Declining Discount Rates: Evidence from the UK

Ben Groom* Phoebe Koundouri† Ekaterini Panopoulou‡
Theologos Pantelidis§

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Abstract

We estimate schedules of declining discount rates for cost benefit analysis in the UK. We highlight the importance of model selection for this task and hence for the evaluation of long-term investments, namely climate change prevention and nuclear build.

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*Department of Economics, University College London, UK.

†Department of Economics, University of Reading, UK and Depatment of Economics, University College London, UK.

‡Department of Banking and Financial Management, University of Piraeus, Greece and Department of Economics, National University of Ireland Maynooth. *Correspondence to:* Ekaterini Panopoulou, Department of Economics, National University of Ireland Maynooth, Co.Kildare, Republic of Ireland. E-mail: apano@may.ie. Tel: 00353 1 7083793. Fax: 00353 1 7083934.

§Department of Banking and Financial Management, University of Piraeus, Greece.

1 Introduction

The discussion of discounting in Cost Benefit Analysis (CBA) has come to the fore once more as policy makers are increasingly required to appraise investments whose costs and benefits accrue in the far distant future. Climate change and nuclear build exemplify this long-term policy arena. Largely in response to the dramatic effects of conventional exponential discounting on welfare changes in the distant future, the discussion has turned to discount rates that decline with the time horizon, Declining Discount Rates or DDRs, and there is now an evolving body of theory.[1] For example, Weitzman (1998) shows that uncertainty and persistence of the discount rate itself provides a rationale for DDRs and of all the theoretical approaches this approach has proven more amenable to implementation, mainly because the informational requirements stop at the characterisation of the uncertainty surrounding the discount rate.[2] In this respect, Newell and Pizer (2003) (N&P, henceforth) characterise interest rate uncertainty by the parameter uncertainty typically encountered in any econometric model. Their model of US interest rates, though simple, yields a working definition and estimation of the Certainty Equivalent forward Rate (CER) for use in CBA. The authors confirm the declining pattern of discount rates and its relation to uncertainty and persistence.

Our view in this paper is that such a simple model is not sufficiently versatile to reproduce the empirical regularities typically found in interest rate series. Our aim therefore is to develop relatively simple econometric models that characterize the past as accurately as possible and offer a flexible framework for the future due to their time-heterogeneity properties. We discuss the in-sample properties of alternative econometric models for the UK interest rates, comment on the properties of the simulated distribution and finally select among them based on their out-of-sample forecasting performance. We exemplify the policy relevance of DDRs and model selection with two UK case studies with long-term impacts: the value of carbon sequestration and the appraisal of nuclear build.

2 Discounting and interest rate models

Discounting future consequences in period t back to the present is typically calculated using the discount factor P_t, where $P_t = \exp(-\sum_{i=1}^{t} r_i)$. When r is stochastic, the expected discounted value of a dollar delivered after t years is:

$$E(P_t) = E\left(\exp(-\sum_{i=1}^{t} r_i)\right) \tag{1}$$

Following Weitzman (1998) we define (1) as the *certainty equivalent discount factor*, and the corresponding *certainty-equivalent forward rate* for discounting between adjacent periods at time t as equal to the rate of change of the expected discount factor:

$$\frac{E(P_t)}{E(P_{t+1})} - 1 = \widetilde{r}_t \tag{2}$$

where $\widetilde{r}_t$ is the forward rate from period t to period $t+1$ at time t in the future.

Our focus is on the determination of the stochastic nature of $\widetilde{r}_t$ through the observed dynamics of the process. Our starting point is the relatively simple $AR(p)$ model employed by N&P, specified as follows:

$$r_t = \eta + e_t, \qquad e_t = \sum_{i=1}^{p} a_i e_{t-i} + \xi_t \tag{3}$$

[1] See e.g. Pearce *et al.* (2003) and the references therein for a detailed discussion of the DDR literature.

[2] For example, the informational requirements do not extend to specific attributes of future generations' risk preferences as would be unavoidable in the case of Gollier (2002a, 2002b).

where $\xi_t \sim N(0, \sigma_\xi^2)$, $\eta \sim N\left(\overline{\eta}, \sigma_\eta^2\right)$ and $\sum_{i=1}^{p} a_i < 1$. However, modeling the interest rate for the very long run through constant coefficient models is likely to be an unrealistic assumption, since a number of factors, such as the economic cycle, oil crises, stock market crises, productivity and technology shocks may account for a time-varying behavior in the data generation process of the interest rate. In this respect, we introduce two models that are time-heterogeneous in the sense that they account for the possibility of time varying parameters and regime changes. Our Regime-Switching (RS) model is one with two regimes as follows:

$$r_t = \eta_k + e_t, \qquad e_t = \sum_{i=1}^{p} a_i^k e_{t-i} + \xi_t \tag{4}$$

where $\xi_t \sim IIDN(0, \sigma_k^2)$, $k = 1, 2$ for the first and second regime, respectively. Each regime incorporates a different speed of mean-reversion, along with a different permanent component, η_k, and error variance. The probability of being in each regime at time t is specified as a Markov 1 process, i.e. it depends only on the regime at time $t-1$, with the matrix of the transition probabilities assumed to be constant.[3] A more convenient way to account for infinite regimes in the interest rate process is through a time varying coefficient model, such as an $AR(1)$ model with an $AR(p)$ coefficient, namely a State Space (SS) model, given by the following system of equations:

$$r_t = \eta + \alpha_t r_{t-1} + e_t, \qquad \alpha_t = \sum_{i=1}^{p} \eta_i \alpha_{t-i} + u_t, \qquad \begin{pmatrix} e_t \\ u_t \end{pmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_e^2 & 0 \\ 0 & \sigma_u^2 \end{bmatrix} \right) \tag{5}$$

3 Empirical Results

3.1 Data and Estimation Results

Our dataset consists of nominal interest rates transformed to real interest rates by subtracting the annual change in the Consumer Price Index for the period 1800 to 2001.[4] To smooth very short-term fluctuations, a 3-year moving average of the real interest rate series is employed and in order to avoid negative interest rates, we use the natural logarithms of the series.

A variety of unit root tests confirmed that the UK real interest rate is a stationary process.[5] Our $AR(4)$ model displays relatively rapid reversion to the implied unconditional mean of 3.32% (see Table 1, Panel A). However, our estimates for the RS model (see Table 1, Panel B) indicate the presence of two distinct regimes (modelled as $AR(2)$ processes). The unconditional means of each are 2.14% and 3.70% and mean reversion is faster in the latter. The first regime has an estimated duration of 4 years, while the second one is more persistent with a duration of 15 years. Overall, the estimates of this model suggest that low interest rate periods are quickly mean-reverting, surrounded by greater uncertainty and transit more often to high interest rates periods which are more persistent and less uncertain. Turning to our SS model, the parameter estimates (see Table 1, Panel C) suggest that the state process is highly persistent, almost a random-walk process, as indicated by the estimate of the autoregressive coefficient. The constant of our model suggests a minimum of 1.31% for the interest rate process.

[3]The matrix of probabilities is as follows:

$$\begin{aligned} \Pr\text{ob}(R_t &= 1 \mid R_{t-1} = 1) = P, \ \Pr\text{ob}(R_t = 2 \mid R_{t-1} = 2) = Q \\ \Pr\text{ob}(R_t &= 2 \mid R_{t-1} = 1) = 1 - P, \ \Pr\text{ob}(R_t = 1 \mid R_{t-1} = 2) = 1 - Q \end{aligned}$$

where R_t refers to the regime at time t.

[4]The nominal interest rate is the United Kingdom 2 1/2% Consol Yield. Data provided by the Global Financial Data, Inc, available at http://www.globalfindata.com.

[5]Unit root tests are not reported for brevity but are available upon request.

3.2 Simulation results and model selection

Based on the estimates presented in Table 1, we simulate 100.000 possible future discount rate paths for each model starting in 2002 and extending 400 years into the future.[6] The expected discount factors and CERs are calculated from equations (1) and (2) and are reported in Tables 2 and 3. We also comment on the empirical distribution of interest rates.

The SS model yields the highest discount factors followed by the RS and AR(4) model. These differences are more pronounced during the first half of the forecast horizon. Only SS sustains some value in the distant future (400 years). Naturally, the corresponding certainty-equivalent discount rates reveal largely the opposite picture. The $AR(4)$ model yields the higher rates during the first half of the sample, while the RS model yields the higher rates in the second half. The SS model gives consistently lower CERs that fluctuate in the range of 2.2% to 1.4%. Turning to the simulated distribution of discount factors, the model with the lowest coefficient of variation (i.e. the ratio of standard deviation over mean) is SS, whereas the AR(4) model yields the highest coefficient.[7] Alternatively, as a measure of uncertainty, we employ the 5% and 95% empirical percentiles. This measure seems to favor the RS model, which has the tightest confidence intervals, suggesting that uncertainty over the expected discount factor is considerably reduced. On the other hand, the percentiles of the SS model are relatively wide.

Evaluating the forecasting performance of our models for the long run is impossible due to limitation of data. However, since forward rates exist for a period of 30 years we undertake a comparison of forecasting performance over this time horizon using available real data. Specifically, we make use of the term structure of the inflation-indexed UK government bonds and use the Mean Square Forecast Error (MSFE) as our selection criterion. For completeness we calculate four modified MSFE criteria by incorporating four kernels[8] which attach different weights to observations based on their proximity to the present. The results are presented in Table 4. Interestingly, the various specifications of the MSFE criterion unanimously rank the SS model first followed by the RS model and then the AR(4) model. In sum, if we select the models on the basis of their ability to characterize the past and their accuracy concerning forecasts of the future we are inclined to prefer the SS model.

4 Policy Implications

In this section we highlight the policy implications of DDRs and model selection by looking at the long-term policy arena. Firstly we follow N&P and consider the present value of carbon sequestration: the removal of 1 ton of carbon from the atmosphere. Secondly, we look at nuclear build in the UK. The two are directly related since nuclear power can benefit from carbon credits under a system of joint implementation and carbon trading (see Pearce *et al.* 2003).

Regarding our first case study, we establish the present value of the removal of 1 ton of carbon from the atmosphere, and hence the present value of the benefits of the avoidance of climate change damages for each of our models.[9] The results, reported in Table 5, suggest that the lower valuation is given by the conventional 3.5% discounting, followed by the AR(4) model. Interestingly, when employing the SS model, the present value of carbon emissions reduction is over 200% larger compared to the case of constant discounting.

Our second case study highlights a sense in which DDRs are limited in accounting for intergenerational equity. We, specifically, consider new nuclear build in the UK which is still being considered as an option to ensure security of energy supply and adherence to Kyoto targets

[6]The process of picking parameters and shocks is available from the authors upon request. Initial values for any lags of the real interest rate necessary for the simulation are set at 3.5 per cent, the rate used for CBA by the UK Treasury (HM Treasury 2003).

[7]The relevant tables are not reported for brevity, but are available upon request.

[8]The Bartlett, the Parzen, the Quadratic-Spectral (QS) and the Tukey-Hanning (TK) kernels are the weighting functions used in our evaluation.

[9]See N&P for the assumptions concerning the modeling of carbon emissions damages.

(UK Performance and Innovation Unit, 2002). Both decommission costs and carbon credits are naturally sensitive to the use of DDRs and in Table 6 we compare the NPV of investment in a nuclear power station using our estimated DDRs.[10] The appraisal shows that although the SS model has significant consequences for the present value of revenues and carbon credits, the present value of decommissioning and operating costs is also increased considerably. In this respect, the NPV of nuclear build is affected only marginally when evaluated using DDRs, although the SS and the RS models increase the NPV of the project by more than 8%.

5 Conclusions

This paper builds on N&P's econometric approach to determining DDRs and emphasises the policy relevance of model selection when dealing with lengthy time horizons. Using UK interest rate data we show that the econometric specification should allow the data generating process to change over time and that the broad class of state space models is appropriate. The policy relevance of our procedure is highlighted in the valuation of carbon sequestration the present value of which is increased by over 200%.

References

[1]Gollier, C. (2002a). Time Horizon and the Discount Rate. *Journal of Economic Theory* 107 (2), 463-473.

[2]Gollier, C. (2002b). Discounting an Uncertain Future. *Journal of Public Economics* 85, 149-166.

[3]HM Treasury, (2003). The Green Book: Appraisal and Evaluation in Central Government: London: HM Treasury.

[4]Newell, R. and W. Pizer (2003). Discounting the Benefits of Climate Change Mitigation: How Much do Uncertain Rates Increase Valuations? *Journal of Environmental Economics and Management* 46 (1), 52-71.

[5]Pearce, D., B. Groom, C. Hepburn and P. Koundouri (2003). Valuing the Future: Recent Advances in Social Discounting. *World Economics* 4 (2), 121-141.

[6]Performance and Innovation Unit (2002). The Energy Review - February.

[7]Weitzman, M. (1998). Why the Far Distant Future Should be Discounted at its Lowest Possible Rate. *Journal of Environmental Economics and Management* 36, 201-208.

[10] We follow the same cost and price assumptions and time horizons for construction, operation and decommissioning as Pearce *et al.* (2003).

Table 1: Estimation Results

Panel A: AR(4) model			
Coefficient	Estimate	Std. Error	t-stat.
n	1.201	0.177	6.777
a_1	1.054	0.058	18.165
a_2	-0.125	0.089	-1.392
a_3	-0.443	0.070	6.308
a_4	0.368	0.035	10.452
σ_ξ^2	0.064	0.005	13.733
Panel B: Regime Switching model			
Coefficient	Estimate	Std. Error	t-stat.
n_1	0.760	0.244	3.117
a_1^1	0.700	0.312	2.249
a_2^1	-0.212	0.312	-0.679
n_2	1.306	0.082	15.892
a_1^2	1.397	0.079	20.573
a_2^2	-0.530	0.058	-9.094
σ_1^2	0.219	0.047	4.694
σ_2^2	0.014	0.002	8.106
P	0.767	0.101	7.543
Q	0.933	0.033	28.617
Panel C: State Space model			
Coefficient	Estimate	Std. Error	t-stat.
n	0.266	0.044	6.091
n_1	0.991	0.002	438.82
$ln(\sigma_e^2)$	-2.503	0.104	-24.049
$ln(\sigma_u^2)$	-6.462	0.594	-10.884

Table 2. Certainty Equivalent Discount Factors

Model Year	3.5% Constant	AR(4)	Regime Switching	State Space
1	0.96618	0.96618	0.96618	0.96618
20	0.50257	0.48208	0.51472	0.61857
40	0.25257	0.23676	0.26746	0.40678
60	0.12693	0.11778	0.13981	0.27722
80	0.06379	0.05912	0.07354	0.19368
100	0.03206	0.02997	0.0389	0.13775
150	0.00574	0.00569	0.00813	0.06172
200	0.00103	0.00115	0.00177	0.02882
250	0.00018	0.00027	0.00041	0.01379
300	0.00003	0.00008	0.0001	0.00669
350	0.00001	0.00003	0.00003	0.00328
400	0.00000	0.00002	0.00001	0.00161

Table 3. Certainty Equivalent Discount Rates

Year/ Model	AR(4)	Regime Switching	State Space
1	3.50	3.50	3.50
20	3.68	3.35	2.22
40	3.58	3.31	2.02
60	3.52	3.28	1.87
80	3.48	3.25	1.76
100	3.43	3.22	1.68
150	3.33	3.14	1.57
200	3.13	3.05	1.51
250	2.77	2.93	1.47
300	2.17	2.75	1.45
350	1.12	2.45	1.43
400	0.39	2.14	1.44

Table 4. Average MSFEs

Model Criterion	AR(4)	Regime Switching	State Space
AMSFE	2.330	1.486	0.195
AMSFE (B)	0.875	0.527	0.135
AMSFE (P)	0.562	0.332	0.132
AMSFE (QS)	0.659	0.407	0.071
AMSFE (TH)	0.818	0.480	0.137

Notes: The weighting functions are as follows: Bartlett(B), Parzen(P), Quadratic-Spectral (QS) and Tukey-Hanning (TH).

Table 5: Value of Carbon Damages

Model	Carbon Values (£/tc)	Relative to Constant Rate
Constant (3.5%)	5.35	—
AR(4)	5.78	7.9%
Regime Switching	6.21	16.1%
State Space	16.73	212.6%

Table 6: The Costs and Benefits of Nuclear Build in the UK

(£/KW)	CAPEX	OPEX	DECOM	Rev/es	C C	NPV	Relative to 3.5%
3.5%	2173	2336	427	4062	228	-646	—
AR(4)	2167	2245	396	3904	215	-689	-6.6%
RS	2178	2401	479	4176	249	-633	8.0%
SS	2196	2973	1126	5170	547	-577	8.9%