first passage time of the Brownian motion $f - m$ to the limits associated with the abandonment of the target zone. Specifically the target zone survives as long as the unregulated fundamental lies within the interval, $a - q \leq f - m \leq b - p$, but the zone is abandoned on the first occasion that the unregulated fundamental reaches the lower (upper) limit. The expected lifetime can be computed following Karatzas and Shreve (1988, p.99) as,

$$E(T|f,m) \approx \left\{ -(f-m) + (a-q) + [(b-p)-(a-q)] \times \frac{1-\exp[-\theta\{(f-m)-(a-q)\}]}{1-\exp[-\theta\{(b-p)-(a-q)\}]} \right\} \Big/ \mu_f \tag{17}$$

Where $\theta = 2\mu_f / \sigma_f^2$. Given the initial reserve stock the key determinant of the expected lifetime is the size of drift of the composite fundamental. This is determined by the assumed size of the underlying drifts and the structure of the model. The drift is much more important than the variance of the composite fundamental because Brownian motion shocks force the exchange rate in either direction, so that the probability of losing reserves is largely offset by that of gaining reserves. A drift pushes the exchange rate in a particular direction and has a much more significant effect on the lifetime.

4. Results

Our main interest is in the channel from the lifetime to bandwidth. However, because the importance of the lifetime depends upon how long the lifetime is, and on the bandwidth that would be chosen if the lifetime were infinite, we separate the results into three subsections. Subsection 4a is an overview of the instantaneous stabilising properties of the band. Subsection 4b discusses the main determinants of the lifetime. Subsection 4c examines the relationship between the expected lifetime and the optimal band.

4a. The optimal band with an infinite lifetime

With an infinite lifetime the optimal band is chosen to minimise some combination of the variances of the price level and output. In figure 2 we plot the variance of output and the contribution of each individual shock against the width of the target zone without considering the lifetime. The parameter values are, $\lambda = 3$, $\eta = .5$, $\alpha = .5$, $\phi = 1$, $\mu_v = \mu_\varepsilon = \mu_\omega = 0$, $\sigma_v = .05$, $\sigma_\varepsilon = .05$, $\sigma_\omega = .05$, giving $\sigma_f^2 \approx 0.0002$.

Figure 2 here

The solid line represents the total variance of output whilst the dashed lines show the contribution of the individual shocks to this total. Widening the band increases the amount of output variability caused by the velocity shock but reduces the amount caused by the demand shock. The supply shock has opposing effects on prices and output, causing its effect on money demand and the nominal exchange rate to be ambiguous. In this example the exchange rate depreciates and widening the band increases the amount of output variability attributable to the supply shock. As in Sutherland (1995), in comparison with a fixed or floating regime a target zone reduces the overall variance of output when a variety of shocks are present. The bandwidth that minimises the variance of output is approximately $\pm 5 \log \%$.

With respect to velocity or goods demand shocks the variance of the price level is a multiple of the variance of output. Thus whatever the target, the optimal band is narrower the larger is the ratio of velocity to good demand shocks. It also means that in the absence of supply shocks, the optimal band is not affected by the relative weights given to output or price stabilisation[3]. If the exchange rate depreciates (appreciates) in response to supply shocks, the optimal band is narrower (wider) for an output target than for a price target whenever supply shocks are present. The exchange rate depreciates if $\phi\eta < 1$ and appreciates if $\phi\eta > 1$. Table 1 summarises the changes in the bandwidth that are required to minimise the variability of the price level or output in response to the individual shocks.

[3] The price variability curve is higher than the output variability curve if $\alpha < 1$ and vice versa. If equal weight is given to price as to output stability and supply shocks are included, the higher the curve the greater its influence on the optimal band.

Table 1: *The Optimal Bandwidth and the Individual Shocks*

Shock type	*Change in band to stabilise the price level*	*Change in band to stabilise output*
Money demand	Narrow	Narrow
Goods demand	Widen	Widen
Supply & $\phi\eta < 1$	Widen	Narrow
Supply & $\phi\eta > 1$	Narrow	Widen

Other than the bandwidth, the main determinants of the amount of price and output variability created by the individual shocks are the slopes of the aggregate supply and goods demand curves. The slope of the goods demand curve is determined by the elasticity of aggregate demand to the real exchange rate, η. We refer to this loosely as the degree of openness. If the fraction of tradables in domestic output is high and if domestic goods have close substitutes, aggregate demand is sensitive to variations in the real exchange rate and the goods demand curve is relatively flat. In the limit goods are homogenous, purchasing power parity holds and the demand curve for goods is horizontal. In this case shocks to goods demand have no effect on the price level, nominal and real exchange rates or output. Conversely, if η is low goods demand shocks are an important source of price and/or output variability. The slope of the aggregate supply curve is determined by the elasticity of supply to unexpected increases in the price level, α. As α increases the supply curve flattens out, and demand shocks have a smaller (larger) effect on the price level (output), whilst supply shocks have a smaller effect on both output and the price level.

To illustrate theses effects figure 3 plots the optimal band for separate price level (shaded line) and output targets (unbroken line) against the slope of the supply curve assuming that $\eta = 1/3$. Figure 4 performs the same operation for $\eta = 3$.

Figure 3 here

With $\eta = 1/3$ the slope of the goods demand curve is -3. A combination of relatively steep goods demand and supply curves produces very different optimal bands for the price and output targets. There are two reasons for this. Firstly with $\eta = 1/3$ real shocks create a lot of systemic variability. However, the steep aggregate supply curve means that most of the variability emanating from the demand side is concentrated on the price level rather than output, and thus the optimal band for output stability depends mainly on the supply shocks. Since $\eta = 1/3$ the exchange rate depreciates in response to a supply shock and output variability is reduced by having a narrower zone, whilst price level variability is reduced by a wider zone. Consequently, the optimal band for output stability is narrow whilst the optimal band for price level stability is wide. For example if $\alpha = .2$, the optimal band for output stability is $\pm .024$ log % whilst the optimal band for the price level target is a free float regime. Increasing α means that supply shocks become less important whilst demand shocks create more output variability but less price variability. The optimal band for the output target first widens and then narrows. Thus initially the dominant effect of increasing α is the reduction in the contribution of supply shocks, but as this becomes small enough, the dominant effect is on velocity shocks and the optimal band starts to narrow. For α large enough the supply shocks become irrelevant, and the optimal band is identical for the price level and output targets. In both cases with $\eta = 1/3$ and given the assumed distribution of shocks, the optimal band is $\approx \pm 12.5 \log \%$.

Figure 4

With $\eta = 3$ the slope of the demand curve is $-1/3$ and goods demand shocks are almost irrelevant. As before, if the aggregate supply curve is steep supply shocks are the dominant source of output variability. Since the exchange rate now appreciates to supply shocks the optimal band for output stability is wide, for $\alpha = .2$ the optimal regime is a free float. In the case of the price level the optimal band is very narrow, both because stabilisation in response to a supply shock calls for a narrower zone, and because goods demand shocks are unimportant. Thus in terms of price stability the optimal band is a fixed exchange rate and this is true for all values of α. As the aggregate supply curve flattens out the supply shock becomes less important, and the optimal band for output narrows. In the limit real shocks do not affect output and the optimal regime is a fixed rate for both price and output stability. An overview of the relationship between the slopes of the supply and goods demand curves and the optimal band is given in table 2. This table refers to the values just discussed and assumes that the variances of the shocks are identical.

Table 2. *Goods supply and demand curves and the optimal bandwidth*

α, η	*Price*	*Output*	*Explanation*
High, high	Very narrow	Very narrow	Only money shocks matter
High, low	Wide	Wide	Goods demand shocks dominate
Low, high	Very narrow	Wide	σ_ε dominates, with $\phi\eta > 1$ output (price) is stabilised by a narrow (wide) zone
Low, low	Wide	Narrow	All shocks matter. Since $\phi\eta < 1$ σ_ε and σ_v create a narrow zone for output

Ignoring the lifetime the optimal band is determined by the distribution of the shocks, the slopes of the aggregate demand and supply curves and the chosen target in the manner outlined by tables 1 & 2.

4b. The lifetime

As previously outlined, given the assumed log excess of reserves the main determinant of the lifetime is the size of the composite drift. If (as in this paper) the drift of the composite fundamental is significantly larger than is the variance, then starting from the mid-point of the band on $f-m$ the probability of abandonment in the opposite direction to the drift is close to zero. The lifetime with no critically high level of reserves is then virtually identical to the lifetime with both high and low critical levels of reserves. With no critically high stock of reserves the mean lifetime is equal to $E(T|f,r)=[(b-a)+(q-p)]/2\mu_f$. The first term is the time bought by the bandwidth, whilst the second term represents the time bought by the excess reserve stock or alternatively, the lifetime of a fixed rate. Substituting for μ_f and separating the underlying sources of the drift yields,

$$E[T|g,r]=[(b-a)+(q-p)]/2\mu_v+\eta[(b-a)+(q-p)]/(1-\phi\eta)2\mu_\varepsilon +\eta[(b-a)+(q-p)]/2\mu_\omega \quad (18)$$

This formula reveals a number of useful results. Firstly, with a pure velocity drift (the first term in square brackets) none of the structural parameters of the model affect the lifetime of the fixed rate. This is because a disturbance originating in the money market is offset by the equal and opposite change in reserves. Under a fixed rate a drift of 10% per year will cause reserves to fall at the same rate. If the initial log excess of reserves is 100% above the critical minimum, the lifetime of the fixed rate is 10 years. Thus unless the band is very wide, the extra lifetime time bought by the bandwidth is likely to be quite small when compared to the lifetime of a fixed rate. It follows that for the lifetime to significantly alter the bandwidth the underlying drift must be reasonably large, and/or the initial log excess of reserves reasonably small.

Secondly the lifetime that corresponds to disturbances originating in the goods market does depend on the values of the structural parameters and consequently the source of the drift is important. In particular with real drifts the lifetime is strongly influenced by the openness parameter η. Openness matters because it determines the amount of exchange rate pressure caused by a real shock. For example under a floating regime, a negative goods demand disturbance causes an exchange rate depreciation that offsets the original disturbance. The size of the depreciation depends upon how sensitive aggregate demand is to the real exchange and this depends upon how open the economy is. Paralleling the discussion of demand shocks, the lower is the degree of openness the larger is the required depreciation. Under a target zone the more specialised is the domestic economy, the greater the frequency of interventions and the earlier the collapse time. The case of a supply drift is more complex but the same general conclusion applies. If the elasticity of aggregate demand is not high, the lifetime will be less in the case of a supply (real) drift than in the case of a velocity drift.

Thirdly (25) also shows that the distribution of the shocks and the slope of the aggregate supply curve do not affect the expected lifetime in an important way. As outlined previously pure Brownian motion shocks tend to cancel one another out and so have a very limited effect on the lifetime. Since the slope of the aggregate supply curve relates surprise movements in the price level to output and the drifts are predictable, the slope of aggregate supply curve does not affect the size of the composite drift and so has only a minimal effect on the lifetime.

4c. The lifetime and the optimal band

From 3a we know when the optimal band will be narrow ignoring the lifetime, and from 3b we know when the lifetime of a given bandwidth will be short. In this section we combine these results to assess the influence of the lifetime on the optimal band. Throughout, the optimal band is chosen to stabilise output alone. In addition to the parameter values used for figure 2 we set the rate of time discount equal to 5% per year $\delta = .05$, and the initial log excess of reserves equal to, $r = (p+q)/2 = (0+200)/2 = 100 \log \%$.

Our first example examines how the size of the fundamental drift affects the optimal band through the lifetime. Increasing the size of the drift alters the position and shape of the S-curve and this changes the instantaneous stabilising properties of the zone. Consequently the optimal band changes irrespectively of any effect via the lifetime[4]. To isolate the effect of the lifetime we also plot the optimal band under an infinite rate of time discount. In the latter case the optimal band is chosen without regard to the

[4] This is easily understood by considering the drift-less case in which the fundamental is uniformly distributed between symmetric upper and lower limits. The asymptotic variance of the exchange rate would be most heavily weighted towards the steepest part of the S-curve at the very centre of the target zone. Adding a positive velocity drift shifts the S-curve upward and the asymptotic variance becomes more heavily weighted toward a flatter part of the S-curve. Given that the average variance (of the exchange rate) is now lower the previous optimal band would be too narrow. Thus adding a drift widens the optimal band even if its effect on the lifetime were minimal.

lifetime so that the gap between the two bands represents the widening that is caused by the finite lifetime. Specifically figure 5 plots the optimal band under $\delta = .05$ and under $\delta \to \infty$ against the size of a velocity drift whilst figure 6 plots the corresponding lifetimes.

Figures 5 & 6

The solid line represents the optimal band under the 5% yearly discount rate whilst the dashed line is the optimal band under the infinite rate of time discount. We refer to the optimal band under the 5% rate of time discount as the long horizon band. If the drift of fundamentals is close or equal to zero, the expected lifetime of the target zone is sufficiently long that the long horizon band and infinitely discounted band are close to one another. For example with a velocity drift of 1% per year the infinitely discounted band is ± 10.6 log % and the expected lifetime is 82.7 years, whilst the long horizon band is ± 11.9 log % and the lifetime is 84.5 years. Increasing the drift size widens both bands and the gap between them also widens. A drift of 10% per year results in a long horizon band of ± 20.5 log %, whilst the infinitely discounted band is ± 13 log %. Percentage-wise including the lifetime as a decision variable widens the bandwidth by 12% when there is a 1% per year annual drift in velocity, and by 58% for a 10% annual drift in velocity. The pure effect of adding a drift is also quite strong. For example, a 10% annual velocity drift widens the infinitely discounted band by 24% from ± 10.5 log % to ± 13 log %.

Figure 6 shows that the lifetime falls as the size of the velocity shocks is increased, and that whilst the lifetime significantly alters the optimal bandwidth the reverse is not true. Even for an annual velocity shock equal to 10% per year the lifetime for the long horizon band at 13.9 years is only 7% more than the 13 years under the infinitely discounted band. This is not particularly significant in either percentage or absolute terms. It is also clear that the expected lifetime in this case is quite long even for quite large monetary drifts. This is not surprising, as outlined in 3b, if the drift were purely monetary then regardless of the slopes of aggregate demand or supply curves, a 100% log excess of reserves and 10% annual drift result in the somewhat long lifetime of 10 years for a fixed exchange rate. Consequently it is unlikely that variations in the bandwidth have a significant effect on the lifetime in percentage terms. Moreover, since the variations in the band are large only when the drift is large this limits the extra lifetime bought in absolute terms.

Although widening the bandwidth increases the lifetime by a seemingly small amount the lifetime significantly influences the optimal band for the following reason. Since the lifetime is not short and the drift is quite large, the average within band variance of the exchange rate is low even for quite wide bands. This means the expected reduction in the within band variance caused by widening the band is small, making it worthwhile to widen the band significantly even though the extra lifetime gained is also small. This is not a product of the approximation. A combination of a moderate drift and a long lifetime mean that on average the exchange rate is very close to the limit of the zone in the direction of the drift, even for a very wide target zone. An alternative way of saying this is to note that even for wide bands, the gap between the free float variance and average within band variance is large, and what matters is not so much the width of the zone but the survival time.

From sections 3a and 3b we know that if the drift is purely monetary, variations in the structural parameters have only a limited effect on the importance of the lifetime via the effect on the incentive to choose a narrow band. With real drifts the structural parameters directly affect the lifetime, and thus affect the importance of the lifetime more significantly. To illustrate this figures 7 & 8 plot the optimal band and lifetime against the degree of openness, assuming a goods demand drift of 10% per year.

Figures 7 & 8 here

The general patterns of these figures reflect how the degree of openness alters the importance of the individual shocks and the lifetime. With a low degree of openness disturbances to goods demand have very significant effects. This causes the long horizon optimal band to be significantly wider than the already wide infinitely discounted optimal band. For example, at $\eta = .2$ the long horizon band is $\pm 250 \log \%$ and the infinitely discounted band is $\pm 78 \log \%$, whilst the lifetime is 8.7 and 5 years respectively. As η increases the goods demand shock become less significant and both bands narrow. Since the goods demand drift also becomes less significant the lifetime increases and the long horizon band and infinitely discounted bands tend to one another. At the benchmark value of $\eta = .5$ the long horizon band is $\pm 45 \log \%$ which is 136% wider than the infinitely discounted band of $\pm 19 \log \%$. This increases the lifetime by 22% from 7.2 to 8.8 years. As η increases beyond $\phi\eta = 1$ the optimal response to a supply shock changes from narrowing to widening the zone. Given that increasing η also increases the importance of supply shocks relative to velocity shocks the long horizon band starts to widen. At $\eta = 2$ the long horizon band is $\pm 71 \log \%$ and the infinitely discounted band $\pm 64 \log \%$. The lifetime in this case is very long both because the infinitely discounted band is wide, and because exchange rate variations are highly effective at offsetting the drift in goods demand. If price stability were to be targeted the infinitely discounted optimal band would be wider for $\phi\eta < 1$ but narrower for $\phi\eta > 1$, reflecting the differential effects of supply shocks on prices and output. Consequently the incentive to choose a narrow zone would be reduced when the lifetime is low, but increased when the lifetime is high, as compared to the case of the output target.

The results depend crucially upon the assumed log excess of initial reserves. The effects of changing this log excess are plotted in figures 9 & 10. The parameter values are as for figure 5 but assume a 10% annual velocity drift. We also report the values for a 10% drift in goods demand.

Figures 9 & 10 here

With zero excess reserves the infinitely discounted band is $\pm 13 \log \%$ and the long horizon band is $\pm 83 \log \%$. The corresponding values for the lifetime are 2.9 and 10.5 years. With a 50% log excess of reserves the long horizon band at $\pm 31 \log \%$ is 138% wider than the infinitely discounted band and this increases the lifetime by 26%, from 8 to 10.1 years. If the underlying drift were to goods demand the lifetime is more important. For example a 50% log excess of reserves produces a long horizon band of $\pm 71 \log$ % that is 284% wider than the infinitely discounted band of $\pm 19 \log$ %, and this increases the lifetime by 66% from 4.7 years to 7.8 years. In the case of goods demand drift even with an excess of log reserves equal to $\pm 250 \log$ % the lifetime still widens the bandwidth by 36% from $\pm 19 \log$ % to ± 26 log % even though the lifetime itself increases by just 2.7% from 14.7 to 15.1 years.

It is noticeable that for a very small log excess of reserves, increasing the reserve stock causes the long horizon band to narrow so much that the lifetime actually falls. This is due to non-linearity of the S-curve. The optimal band balances the gain from tightening the band, the difference between within band and free float volatility, against the reduction in the lifetime. Narrowing the band flattens the S-curve and this both increases the gain from tightening the zone, and reduces the loss in lifetime that occurs from a further tightening. If this improvement in the trade off is large enough in terms of a lower variance of the target variable (output) it becomes worthwhile to sacrifice lifetime in exchange for the increased stability on offer. When the initial zone is very wide this improvement in the trade off is quite large but it falls as the band narrows so that in general adding reserves leads to a narrower zone and a higher lifetime.

Thus far the results suggest that unless the drift in fundamentals is small the lifetime will have a significant effect on the optimal bandwidth in both absolute and percentage terms. The reverse is not true unless there is a real drift and a low elasticity of aggregate demand to the real exchange rate, and/or the initial log excess of reserves is much less than the 100% we have used as the benchmark example.

One way of estimating the choosing the initial log excess of reserves would be to consider the realism of the estimated lifetime of the fixed rate. The lifetime of the fixed rate depends on just two parameters, the initial log excess of reserves and the size of the composite drift. The latter is determined by the size of the underlying drift and in the case of real drifts, the degree of openness. Our basic parameterisation assumes an initial reserve stock 2.7 times the size of the speculative attack that destroys the zone and a 10% annual drift. With $\eta = .5$ this gives a lifetime for a fixed rate of 10 years for the monetary drift, and 5 years for the drift in goods demand. An underlying drift of 10% implies that $\mu_f = 10\%$ per year for the monetary drift and $\mu_f = 20$ % per year for the goods demand drift. Since μ_f measures the average annual exchange rate depreciation under a pure free float, for those countries subject to periodic crises these values are plausible. The amount of initial excess reserves depends upon the access to foreign borrowing and the political commitment to the target zone regime. This makes the appropriate initial excess of reserves more difficult to assess. An alternative way to choose the initial excess reserves would be to contrast the calculated lifetime with the study by Klein & Marion (1997) of 61 exchange rate pegs in Latin American countries in which the average peg lasted just 2.7 years[5]. Given a drift such that $\mu_f = 10\%$ per year we choose the initial excess of log reserves such that the lifetime is, $E(T|f,r)\big|_{b=a} = (q-p)/.2 = 3$. Given a lifetime of a fixed rate equal to 3 years our final example analyses how altering the interest elasticity of money demand λ, alters the importance of the lifetime.

Figures 11 & 12 here

With λ very low the effect of the zone on the exchange rate is quite weak and both bands are very narrow. In percentage terms the long horizon band at 4.65 log % is 49% wider than the infinitely discounted band of 3.11 log %. Since the absolute values are so small this raises the lifetime by just 5% from 3.65 to 3.85 years. Increasing λ widens both bands and the gap between the two grows. Since the old bandwidth was optimally chosen and increasing λ leads to greater exchange rate stability, the old band is now too tight so that the infinitely discounted band widens as λ increases. The long horizon band widens more than proportionately indicating that

[5] The standard deviation was 49 months and the median duration just 10 months. That is a lot of pegs collapsed within the first year whilst a significant number of pegs, having gained some credibility survived for considerably longer than 32 months.

the lifetime is having a bigger influence the larger is λ. The reason that the lifetime matters more is that the amount of stability offered by the zone increases, and this makes it more important that it survives. Consequently the lifetime has a bigger influence on the optimal band the larger is the value of λ. An interest elasticity of money demand equal to 20% produces a long horizon band of ± 29 log % that is 190% wider than the infinitely discounted band of ± 10 log %. In terms of the lifetime the big variations in the bandwidth do increase the lifetime significantly if the interest elasticity of money demand is not low. For example, if $\lambda = 20\%$ the lifetime under the long horizon band at 7.42 years is 40% more than the 5.3 years under the infinitely discounted band. At $\lambda = 30\%$ these values are 9.4 years, 56%, and 6 years respectively. In this case in which we set the lifetime of the fixed rate equal to 3 years, including the lifetime in the determination of the optimal band causes variations in the bandwidth that are large enough to buy significant amounts of extra lifetime in absolute and percentage terms.

The results depend upon the approximation to the within band variance of the target zone and the assumed rate of time discount. The latter affects our results in an obvious way. Increasing the rate of time discount causes the long horizon band to converge to the infinitely discounted band and the expected lifetime to fall accordingly. The within band approximation almost certainly causes our results to understate the importance of the lifetime. As outlined previously the larger the drift and the longer the lifetime, the lower is the average variance of the exchange rate and the less it responds to changes in the bandwidth. Since the lifetime is only really low when the drift is very large the exchange rate must on average be close to the bands edges and the average within band variance would be lower than implied by our approximation. What matters in this case is not so much how wide the band is but rather that the band survives.

Conclusions

Several papers analyse the optimality of an exchange rate target zone assuming the zone to be infinitely lived. In this paper the stock of available reserves is limited and we examine the expected lifetime, and the extent to which it influences the optimal band. Our results show, following Dumas & Svensson (1994), that in the absence of a drift in fundamentals the lifetime of a target zone is extremely long and not a significant determinant of the optimal band. However, if the drift is at least moderate the expected lifetime significantly widens the optimal band, even if the amount of extra lifetime bought is a small fraction of the total lifetime. The reason is that with a combination of a long lifetime and moderate drift, widening the bandwidth has very little effect on the instantaneous stabilising properties of the target zone, and what matters is that the zone survives. Even though the lifetime tends to widen the bandwidth significantly, the amount of extra lifetime bought is not large in absolute terms, and for countries with good access to foreign borrowing the extra lifetime bought is small in percentage terms. The extra lifetime bought is much more significant in percentage terms when there is a large real drift and a low degree of openness, and/or if access to foreign borrowing is more limited. In particular if the initial reserve stock is chosen to match the lifetime of a fixed rate for countries subject to frequent crises, the extra lifetime bought is a very significant fraction of the total lifetime. Finally it should be reiterated that stabilisation without intervention, and hence the extra lifetime is the unique feature of the target zone.

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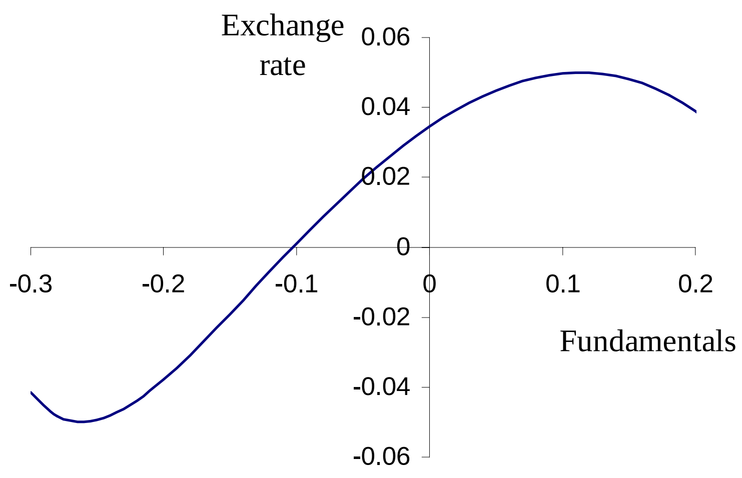

Figure 1: A Target Zone

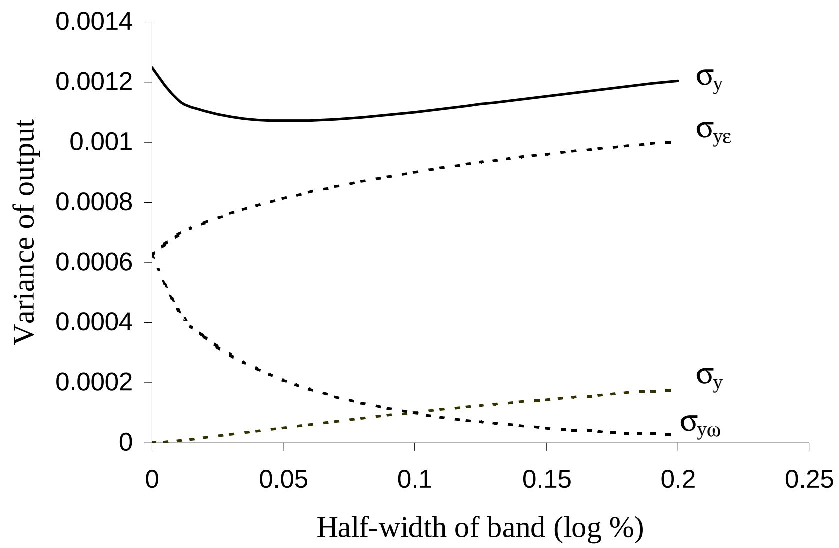

Figure 2: Shocks, output variability and the bandwidth

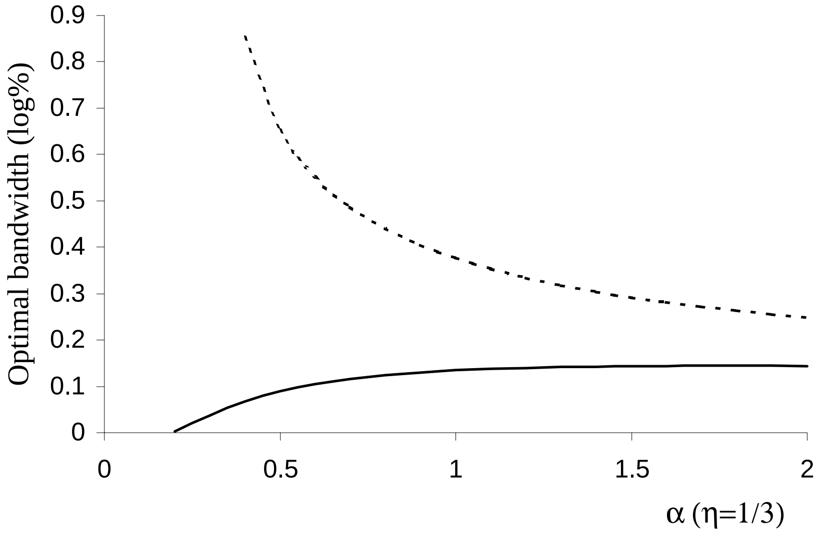

Figure 3: The supply curve and optimal band

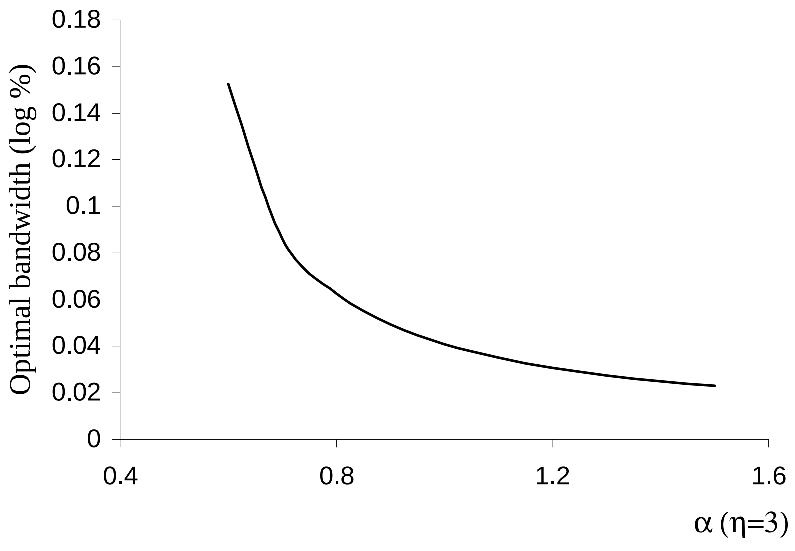

Figure 4: The supply curve and optimal band

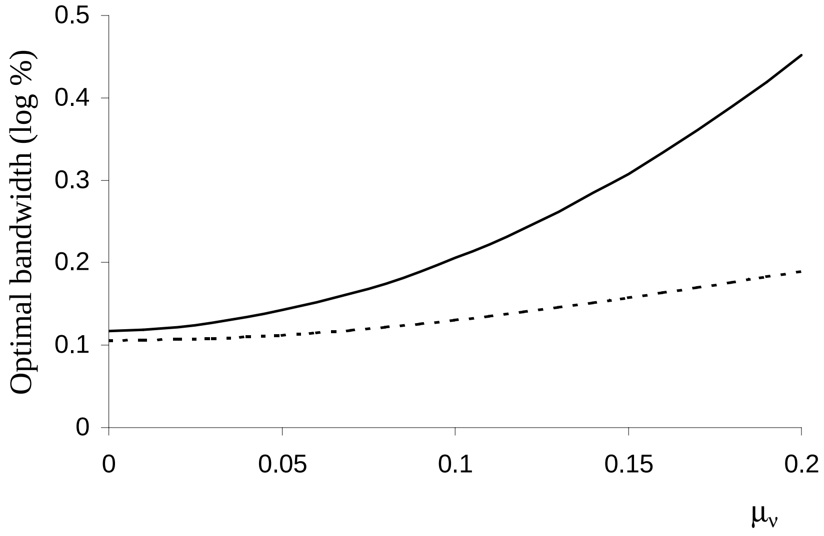

Figure 5: The optimal band and a monetary drift

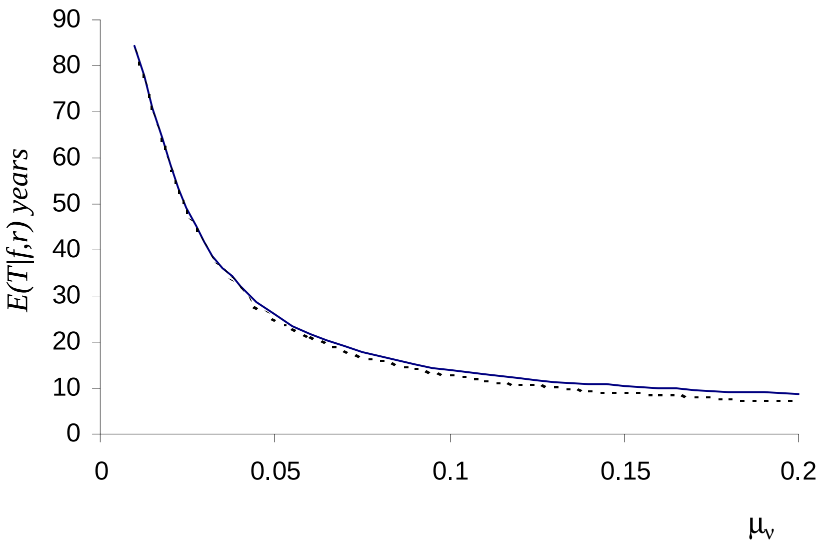

Figure 6: The lifetime and a monetary drift

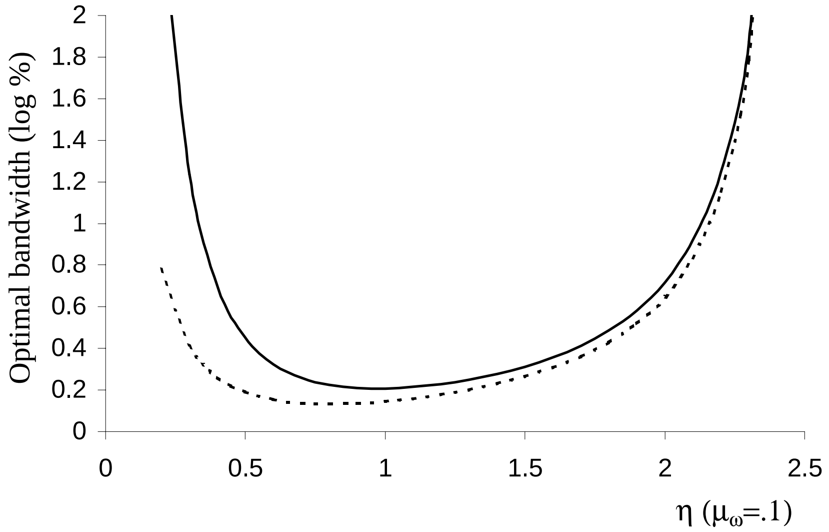

Figure 7: The optimal band and openness

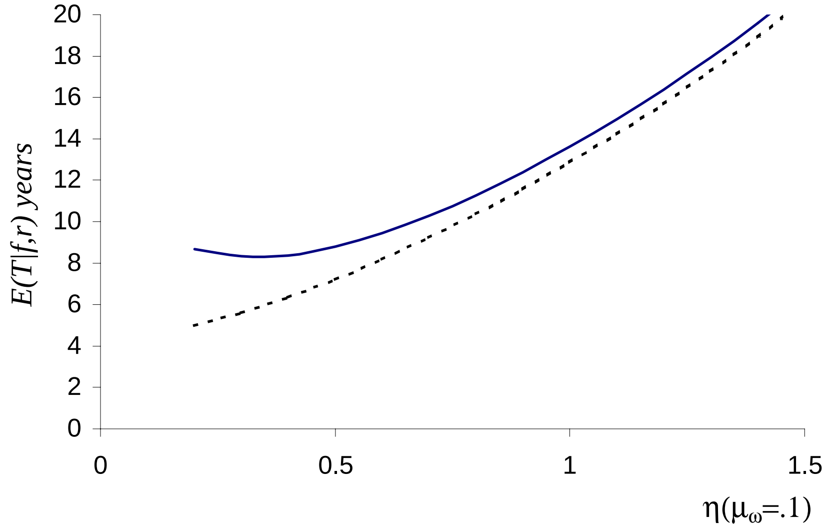

Figure 8: The lifetime and openness

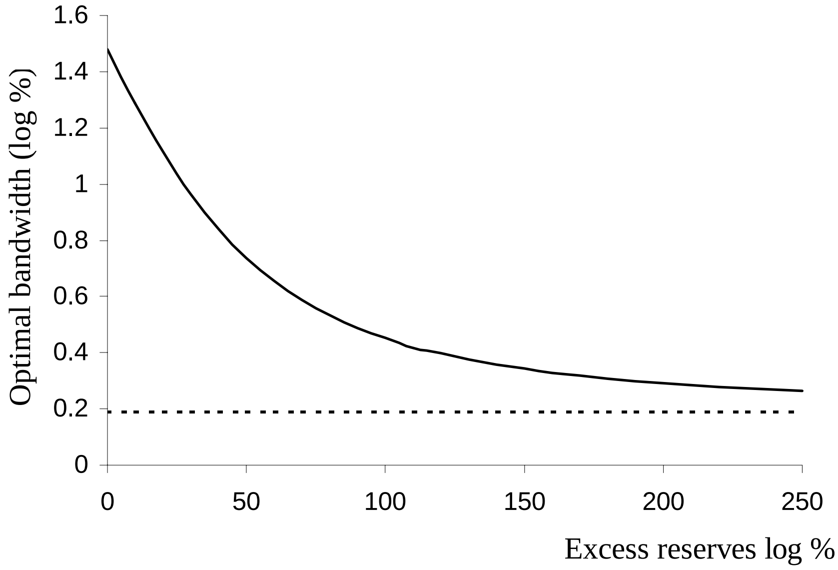

Figure 9: The optimal band and excess reserves

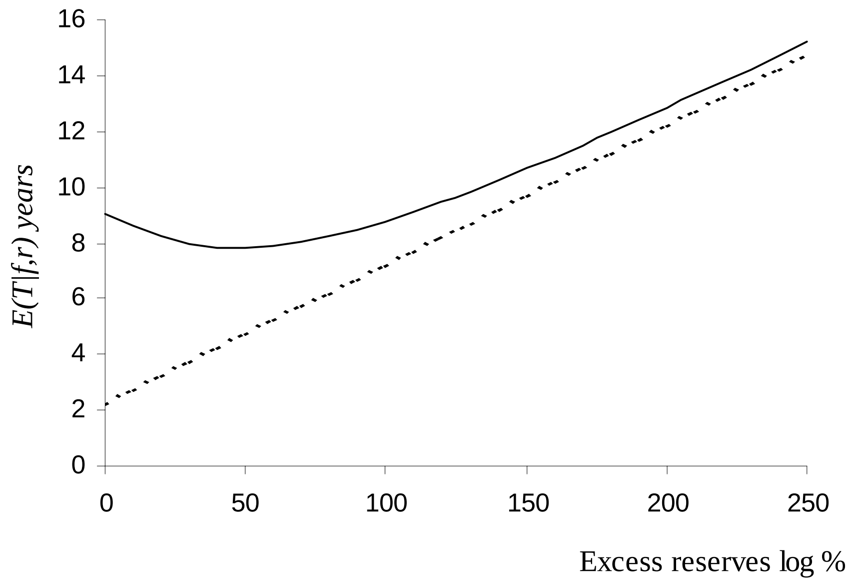

Figure 10: The lifetime and excess reserves

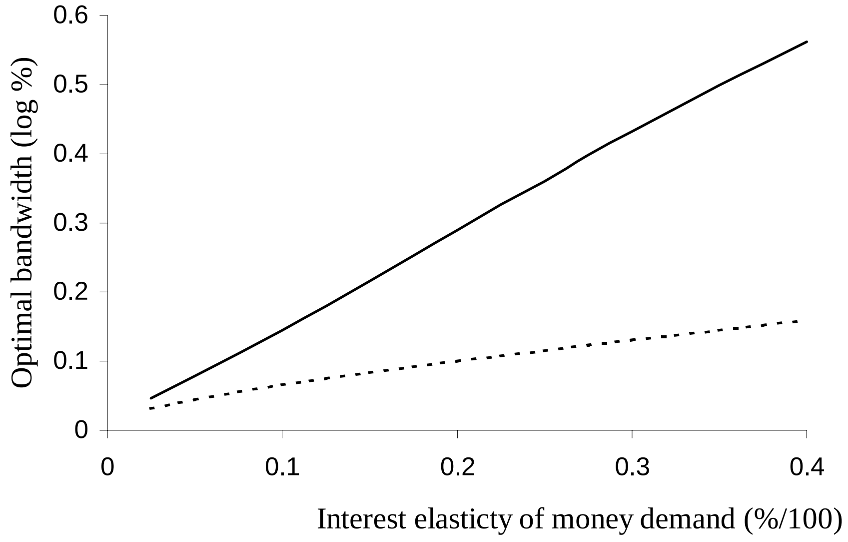

Figure 11: The optimal band and the honeymoon effect

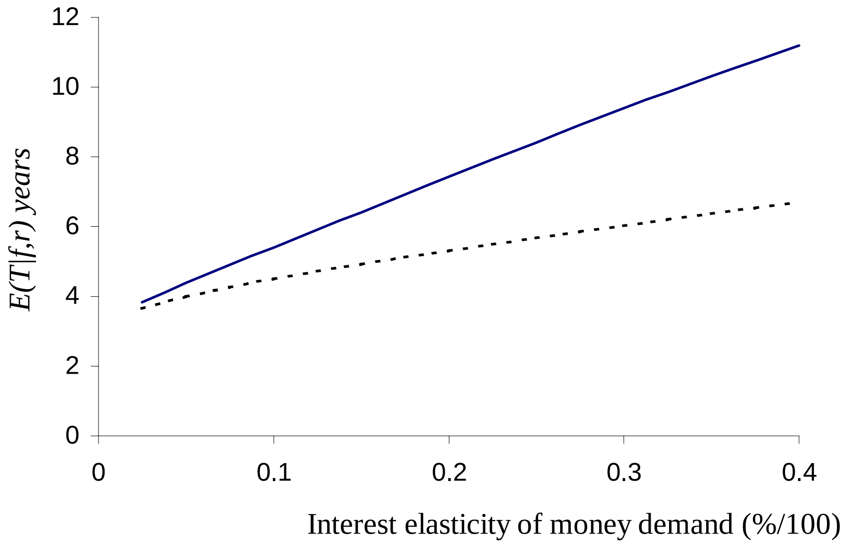

Figure 12: The lifetime and honeymoon effect