

# Long-Run Cash-Flow and Discount-Rate Risks in the Cross-Section of US Returns\*

Michail Koubouros<sup>†</sup>, Dimitrios Malliaropoulos<sup>‡</sup>, Ekaterini Panopoulou<sup>§</sup>

This version: May 2005

## Abstract

This paper decomposes the overall market (CAPM) risk into parts reflecting uncertainty related to the long-run dynamics of portfolio-specific and market cash flows and discount rates. We decompose market betas into four sub-betas (associated with assets' and market's cash flows and discount rates) and we employ a discrete time version of the I-CAPM to derive a four-beta model. The model performs well in pricing average returns on single- and double-sorted portfolios according to size, book-to-market, dividend-price ratios and past risk, by producing high estimates for the explained cross-sectional variation in average returns and economically and statistically acceptable estimates for the coefficient of relative risk aversion.

**JEL:** *G11, G12, G14*

**Keywords:** *CAPM, cash-flow risk, discount-rate risk, VAR-GARCH, BEKK, asset pricing*

---

\*We are grateful to Gikas Hardouvelis, Jack Meyer, Dimitrios Thomakos and the participants at the University of Piraeus-Athens Derivatives Stock Exchange Seminar for their helpful comments and suggestions. We acknowledge financial support from the Greek Ministry of Education and the European Union under "Pythagoras" grant. The usual disclaimer applies.

<sup>†</sup>Corresponding author: University of Peloponnese, Department of Economics, Terma Karaiskaki, 22 100 Tripolis, Greece. Phone: (+30) 2710-230129, fax: (+30) 2710-230139, e-mail: [m.koubouros@uop.gr](mailto:m.koubouros@uop.gr).

<sup>‡</sup>Department of Banking and Financial Management, University of Piraeus, and National Bank of Greece.

<sup>§</sup>Department of Economics, National University of Ireland, Maynooth.

# 1 Introduction

Since the original statement of the Sharpe-Lintner one-factor Capital Asset Pricing Model (CAPM), there is a considerable ongoing debate on whether its single risk measure, the market beta, can adequately describe the cross-section of average returns on individual stocks and portfolios sorted according to risk measures and firm-specific characteristics. Numerous studies have shown that the single beta CAPM, at least in its unconditional form, performs poorly, since the cross-sectional variation in unconditional market betas cannot match the observed spread in average excess returns.<sup>1</sup>

In this paper, we decompose the market systematic risk (CAPM beta) of common stocks into four long-run risk components related to the covariance of unexpected changes in stock-specific cash-flows and discount rates with unexpected changes in market-wide cash-flows and discount rates. Further, we empirically test whether these sources of risk are priced using a discrete time version of the intertemporal asset pricing model of Merton (1973), recently developed by Campbell (1993, 1996).

Our paper is related to the work of Campbell (1991), Campbell and Mei (1993), Campbell and Vuolteenaho (2005) and Campbell, Polk and Vuolteenaho (2003). In a novel paper, Campbell (1991) shows that unexpected stock returns can be decomposed into the discounted sum of revisions in expectations about future cash flows and future discount rates. Campbell and Mei (1993) extend this analysis by studying the behavior of asset specific cash-flow and discount-rate components of portfolio betas but do not provide any evidence on whether these parts of systematic risk carry individual risk prices.

More recently, Campbell and Vuolteenaho (2005) show that the market beta can be decomposed into a relatively “bad” cash-flow beta, reflecting news about the market’s future cash flows, and a relatively “good” discount rate beta, reflecting news about the market’s future discount rates. They argue that the two components of return innovations have different implications for the rational investor. Since shocks to market cash flows and market discount rates represent permanent and temporary shocks to overall wealth respectively, rational conservative investors are particularly averse to the former and require a premium which is a multiple of their attitude towards risk. As a result, discount rate betas are relatively “good” betas with low risk prices, whereas cash flow betas are “bad” betas with high risk prices. Empirically, Campbell and Vuolteenaho find that small stocks and value stocks have considerably higher cash-flow (“bad”) betas than growth stocks and large stocks, and this can explain their higher average returns. However, they restrict their analysis by assuming that “good” and “bad” betas are independent of whether the innovation in individual returns is due to unexpected changes in future

---

<sup>1</sup>For a recent review on the CAPM see, among others, Fama and French (2004).

cash-flows or discount rates of the company.

In a paper closest to ours, Campbell, Polk and Vuolteenaho (2003) decompose the overall market beta into four betas which reflect the covariance of unexpected changes in stock-specific cash-flows and discount rates with unexpected changes in market-wide cash-flows and discount rates. This decomposition of the market beta allows the authors to answer the question whether the high “bad” beta of small and value stocks and the high “good” beta of growth stocks and large stocks are attributable to their cash flows or their discount rates. Campbell, Polk and Vuolteenaho estimate sample betas for growth and value portfolios and show that growth portfolios’ cash flows are particularly sensitive to temporary movements in aggregate stock prices (driven by market-wide shocks to discount rates) while value portfolios’ cash-flows are highly correlated with temporary movements in market returns (driven by market-wide shocks to cash-flows). However, they do not test the asset pricing implications of this four factor model, leaving the question unanswered as to what economic forces determine the risk prices associated with these four sources of risk.

Our four-beta model aims in investigating whether these four components of the overall market beta are priced according to a standard asset pricing model that identifies changes in expectations about future cash flows and future discount rates as the long-run risk factors that can explain the cross-section of mean returns. Using the discrete time version of Merton’s (1973) Intertemporal CAPM (I-CAPM) proposed by Campbell (1993, 1996), our structural four-beta model shows considerable in-sample success in pricing average returns on single- and double-sorted portfolios according to market capitalization, book-to-market, dividend-price ratios and risk. The model generates low and insignificant pricing errors, high estimates for the explained cross-sectional variation in average returns and statistically and economically acceptable estimates for the degree of relative risk aversion. We find that, as predicted by economic theory, permanent shocks to market returns are the main determinant of the overall risk premium, their covariances with both portfolio cash-flow and discount-rate dynamics earn equilibrium risk premia that are indistinguishable from zero, but the premia associated with asset-specific cash-flow news are greater than those linked to asset-specific discount-rate news. More importantly, we provide evidence that the coefficient of proportionality between the two premia is equal to the constant coefficient of relative risk aversion, as predicted by theory.

The remainder of the paper is as follows: Section 2 provides the theoretical decomposition of total market risk into four parts: cash-flow and discount-rate portfolio risks associated with market’s cash-flow and discount-rate dynamics. Also, it develops the asset pricing framework that will be used for estimation. Section 3 describes the data set and the econometric model used to extract the news components of unexpected returns.

Section 4 presents the empirical results. Finally, section 5 concludes.

## 2 Decomposing Risk and Return

### 2.1 Cash-Flow and Discount-Rate Risk

The starting point of our analysis is the decomposition of the unexpected return, developed by Campbell and Shiller (1988) and further expanded by Campbell (1991). We define the one-period holding real gross return on asset  $i$  as  $r_{i,t+1} = \log(P_{i,t+1} + D_{i,t+1}) - \log(P_{i,t})$ , where  $P_{i,t+1}$  is the real stock price measured at the end of period  $t + 1$  (ex-dividend) and  $D_{i,t+1}$  is the real dividend payment during this period. Approximating this return with a first-order Taylor expansion around the, assumed constant, mean log dividend-price ratio,  $\bar{\delta}_i = E[\log(d_{i,t} - p_{i,t})]$ , we obtain:

$$r_{i,t+1} \approx k_i + \rho_i p_{i,t+1} - p_{i,t} + (1 - \rho_i) d_{i,t+1}, \quad (1)$$

with

$$k_i = -\log(\rho_i) - (1 - \rho_i) \log[(1/\rho_i) - 1],$$

and

$$\rho_i = 1/[1 + \exp(\bar{\delta}_i)]$$

being firm-specific constants. Campbell (1991), using this approximation of log returns, goes one step further and derives a decomposition of the unexpected return,  $e_{i,t+1} = r_{i,t+1} - E_t[r_{i,t+1}]$ , into revisions in expectations about future dividend growth rates (that is growth rates of future cash flows) and revisions in expectations about future log returns (that is future discount rates):

$$e_{i,t+1} = N_{i,t+1}^C - N_{i,t+1}^D, \quad (2)$$

with  $N_{i,t+1}^C$  and  $N_{i,t+1}^D$  defined as:

$$N_{i,t+1}^C = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho_i^j \Delta d_{i,t+1+j},$$

and

$$N_{i,t+1}^D = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho_i^j r_{i,t+1+j},$$

respectively. The above sums can be viewed as representing cash-flow and discount-rate “news” for the investor, since any upward or downward revision in her expectations at

time  $t + 1$  must be consistent with the arrival of new valuable information at time  $t + 1$ . Moreover, as Campbell and Shiller (1988a, 1993), and Campbell (1991) argue, equation (2) must be considered as a consistent model of expectations, since a positive (negative) unexpected return today must be only associated with an upward (downward) revision in expectations about future cash-flows, a downward (upward) revision in expectations about future returns, or a combination of the two. That is, although equation (2) does not restrict the generating mechanism of expectations or the asset pricing model that derives equilibrium expected returns, it restricts the way through which changing expectations due to “good” or “bad” news affect unexpected returns on any asset if investors’ expectations are to be consistent with the observed asset prices.

As, among others, Campbell and Vuolteenaho (2005) and Campbell, Polk and Vuolteenaho (2003) argue, the two components of unexpected returns can be viewed as permanent and transitory shocks to the value of the underlying asset. A positive unexpected return caused by an upward revision in cash-flow expectations represents a permanent positive effect on the value of the asset since it is never reversed subsequently, whereas a positive unexpected return generated from a downward revision in expectations about future returns can be viewed as a temporary shock to the asset price, since the capital gain today is at a cost of lower future investment opportunities. In the case where the underlying asset is the total wealth portfolio held by investors, these effects can be viewed as permanent and temporary movements in total wealth.

We now turn to link the sources of time variation in asset returns with the associated sources in the total wealth portfolio. Following Campbell and Shiller (1993), we define the “market” or CAPM beta as the ratio of the conditional covariance of asset’s and market’s unexpected returns divided by the conditional variance of market unexpected returns:

$$\beta_{im,t} = \frac{\text{Cov}_t(e_{i,t+1}, e_{m,t+1})}{\text{Var}_t(e_{m,t+1})}, \quad (3)$$

where  $\text{Var}_t(\cdot)$  and  $\text{Cov}_t(\cdot)$  are the conditional, at time  $t$ , variance and covariance operators, respectively. Given that the current innovation in returns on both the asset  $i$  and the market portfolio can be written as the sum of cash-flow and (the negative of) discount-rate news (equation (2)), we obtain the following decomposition of the conditional market sensitivity  $\beta_{im,t}$  which can be now written as the sum of four conditional “beta-like measures” of systematic risk:

$$\begin{aligned} \beta_{im,t} &= \frac{\text{Cov}_t(N_{i,t+1}^C - N_{i,t+1}^D, N_{m,t+1}^C - N_{m,t+1}^D)}{\text{Var}_t(e_{m,t+1})} \\ &= \beta_{i,CC,t} + \beta_{i,CD,t} + \beta_{i,DC,t} + \beta_{i,DD,t}, \end{aligned} \quad (4)$$

where the individual components of total market risk,  $\beta_{i,CC,t}$ ,  $\beta_{i,CD,t}$ ,  $\beta_{i,DC,t}$ , and  $\beta_{i,DD,t}$ , are defined as:

$$\beta_{i,CC,t} = \frac{\text{Cov}_t(N_{i,t+1}^C, N_{m,t+1}^C)}{\text{Var}_t(e_{m,t+1})}, \beta_{i,CD,t} = \frac{\text{Cov}_t(N_{i,t+1}^C, -N_{m,t+1}^D)}{\text{Var}_t(e_{m,t+1})},$$

and

$$\beta_{i,DC,t} = \frac{\text{Cov}_t(-N_{i,t+1}^D, N_{m,t+1}^C)}{\text{Var}_t(e_{m,t+1})}, \beta_{i,DD,t} = \frac{\text{Cov}_t(-N_{i,t+1}^D, -N_{m,t+1}^D)}{\text{Var}_t(e_{m,t+1})} \quad (5)$$

These “beta-like” ratios in (5) are not the traditional conditional sensitivities used in APT models. These models identify betas to be the univariate slope coefficient of a regression of unexpected returns on the unexpected component (or return) of the risk factor (or factor mimicking portfolio). Rather, the “beta-like” measures of systematic risk in (5) represent the part of total market (CAPM) risk attributed to portfolio and market shocks to time-varying economic fundamentals and shocks to time-varying returns.

## 2.2 Pricing cash-flow and discount rate risk

The approach of decomposing and pricing the sources of systematic risk is not new to the finance literature. Campbell and Mei (1993) decompose the unexpected component of assets’ returns into cash-flow and discount-rate news and examine their covariation with market total unexpected return. However, they do not consider shocks to market portfolio returns in their calculations and thus they work with  $(\beta_{i,CC,t} + \beta_{i,CD,t})$  and  $(\beta_{i,DC,t} + \beta_{i,DD,t})$  as representing aggregate risk quantities. Campbell and Vuolteenaho (2005) examine the opposite story and, while they do not split the full unexpected return on the asset, they decompose the market return innovation into permanent and transitory shocks. Further, using a discrete time variant of Merton’s (1973) I-CAPM (see Campbell (1993, 1996)) they show that their two-beta model performs well in describing the cross section of average returns on size-value and risk-loading sorted portfolios.<sup>2</sup> In a recent paper, Campbell, Polk and Vuolteenaho (2003) decompose the market beta in a similar way to ours and study the properties of sub-betas of value and growth stocks but they do not investigate the asset pricing implications of this decomposition. That is, they do not estimate the individual risk premia associated with the asset-specific return shocks driven by either changing expectations about future fundamentals and/or returns. Filling this gap is the main purpose of the present study.

In order to derive testable restrictions on the premia associated with the cash-flow and discount rate risks in (4) we need a risk story. For this purpose, we employ the

---

<sup>2</sup>These betas correspond to  $\beta_{i,C,t} = \beta_{i,CC,t} + \beta_{i,DC,t}$  and  $\beta_{i,D,t} = \beta_{i,CD,t} + \beta_{i,DD,t}$  in our decomposition in (5).

recursive utility framework provided by Epstein and Zin (1989, 1991) and Weil (1989). The lifetime utility function of the investor is given by the recursive utility function  $U_t$ , defined over current real consumption and future expected utility of real consumption:

$$U_t [C_t, E_t (U_{t+1})] = \left[ (1 - \delta) C_t^{\frac{1-\gamma}{\theta}} + \delta E_t (U_{t+1}^{1-\gamma})^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}}, \quad (6)$$

where  $C_t$  is current real consumption at time  $t$ ,  $0 < \delta < 1$  is the subjective discount factor,  $\gamma > 0$  is the constant coefficient of relative risk aversion (CRRA),  $\theta$  is a parameter defined as  $\theta = (1 - \gamma)/(1 - \sigma^{-1})$ , and  $\sigma > 0$  is the elasticity of intertemporal substitution (EIS) between current and expected future consumption. Equation (6) has the advantage of breaking the tight link between CRRA and EIS given by power utility ( $\gamma = \sigma^{-1}$ ), thus, disconnecting investors' risk attitude across states of nature and across time.<sup>3</sup> The consumer is assumed to finance all her consumption plan entirely from her total real wealth  $W_t$ , given the following dynamic budget constraint:

$$W_{t+1} = (1 + R_{m,t+1})(W_t - C_t), \quad (7)$$

where  $R_{m,t+1}$  is the net real return on total wealth (or the market portfolio,  $m$ ). Epstein and Zin (1989) solve for the optimal portfolio and consumption policies and show that the following set of conditional moment restrictions hold for each asset  $i$ :

$$E_t \left[ \beta^\theta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\sigma}} (1 + R_{m,t+1})^{\theta-1} (1 + R_{i,t+1}) \right] = 1 \quad (8)$$

The above set of non-linear moment restrictions can be linearized using the assumption of joint conditional log-normality of asset returns and consumption in the spirit of Hansen and Singleton (1983). Campbell (1993, 1996) goes one step further and, using these strong assumptions along with the dynamic budget constraint in (7), derives the following cross-sectional linear restrictions on assets' risk premia:<sup>4</sup>

$$E_t [R_{i,t+1}] - R_{f,t+1} = \gamma \text{Cov}_t(e_{i,t+1}, e_{m,t+1}) + (\gamma - 1) \text{Cov}_t(e_{i,t+1}, N_{m,t+1}^D), \quad (9)$$

<sup>3</sup>For a discussion of the properties of this specification see Campbell (2003) and the references therein.

<sup>4</sup>Campbell (1993, 1996) discusses how to handle heteroscedasticity of returns. Equation (8) approximately holds even if returns are heteroskedastic if one assumes that the elasticity of intertemporal substitution is equal to unity. Since our aim is to test the unconditional version of our the model, we employ this assumption (see also Guo (2003)).

which using equation (2) for any individual asset as well as the market portfolio,  $m$ , gives:

$$E_t [R_{i,t+1}] - R_{f,t+1} = \gamma \text{Cov}_t(N_{i,t+1}^C, N_{m,t+1}^C) + \text{Cov}_t(N_{i,t+1}^C, -N_{m,t+1}^D) \\ + \gamma \text{Cov}_t(-N_{m,t+1}^D, N_{m,t+1}^C) + \text{Cov}_t(-N_{i,t+1}^D, -N_{m,t+1}^D) \quad (10)$$

The left part of equations (9) and (10) represent the risk premium in simple returns which are equal to  $E_t [r_{i,t+1}] - r_{f,t+1} + \frac{1}{2} \text{Var}_t(e_{i,t+1})$ ; an expression resulting from the log linearization of the first-order condition in (8). The covariance-risk representation of the equity premium in (10) can have a “beta-like-premium” representation (see, for example, Cochrane (2001)). Multiplying and dividing each conditional covariance term in (10) by the conditional variance of market’s unexpected returns,  $\text{Var}_t(e_{m,t+1})$ , we obtain the following representation for the risk premium on any risky asset  $i$ :

$$E_t [R_{i,t+1}] - R_{f,t+1} = \lambda_{0,t} + \lambda_{CC,t} \beta_{i,CC,t} + \lambda_{CD,t} \beta_{i,CD,t} + \lambda_{DC,t} \beta_{i,DC,t} + \lambda_{DD,t} \beta_{i,DD,t}, \quad (11)$$

where  $\lambda_{0,t}$  represents the conditional Jensen’s alpha, the rest of the  $\lambda$ s represent time-varying prices of beta risks, defined as  $\lambda_{CC,t} = \lambda_{DC,t} = \gamma \text{Var}_t(e_{m,t+1})$  and  $\lambda_{CD,t} = \lambda_{DD,t} = \text{Var}_t(e_{m,t+1})$ , respectively, and the betas are defined similarly to (5). Equation (11) states that the required risk premium on asset  $i$  is jointly determined by the covariances of asset’s shocks to cash flows and discount rates with the corresponding components of the total market innovation. Similarly to Campbell and Vuolteenaho (2005), a conservative risk-averse investor with  $\gamma > 1$  demands a higher risk price for risks associated with market cash flow uncertainty ( $\beta_{i,CC,t}$  and  $\beta_{i,DC,t}$ ) rather than for risks linked to shocks to market returns ( $\beta_{i,CD,t}$  and  $\beta_{i,DD,t}$ ), since any positive (negative) shock to market discount rates is at a benefit (cost) of worse future investment opportunities, whereas the investor is never compensated later for every positive (negative) shock to dividends. Hence, the beta prices of market cash-flow risk,  $\lambda_{CC}$  and  $\lambda_{DC}$ , are a  $\gamma$  multiple of the beta risk prices of market discount-rate risk,  $\lambda_{CD}$  and  $\lambda_{DD}$ , respectively.

We are interested in studying average returns for a long sample of U.S. stock market and macroeconomic data in order to get comparable results to the literature of the unconditional CAPM and, more importantly, to the empirical findings of the two-beta model of Campbell and Vuolteenaho (2005). Using the methodology described in the next section, we proceed with an unconditional version of (11):

$$E [R_{i,t+1}] - R_{f,t+1} = \lambda_0 + \lambda_{CC} \beta_{i,CC} + \lambda_{CD} \beta_{i,CD} + \lambda_{DC} \beta_{i,DC} + \lambda_{DD} \beta_{i,DD} \quad (12)$$



### 3 Data and Empirical Methodology

We study monthly US asset and macroeconomic data from December 1928 to December 2001 (877 monthly observations). Our data consist of different sets of common stock portfolios sorted on various firm-specific characteristics and risk measures, and a set of economy-wide variables that serve as instruments. Following common practice, these variables have been selected under the assumption that they forecast future returns.

The test assets include monthly excess returns on (a) 25 size-BE/ME sorted portfolios from CRSP, corresponding to the Davis, Fama and French (2001) data file, (b) 20 risk-sorted portfolios provided by Campbell and Vuolteenaho (2005),<sup>5</sup> and (c) a set of 10 book-to-market, 10 dividend-price ratio and 10 size sorted portfolios (30 in total). The value-weighted CRSP portfolio serves as the market portfolio of all traded wealth.<sup>6</sup> Although our model in (12) is written in real log returns, we assume that for the monthly test interval we employ, inflation rates are almost fully forecastable, and thus we proxy real log returns with nominal log returns.

Variables that have been successful in predicting the future state of the economy and asset returns are used to generate cash-flows and discount rate news through the VAR specification in (13). Following common practice, we use the following variables: (a) the log excess market return  $r_m - r_f$ , defined as the difference between the log return on the value-weighted CRSP stock index portfolio and the log return on the risk-free rate, constructed by CRSP from T-bills with approximately 3 month maturity, (b) the log price-earnings ratio,  $p - e$ , taken from Shiller (2000) and defined as the log of the S&P 500 index, scaled by the 10-year moving average of aggregate earnings of companies in the S&P 500 index, (c) the term yield spread,  $TY$ , constructed by Global Financial Data and defined as the yield differential between ten-year taxable bonds and short-term taxable notes, and (d) the small-stock value spread,  $VS$ , defined as the difference between the log (BE/ME) of the small high-BE/ME portfolio and the log (BE/ME) of the small low-BE/ME portfolio.<sup>7</sup>

Measuring cash flow news and discount rate news, as the main sources of risk, is central in our methodology. We follow Campbell (1991) and estimate the cash-flow-news and discount-rate-news series using a first-order vector autoregressive (VAR) model. We

---

<sup>5</sup>Campbell and Vuolteenaho (2005) sort common stocks into 20 portfolios according to their past loadings on the market return and innovations on the VAR variables. The purpose of their strategy is to generate portfolios with large spread in these loadings and thus overcome Daniel and Titman's (1997) observation that sorting only on firm characteristics could generate a spurious link between premia and risk measures.

<sup>6</sup>The returns on book-to-market, size, and dividend-price sorted portfolios are available at Kenneth's French web site [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html).

<sup>7</sup>The returns on the 20 risk sorted portfolios and the state variables  $r_m - r_f$ ,  $TY$  and  $VS$  are kindly provided by Tuomo Vuolteenaho and correspond to those used in Campbell and Vuolteenaho (2005).

first estimate expected returns and the revisions in expectations about future returns ( $E_t[r_{t+1}]$  and  $(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho_i^j r_{t+1+j}$ , respectively) and then we use  $r_{t+1}$  and equation (2) to back out the cash-flow news. This practice has an important advantage as it relies only on the dynamics of expected returns and there is no need for modelling the dynamics of dividends. The latter are derived by the VAR estimates and the realizations of returns and state variables.

We assume that the data are generated by the following VAR model:

$$Y_{t+1} = C + AY_t + U_{t+1}, \quad (13)$$

where  $Y_{t+1} = (r_{i,t+1}, Y_{1,t+1}, \dots, Y_{m,t+1})$  is a  $m \times 1$  vector of variables containing returns as its first element and  $(m - 1)$  variables which have predictive power for returns,  $C$  is a  $m \times 1$  vector of constants and  $A$  is a  $m \times m$  matrix of constants. In order to allow for heteroscedasticity of the VAR residuals, we assume that the  $m \times 1$  error vector  $U_t$  is given by:

$$U_t = H_t^{1/2} z_t, \quad z_t \sim \text{i.i.d.}(0, I_m), \quad (14)$$

where  $H_t$  is the conditional covariance matrix and the innovations sequence  $\{z_t\}$  follows an  $m$ -variate standard Gaussian distribution. The conditional covariance matrix,  $H_t$ , is specified as a first-order diagonal BEKK model as suggested by Engle and Kroner (1995):

$$H_t = D'D + MU_{t-1}U'_{t-1}M' + GH_{t-1}G', \quad (15)$$

where  $D$  is a lower triangular  $m \times m$  matrix of constant parameters and  $M$  and  $G$  are diagonal  $m \times m$  matrices of constant parameters. The diagonality of  $M$  and  $G$  ensures that  $h_{jt} = k_j + \mu_j^2 u_{jt-1}^2 + g_j^2 h_{jt-1}$ , for each  $j = 1, \dots, m$ , i.e. the innovations  $u_{jt}$  follow univariate GARCH(1,1) processes. Provided that the data are generated by the process as specified in equations (13)-(15), the standardized residuals vector:

$$z_t = \frac{U_t}{H_t^{1/2}} \sim \text{i.i.d.}(0, I_m)$$

has the property of a multivariate i.i.d. process. We estimate (13)-(15) for the market return and for each individual portfolio return. We then compute cash-flow and discount rate news as linear functions of the  $t + 1$  vector of standardized innovations,  $z_{t+1}$ :

$$N_{t+1}^D = e1'\lambda z_{t+1} \text{ and } N_{t+1}^C = (e1' + e1'\lambda)z_{t+1}, \quad (16)$$

where  $e1$  is a  $m \times 1$  vector with the first element equal to unity and the remaining

elements equal to zero. The mapping of the shock vector to the news vectors is given by  $\lambda \equiv \rho A(I_m - \rho A)^{-1}$ . The term  $e1'\lambda$  in (16) captures the long-run significance of each individual VAR shock to discount-rate expectations. The greater the absolute value of a variable's coefficient in the return prediction equation (the top row of  $A$ ), the greater the weight the variable receives in the discount-rate-news formula. More persistent variables should also receive more weight, which is captured by the term  $(I_m - \rho A)^{-1}$ . Since we use standardized residuals to compute news, the forecasting ability of each economic variable is filtered through the conditional variability derived from the GARCH(1,1) model. As a result, shocks to state variables that are expected to be volatile in the future have high conditional volatility and, hence, are of less importance in the construction of "news" series since the investor judges that these variables are more "risky" as predictive instruments.

## 4 Empirical Evidence

### 4.1 Estimation of News Components for Market Portfolio

Table 1 reports parameter estimates for the market VAR model. Our estimates suggest that the state variables have some predictive power for stock market excess returns. Specifically, monthly market returns display some degree of reversal towards their mean with a statistically significant coefficient of 0.093. The effect of the term yield spread on market returns is positive and significant, a finding consistent with Keim and Stanbaugh (1986), Campbell (1987), Fama and French (1989) and Campbell and Vuolteenaho (2005). The remaining state variables, namely the log price-to-earnings ratio and the small-stock value spread, negatively predict the market return, confirming previous results by e.g. Campbell and Shiller (1988a, 1988b, 1998), Rozeff (1984), Fama and French (1988, 1989), Eleswarapu and Reinganum (2002) and Brennan, Wang and Xia (2001)). The remaining columns of Table 1 summarize the dynamics of the state variables. We do not comment on the remaining equations separately as our estimates coincide with those in Campbell and Vuolteenaho (2005). The last two rows of Table 1 report the ARCH-LM tests for heteroskedasticity in the VAR residuals. The statistics provide evidence for the existence of strong second-order dependence in the error terms.

We model the second moments of the error vector  $U_t$  generated by the VAR model as GARCH (1,1) processes, i.e.,

$$h_{jt} = k_j + \mu_j^2 u_{jt-1}^2 + g_j^2 h_{jt-1} \quad (17)$$

where  $h_{jt}$ ,  $j = 1, \dots, 4$ , is the conditional variance of the  $j^{\text{th}}$  variable's innovations,  $u_{jt}$ ,

and  $k, \mu, g$  are constant parameters. The coefficient  $g_j^2$  measures the extent to which a volatility shock today feeds into next period's volatility, while  $\mu_j^2 + g_j^2$  measures the rate at which this effect dies out over time. By accounting for time-varying volatility, we ensure that the distribution of the error vector  $U_t$ , conditional on its past history, is normal, or, equivalently, the standardized residuals of the GARCH (1,1) models,  $z_{jt} = u_{jt}/\sqrt{h_{jt}}$ , are normal. These normal shocks are then fed into the mapping functions  $e1'\lambda$  and  $e1' + e1'\lambda$ , to retrieve cash-flow and discount rate news. It should be noted that the mapping functions are invariant to the GARCH specification of our model due to the fact that the OLS estimates of the parameters of the VAR model are consistent even in the presence of heteroskedastic errors.

Table 2 reports estimation results of the univariate GARCH(1,1) models for the error vector  $U_t$ . The GARCH parameter estimates ( $\mu_j^2, g_j^2$ ) are highly significant, with  $\mu_j^2 + g_j^2 > 0.95$ , suggesting strong volatility clustering and in some cases nearly integrated GARCH processes. The adequacy of the GARCH (1,1) model is supported by the LM test in the standardized residuals, reported in the last two rows of the table, which rejects any remaining second-order dependence.

Table 3 summarizes the behavior of implied cash-flow news and discount-rate news components of market excess returns. The top panel shows that the standard deviation of discount rate news is twice the standard deviation of cash-flow news. This finding is consistent with Campbell (1991) and Campbell and Vuolteenaho (2005). However, in contrast to Campbell and Vuolteenaho (2005), but in line with Campbell (1991 and 1996), the two components of return exhibit some degree of correlation (0.621). The bottom panel of Table 3 reports correlations of cash-flow and discount-rate news with innovations in market excess returns and state variables. Discount-rate and cash-flow news are negatively correlated with innovations in the market excess return, the price-earnings and value spread. In contrast, innovations to the term spread are uncorrelated with discount rate and cash-flow news.

## 4.2 Estimation of Stock-Specific News Components and Betas

The VAR-GARCH methodology presented in Section 2 has been applied to every single portfolio under consideration, using the same economy-wide state variables, in order to extract portfolio-specific cash-flow and discount rate news. Since data on dividend yields of individual portfolios are not available to us, we follow Campbell and Mei (1993), and proxy individual discount factors,  $\rho_i$  in equation (2), with the full-sample estimate of the discount factor of the market portfolio,  $\bar{\rho}_m = 0.9957$ .<sup>8</sup>

---

<sup>8</sup>We do not report VAR estimates for individual portfolios. These results are available upon request.

The standardized innovations of the state variables are used to study the systematic risks and their relationship with average returns on portfolios of common stocks sorted on firm characteristics and risk. Empirical measures of the cash flow and discount rate betas in (4) are derived using a methodology similar to this employed in Campbell and Vuolteenaho (2005) to ensure that our sample estimates are not affected by non-synchronous trading (especially in the early years of our sample) and under-reaction of stock prices to changes in the market index (especially for large stocks).<sup>9</sup> Our four sample betas, that will be used in the cross-sectional regressions, are defined as the “sum” of contemporaneous, one lag and two lags of the full-sample covariances of portfolio news at  $t + 1$  with market news, divided by the time  $t + 1$  full-sample variance of standardized market return innovations,  $\widehat{\text{Var}}(z_{m,t+1})$ . For example, the betas associated with shocks to assets’ cash-flows and revisions in market fundamentals in (5) are estimated as follows:

$$\hat{\beta}_{i,CC} = \frac{\widehat{\text{Cov}}(N_{i,t+1}^C, N_{m,t+1}^C)}{\widehat{\text{Var}}(z_{m,t+1})} + \frac{\widehat{\text{Cov}}(N_{i,t+1}^C, N_{m,t}^C)}{\widehat{\text{Var}}(z_{m,t+1})} + \frac{\widehat{\text{Cov}}(N_{i,t+1}^C, N_{m,t-1}^C)}{\widehat{\text{Var}}(z_{m,t+1})}, \quad (18)$$

and all the remaining betas in (5) are estimated accordingly.

### 4.3 The Cross-Section of Cash-Flow and Discount-Rate Risks

Tables 4 and 5 report the estimated betas given our definition in (18) for the 25 double sorted portfolios according to size (market value) and book-to-market, and the set of the 30 size, BE/ME and dividend-price ratio sorted portfolios. The main characteristic of our results is that our methodology generates almost no spread in the overall market risk  $\beta_{i,m}$  (the sum of individual cash-flow and discount-rate betas defined in (4)) as the literature on the failure of the static CAPM argues. However, in all sub-betas there is a considerable spread (both in single- and double-sorted portfolios), indicating the conflicting role of cash-flow and discount-rate risk in explaining the cross-sectional predictability in average returns.

The observed spread in the two aggregate “bad” (cash-flow) and “good” (discount-rate) betas confirm the story argued by Campbell and Vuolteenaho (2005) that value stocks have relatively high cash-flow betas while growth stocks have relatively high discount-rates betas – see Panels E and F of Table 4. The difference between value and growth cash-flow betas ( $\beta_{i,C}$ ) ranges from 0.025 to 0.09 (from the smallest to the largest decile) while at the same time the difference in discount-rate betas ( $\beta_{i,D}$ ) ranges from -0.09 to -0.151. Most importantly, most cross-sectional variation in market bad

---

<sup>9</sup>See Scholes and Williams (1977) and Dimson (1979) for the effects of non-synchronous trading and McQueen, Pinegar and Thorly (1996) and Peterson and Sanger (1995) for the under-reaction pattern of stock prices.

risk comes from the cross-sectional variation in portfolio economic fundamentals rather than in revisions in expectations about future discount factors. However, there is no clear spread in the two components of relatively good risk. While for the fundamental component of market total discount-rate risk of growth stocks (with the exception of the small deciles) there is a positive difference with that of value stocks (differences across value range from -0.409 to -1.277), for the discount-rate component ( $\beta_{i,DD}$ ) the results are mixed.

Table 5 illustrates the estimates of the beta decomposition for three sets of 10 portfolios sorted on size, BE/ME and D/P respectively. In all cases, and although there is a considerable spread in all risks, we observe that the estimated sensitivities of portfolios' cash flows with both market cash flows and discount rates exhibit greater spreads as compared to the sensitivities of portfolios' discount rates with both market cash flows and discount rates. Thus, the observation for value stocks, originally made by Campbell and Vuolteenaho (2005) and Campbell, Polk and Vuolteenaho (2003), is also quite clear in our calculations and, furthermore, we provide evidence on the importance of the beta decomposition using value, size and dividend-price ratio single-sorted portfolios.

#### 4.4 Are Asset-Specific Cash-Flow and Discount-Rate Risks Priced?

Having estimated the full-sample cash-flow and discount rate betas given our specification of the return and variance generating processes in (13) and (17), respectively, we proceed with cross-sectional asset pricing tests to evaluate the ability of our four-beta model to capture cross-sectional variation in average portfolio returns. We study the unconditional asset pricing model in (12) and we assume that the market portfolio is a good proxy for the total wealth portfolio in the economy.

The model is tested against the traditional CAPM (where only the full market beta,  $\beta_{i,m}$ , matters) and the two-beta (both  $\beta_{i,C}$  and  $\beta_{i,D}$  matter) I-CAPM model recently developed by Campbell and Vuolteenaho (2005). We consider the following cross-sectional regression for our four-beta model:

$$E_T [R_i^e] = \lambda_0 + \lambda_{CC}\hat{\beta}_{i,CC} + \lambda_{CD}\hat{\beta}_{i,CD} + \lambda_{DC}\hat{\beta}_{i,DC} + \lambda_{DD}\hat{\beta}_{i,DD}, \text{ for } i = 1, \dots, N, \quad (19)$$

and we test our specification against the popular static, single-beta, CAPM:

$$E_T [R_i^e] = \lambda_0 + \lambda_m\hat{\beta}_{i,m}, \text{ for } i = 1, \dots, N, \quad (20)$$

and the two-beta I-CAPM:

$$E_T [R_i^e] = \lambda_0 + \lambda_C \hat{\beta}_{i,C} + \lambda_D \hat{\beta}_{i,D}, \text{ for } i = 1, \dots, N, \quad (21)$$

In all models (19) to (21),  $E_T [R_i^e]$  denotes the full-sample estimate of the mean risk premium defined as the sample mean return on each portfolio in excess of the risk-free interest rate. We estimate the unconditional unrestricted prices of beta risks for all models (“factor models”) as well as the following restricted version of the four-beta model in (19):

$$E_T [R_i^e] = \lambda_0 + \gamma \lambda \hat{\beta}_{i,CC} + \lambda \hat{\beta}_{i,CD} + \gamma \lambda \hat{\beta}_{i,DC} + \lambda \hat{\beta}_{i,DD}, \text{ for } i = 1, \dots, N \quad (22)$$

This last version enables us to estimate the coefficient of relative risk aversion  $\gamma$  and test for the theoretically implied equality across risk prices associated with cash-flow and discount-rate betas ( $\lambda_{CC} = \lambda_{DC}$  and  $\lambda_{CD} = \lambda_{DD}$ ).

Panels A to D of Table 6 present the empirical findings. For each test, the table reports the mean, standard error and  $t$ -statistic for each estimate, as well as the adjusted  $R^2$  of the regression. Also, we conduct an  $F$ -test that all the coefficients except the constant,  $\lambda_0$ , are jointly equal to zero and we report the value and the  $p$ -value of the test. For the two-factor and four-factor models we run the regressions in two steps. First, we regress average excess returns on a constant and the two and four betas respectively. The results are illustrated in the second and fourth column in each table. Given that the asset pricing restriction implies that the average pricing error in all models (19) to (21) must be equal to zero (under the null hypothesis that the model is correctly specified and the sources of risk (i.e. betas) provide a full description of the cross-sectional variation in average returns), we conduct a Wald test that  $\lambda_0$  and the less statistically significant premium are jointly zero. If the test rejects the null hypothesis, we re-estimate the regression ignoring the constant given that the price of beta risk under consideration gets a lower  $p$ -value. The results of these experiments appear in the third and fifth column in all tables. Finally, for the four-beta model in (22), we report a  $\chi^2$  statistic that tests for equality across premia as well as the estimated value (along with the  $p$ -value) of the coefficient of relative risk aversion,  $\gamma$ , and  $\lambda$ .

Panel A reports the empirical findings for the 25 size-BE/ME double sorted common stock portfolios. Similarly to Fama and French (1992), the traditional static CAPM performs poorly and explains almost none of the cross-sectional variation in average returns resulting a low adj.- $R^2$  equal to 3.4% and a highly significant average pricing error equal to 0.029 per month ( $\hat{t} = 1.901$ ). We then ask whether the two-beta and four beta decompositions in (19) and (21) with unrestricted prices of beta risk can improve the empirical validity of the standard static CAPM and it is clear that they both

do. The two-factor model performs quite well and generates insignificant pricing error ( $\hat{\lambda}_0 = -0.003, \hat{t} = -0.229$ ) and statistically significant premia with the premium associated with market cash flow risk being considerably higher than the premium associated with market's discount rate risk ( $\hat{\lambda}_C = 0.077$  and  $\hat{\lambda}_D = 0.013$  with  $\hat{t} = 5.296$  and  $14.233$  respectively). A high adj.- $R^2$  of 42.4% shows that much of the cross-sectional variation in average returns is explained. These results are in line with Campbell and Vuolteenaho (2005). The four-factor I-CAPM model performs even better. When the highly insignificant constant  $\lambda_0$  ( $\hat{t} = -0.121$ ) is removed, the model in its restricted version yields a highly statistically significant and economically reasonable estimate for the RRA coefficient ( $\hat{\gamma} = 5.755$  with  $\hat{t} = 7.878$ ) and a higher adj.- $R^2$  of almost 55%. Also, the model yields the predicted difference between the level of risk prices for the components of market cash-flow and discount-rate risk: the premia associated with market cash-flows ( $\lambda_{CC}$  and  $\lambda_{DC}$ ) are 5 to 6 times higher than those associated with market discount rates ( $\lambda_{CD}$  and  $\lambda_{DD}$ ). However, we can not establish a clear statistical relationship of equality between the two pairs since the equality hypothesis  $\lambda_{CC} = \lambda_{DC}$  cannot be rejected at the low 2% level of significance and the equality hypothesis  $\lambda_{CD} = \lambda_{DD}$  is rejected even for lower levels of significance.

Panel B of Table 6 reports our model estimates for three sets of 10 portfolios sorted according to BE/ME, D/P and market value. This experiment with single-sorted portfolios may provide us with better empirical evidence on the observed pattern of mean excess returns on value and size portfolios and the cross-sectional variation in fundamental and discount-rate risks. Our model again improves the ability of the disappointing static CAPM and the well performing two-beta I-CAPM to capture the spread in mean asset premia. The proportion of cross-sectional variability explained increases from 46.4% (two-beta model) to an impressive 83.1%, while the pricing error is still highly insignificant ( $\hat{t} = -0.566$ ). Most importantly, and even when the insignificant constant is included in the regression, all the slope coefficients (except  $\lambda_{CD}$  ( $\hat{t} = 1.69$ )) are significant, indicating that the approach of decomposing cash flow and discount rate market risks yields interesting insights for the determination of average risk premia. Once  $\lambda_0$  is removed, all four risk prices are highly significant and the high estimated values for  $\lambda_{CC}$  and  $\lambda_{DC}$  (0.019 and 0.023, respectively) provide further support on the results presented by Campbell and Vuolteenaho (2005) and Campbell, Polk and Vuolteenaho (2003). They argued that value and small stocks have considerably higher cash-flow betas than large and growth stocks and this can explain their high average returns. We extend their results by showing that the sign and magnitude of our estimated beta-risk prices of the decomposed cash-flow market risk are in line with a rational asset pricing model for a long-lived conservative investor. This investor requires a higher premium per unit of market cash-flow risk than



for market discount-rate risk. Further, the factor of proportionality that is restricted to be equal to the coefficient of relative risk aversion is both economically and statistically significant ( $\hat{\gamma} = 5.304, \hat{t} = 9.713$ ). For this group of portfolios, although we again cannot reject the equality hypothesis for the market discount rate premia  $\lambda_{CD}$  and  $\lambda_{DD}$ , we can safely accept it for the cash flow premia ( $\chi^2 = 0.846, p = 0.357$ ). Overall, for our four-beta specification in (22) the spread in size and value portfolios seems to be not puzzling.

We also test the empirical validity of our four-factor model using the 25 BE/ME portfolios as well as 20 risk portfolios sorted on market betas and betas associated with innovations to the state variables.<sup>10</sup> The approach of sorting stocks according to past risk rather than firm-specific characteristics can gauge the impact of data snooping on empirical findings that reveal relationships between characteristic-sorted portfolio trading strategies and average returns. Panel C of Table 6 shows that the static CAPM still performs badly and generates a very low adj.- $R^2$  of  $-0.9\%$ , statistically significant pricing errors ( $\hat{\lambda}_0 = 0.014$  with  $\hat{t} = 1.665$ ) and an insignificant and economically rejected point estimate for the market premium ( $\hat{\lambda}_m = -0.007, \hat{t} = -0.769$ ). Thus, market beta, as a single aggregate risk measure, fails to capture the cross-sectional spread in returns. The four-beta model captures a large part of the cross-sectional average return variation and, compared to the two-beta specification, impressively increases the percentage of explained cross-sectional return variability from 44% to almost 61%. It produces even more insignificant pricing errors and results in a significant CRRA value ( $\hat{\gamma} = 5.788, \hat{t} = 11.130$ ). It is interesting that the observed pattern in cash-flow and discount-rate prices of risk is in line with our previous tests: still, risk prices associated with the two components of market cash-flow risk ( $\hat{\lambda}_{CC} = 0.062, \hat{\lambda}_{DC} = 0.072$ ) are much higher than the risk prices associated with market discount-rate risk ( $\hat{\lambda}_{CD} = 0.008, \hat{\lambda}_{DD} = 0.013$ ).

For experimental reasons we include the 5 sets of portfolios (25 size-BE/ME, 20 risk, 10 BE/ME, 10 D/P and 10 size sorted) in one cross-sectional regression. Panel D in Table 6 illustrates the results. Similarly to the previous empirical findings, the market overall beta,  $\beta_{im}$ , explains none of the cross-sectional variation in average returns. The two-beta model again results insignificant pricing errors and significant risk prices for the aggregate market cash-flow and discount-rate risk with the latter being much higher (0.068 and 0.012 respectively) as predicted by economic theory. However, our four-beta specification in (22) increases the ability of the two-beta model by more than 20% in terms of explanatory power. The point estimate of the CRRA is significant, economically acceptable and similar to the one generated from the previous samples ( $\hat{\gamma} = 5.594, \hat{t} = 14.640$ ). However,

---

<sup>10</sup>For recent studies that relates loadings of unexpected returns on innovations of state variables and the global size and book-to-market premia of the Fama-French (1996) three factor model see Petkova (2002) and the references therein.

as in all previous tests, we cannot accept the hypothesis that the risk prices concerning markets cash-flows and market discount-rate risks are equal.

## 5 Conclusions

This paper builds on the decomposition of the overall market, or CAPM, risk into parts reflecting time variation related to the dynamics of portfolio-specific and aggregate market cash flows and discount rates. Extending the methodology of Campbell (1991), Campbell and Mei (1993), Campbell and Vuolteenaho (2005) and Campbell, Polk and Vuolteenaho (2003) we decompose market betas into four sub-betas, two associated with market cash-flows and two with market discount-rates. The approach used attempts to fill the gap between the time-series predictability of returns and the cross-sectional variation in average returns due to differences in risk. Using a VAR-GARCH(1,1) approach and a discrete time version of Merton's I-CAPM, we ask whether these parts of overall risk related to innovations to state variables that are related to changes in expectations about future dividends and future returns are rationally priced.

Our four-beta model performs well in pricing average returns on single- and double-sorted portfolios according to market capitalization, book-to-market, dividend-price ratios and past risk, by producing insignificant pricing errors, high estimates for the explained cross-sectional variation (which in many cases exceeds 80%) in average monthly returns and both economically and statistically acceptable estimates for the coefficient of relative risk aversion (values range between 5 and 6). We find that the risks associated with permanent shocks to market returns, as these are described by the two market cash-flow betas, earn higher unconditional risk prices compared to the risk prices associated with market discount-rate risks, but all four components are required in order to improve the ability of the static CAPM to capture the cross-sectional variation of mean premia.

## References

- [1] Bollerslev Tim (1986), Generalized autoregressive conditional heteroskedasticity, *Journal of Econometrics* 31, 307-328.
- [2] McCurdy Thomas and Morgan (1998), Testing the martingale hypothesis in Deutchmark futures with models specifying the form of heteroskedasticity, *Journal of Applied Econometrics* 3, 187-202.
- [3] Brennan Michael, Ashley Wang and Yihong Xia (2001), A simple model of intertemporal capital asset pricing model and its implications for the Fama-French three factor model, unpublished paper, UCLA
- [4] Campbell John (1991), A variance decomposition of stock returns, *Economic Journal*, 101, 157-159
- [5] Campbell, John (1993), Intertemporal asset pricing without consumption data, *American Economic Review* 83, 487-512.
- [6] Campbell John (1996), Understanding risk and return, *Journal of Political Economy*, 104, 298-345.
- [7] Campbell John (2002), Consumption based asset pricing, unpublished paper, Harvard University.
- [8] Campbell John and Jiapping Mei (1993), Where do betas come from? Asset pricing dynamics and the sources of systematic risk, *Review of Financial Studies* 6, 567-592
- [9] Campbell, John, Christofer Polk and Tuomo Vuolteenaho (2003), Growth or glamour, unpublished paper, Harvard University.
- [10] Campbell, John and Robert Shiller (1988a), The dividend-price ratio and expectations about future dividends and discount factors, *Review of Financial Studies* 1, 195-228
- [11] Campbell, John and Robert Shiller (1988a), Stock prices, earnings and expected dividends, *Journal of Finance*, 43, 661-676.
- [12] Campbell, John and Robert Shiller (1998b), Valuation ratios and the long-run stock market outlook, *Journal of Portfolio Management* 24 (2), 11-26.
- [13] Campbell, John and Tuomo Vuolteenaho (2005), Bad beta good beta, forthcoming *American Economic Review*.
- [14] Chako, George and Luis Viceira (1999), Dynamic consumption and portfolio choice with stochastic volatility in incomplete markets, NBER working paper no. 7377.
- [15] Chen, Joseph (2003), Intertemporal CAPM and the cross-section of stock returns, unpublished paper, USC.
- [16] Cochrane, John (2001), *Asset Pricing*, Princeton University Press, Princeton NJ.

- [17] Daniel, Kent and Sheridan Titman (1997), Evidence on the characteristics of cross-sectional variation in stock returns, *Journal of Finance*, 52, 1-33.
- [18] Davis James, Eugene Fama and Kenneth French (2000), Characteristics, covariances and average returns: 1929-1997, *Journal of Finance* 55, 389-406.
- [19] Dimson, Elroy (1979), Risk measurement when shares are subject to infrequent trading, *Journal of Financial Economics* 7, 197-226.
- [20] Eleswarapu, Venkat and Marc Reinganum (2002), The predictability of aggregate stock market returns: evidence based on glamour stocks, forthcoming *Journal of Business*.
- [21] Epstein, Lawrence and Stanley Zin (1989), Substitution, risk aversion and the temporal behavior of consumption and asset returns: a theoretical framework, *Econometrica* 57, 937-969.
- [22] Epstein, Lawrence and Stanley Zin (1991), Substitution, risk aversion and the temporal behavior of consumption and asset returns: an empirical investigation, *Journal of Political Economy* 99, 263-286.
- [23] Fama, Eugene and Kenneth French (1988), Dividend yields and expected stock returns, *Journal of Financial Economics* 22, 3-27.
- [24] Fama, Eugene and Kenneth French (1989), Business conditions and expected returns on stocks and bonds, *Journal of Financial Economics* 25, 23-49.
- [25] Fama, Eugene and Kenneth French (1992), The cross-section of expected stock returns, *Journal of Finance* 2, 427-465.
- [26] Fama, Eugene and Kenneth French (1993), Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3-56.
- [27] Fama, Eugene and Kenneth French (2004), The Capital Asset Pricing Model: Theory and Evidence, unpublished paper, University of Chicago.
- [28] Guo, Hui (2003), Time-varying risk premia and the cross-section of stock returns, unpublished paper, Federal Reserve bank of Saint Louis.
- [29] Hansen and Singleton (1983), Stochastic Consumption, Risk Aversion and the Temporal Behavior of Asset Returns, *Journal of Political Economy* 91, 249-268.
- [30] Lintner, John (1965), The valuation of risky assets and the selection of risky investments in stock portfolios and capital budgets, *Review of Economics and Statistics* 47, 13-37.
- [31] McQueen, Grant, Michael Pinegar and Steven Thorley (1996), Delayed reaction to good news and the cross-autocorrelation of portfolio returns, *Journal of Finance* 51, 889-919

- [32] Merton, Robert (1973), An intertemporal capital asset pricing model, *Econometrica* 41, 867-887.
- [33] Peterson James and Gary Sanger (1995), Cross-autocorrelations, systematic risk and the period of listing, unpublished paper, University of Notre Dame.
- [34] Petkova, Ralitsa (2002), Do do the Fama-French factors proxy for innovations in predictive variables?, unpublished paper, University of Rochester.
- [35] Rozeff (1984), Dividend Yields are Equity Risk Premiums, *Journal of Portfolio Management* 10, 68-75
- [36] Scholes , Myron and Joseph Williams (1977), Estimating betas from nonsynchronous data, *Journal of Financial Economics* 5, 309-327.
- [37] Sharpe, William (1964), Capital asset prices: a theory of market equilibrium under conditions of risk, *Journal of Finance* 19, 425-442
- [38] Shiller, Robert (2000), *Irrational Exuberance*, Princeton University Press, Princeton, N.J.

**Table 1. VAR estimates for market portfolio**

Each column of the table corresponds to a different equation of the VAR model. The first five rows report coefficients on the three state variables plus a constant and lagged values of the excess market return. OLS standard errors are reported in parentheses below coefficients. The table also reports the  $R^2$  and  $F$  statistics for each equation and a Lagrange Multiplier test for heteroscedasticity of the VAR residuals up to four lags.

	$r_{m,t+1}$	$TY_{t+1}$	$p_{t+1} - e_{t+1}$	$VS_{t+1}$
constant	0.062 (0.020)	0.046 (0.097)	0.019 (0.013)	0.014 (0.017)
$r_{m,t}$	0.093 (0.034)	0.033 (0.165)	0.519 (0.022)	-0.008 (0.029)
$TY_t$	0.006 (0.003)	0.880 (0.016)	0.002 (0.002)	0.002 (0.003)
$p_t - e_t$	-0.015 (0.005)	-0.036 (0.026)	0.994 (0.004)	0.000 (0.005)
$VS_t$	-0.012 (0.006)	0.082 (0.028)	-0.003 (0.004)	0.991 (0.005)
$R^2$	2.6%	82.4%	99.1%	98.4%
$F$ -stat.	5.713	1020.446	22898.7	13400.13
LM Test for Heteroscedasticity (ARCH Test: lag = 4)				
	$\hat{u}_{r_m}$	$\hat{u}_{TY}$	$\hat{u}_{p-e}$	$\hat{u}_{VS}$
$F$ -stat.	25.071	24.652	13.264	22.841
$p$ -value	[0.000]	[0.000]	[0.000]	[0.000]

**Table 2: Estimates of univariate GARCH(1,1) models of market portfolio VAR innovations**

The table reports estimates of GARCH(1,1) models for the conditional variance of VAR innovations for the market portfolio, equation (17). Column 2 refers to innovations of market excess returns and columns 3-5 to innovations of the yield-spread ( $TY$ ), the log price-earnings ratio ( $p - e$ ) and the value spread ( $VS$ ), respectively. The last rows report the results of a Lagrange Multiplier test for heteroscedasticity of the standardized residuals,  $z_{jt} = u_{jt}/\sqrt{h_{jt}}$ , up to four lags. Robust standard errors are reported in parentheses below coefficients. Probability values are given in brackets.

Parameter	$h_{r_m,t}$	$h_{TY,t}$	$h_{p-e,t}$	$h_{VS,t}$
$k_j$	$5.75e - 05$ ( $1.98e - 05$ )	$9.39e - 05$ ( $4.11e - 05$ )	$3.08e - 05$ ( $9.76e - 06$ )	$0.000$ ( $1.76e - 05$ )
$\mu_j^2$	$0.107$ ( $0.017$ )	$0.135$ ( $0.019$ )	$0.082$ ( $0.017$ )	$0.048$ ( $0.007$ )
$\delta_j^2$	$0.877$ ( $0.017$ )	$0.805$ ( $0.014$ )	$0.892$ ( $0.022$ )	$0.905$ ( $0.014$ )
LM Test for Heteroscedasticity (ARCH Test: lag = 4)				
	$\hat{z}_{r_m}$	$\hat{z}_{TY}$	$\hat{z}_{p-e}$	$\hat{z}_{VS}$
$F$ -stat.	$0.331$	$0.890$	$0.586$	$0.223$
$p$ -value	[ $0.857$ ]	[ $0.469$ ]	[ $0.673$ ]	[ $0.926$ ]

**Table 3: Market portfolio cash-flow and discount-rate news**

The table reports the estimated covariance matrix (upper-left) and the correlation matrix with st. dev. (upper-right) of estimated market portfolio cash-flow and discount rate news using equations (13) to (16), the correlations of innovations of state variables with market news (lower-left) and the mapping functions defined in (16).

Covariance matrix of news			News corr/st.d.	
	$N_m^C$	$N_m^D$		
$N_m^C$	0.385	0.486	$N_m^C$	0.621
$N_m^D$	0.486	1.590	$N_m^D$	1.262
Correlations of innovations with news			Functions	
Innovations/News	$N_m^C$	$N_m^D$		
$r_m - r_f$	-0.162	-0.874	$r_m - r_f$ shock	0.599
$TY$	0.039	0.011	$TY$ shock	0.010
$p - e$	-0.692	-0.953	$p - e$ shock	-0.889
$VS$	-0.377	-0.221	$VS$ shock	-0.263



**Table 4. Cash-flow and discount-rate betas for 25 book-to-market portfolios**

The table reports sample estimates of cash-flow and discount rate betas for the 25 size-BE/ME portfolios defined in (4) and calculated according to (18). Estimates for the “bad” and “good” betas are defined as  $\hat{\beta}_{i,C} = \hat{\beta}_{i,CC} + \hat{\beta}_{i,DC}$  and  $\hat{\beta}_{i,D} = \hat{\beta}_{i,CD} + \hat{\beta}_{i,DD}$  and the estimates for the full market beta are defined as the sum  $\hat{\beta}_{i,m} = \hat{\beta}_{i,CC} + \hat{\beta}_{i,DC} + \hat{\beta}_{i,CD} + \hat{\beta}_{i,DD}$ . “Diff.” refers to the difference between the extreme cell.

Panel A. Cash-flow cash-flow betas ( $\hat{\beta}_{i,CC}$ )						
	Growth	2	3	4	Value	Diff.
Small	0.753	0.732	0.476	0.409	0.641	-0.112
2	0.408	0.335	0.398	0.497	0.690	0.282
3	0.370	0.348	0.335	0.453	0.582	0.212
4	0.235	0.316	0.422	0.395	0.694	0.460
Large	0.139	0.242	0.368	0.418	0.471	0.332
Diff.	0.614	0.490	0.108	-0.009	0.169	

  

Panel B. Cash-flow discount-rate betas ( $\hat{\beta}_{i,CD}$ )						
	Growth	2	3	4	Value	Diff.
small	-1.711	-1.397	-1.273	-1.107	-1.640	0.071
2	-1.141	-0.936	-1.003	-1.258	-1.549	-0.409
3	-1.205	-0.787	-0.673	-0.931	-1.218	-0.013
4	-0.475	-0.694	-0.817	-0.774	-1.435	-0.960
Large	-0.069	-0.200	-0.401	-0.683	-1.347	-1.277
Diff.	-1.642	-1.197	-0.873	-0.424	-0.293	

  

Panel C. Discount-rate cash-flow betas ( $\hat{\beta}_{i,DC}$ )						
	Growth	2	3	4	Value	Diff.
Small	-0.818	-0.802	-0.527	-0.461	-0.681	0.137
2	-0.505	-0.413	-0.459	-0.541	-0.727	-0.222
3	-0.473	-0.421	-0.379	-0.489	-0.607	-0.134
4	-0.335	-0.382	-0.461	-0.423	-0.719	-0.384
Large	-0.252	-0.325	-0.412	-0.438	-0.493	-0.242
Diff.	-0.566	-0.477	-0.115	-0.023	-0.187	

  

Panel D. Discount-rate discount-rate betas ( $\hat{\beta}_{i,DD}$ )						
	Growth	2	3	4	Value	Diff.
Small	2.696	2.409	2.271	2.075	2.592	-0.104
2	2.192	1.959	1.972	2.194	2.492	0.300
3	2.215	1.809	1.632	1.876	2.113	-0.102
4	1.503	1.679	1.789	1.717	2.373	0.871
Large	1.106	1.231	1.315	1.562	2.232	1.126
Diff.	1.589	1.178	0.957	0.513	0.360	

Panel E. Cash-flow betas ( $\hat{\beta}_{i,C}$ )

	Growth	2	3	4	Value	Diff.
Small	-0.065	-0.071	-0.051	-0.052	-0.040	0.025
2	-0.097	-0.078	-0.061	-0.044	-0.037	0.059
3	-0.103	-0.073	-0.044	-0.036	-0.025	0.078
4	-0.100	-0.067	-0.039	-0.028	-0.024	0.076
Large	-0.112	-0.083	-0.044	-0.020	-0.022	0.090
Diff.	0.047	0.012	-0.007	-0.031	-0.018	

Panel F. Discount-rate betas ( $\hat{\beta}_{i,D}$ )

	Growth	2	3	4	Value	Diff.
Small	0.985	1.012	0.998	0.968	0.952	-0.032
2	1.052	1.023	0.969	0.935	0.943	-0.109
3	1.010	1.023	0.959	0.945	0.895	-0.115
4	1.028	0.985	0.971	0.943	0.938	-0.090
Large	1.037	1.031	0.914	0.879	0.886	-0.151
Diff.	-0.052	-0.019	0.084	0.088	0.067	

Panel G. Market betas ( $\hat{\beta}_{i,m}$ )

	Growth	2	3	4	Value	Diff.
Small	0.919	0.941	0.947	0.916	0.912	-0.007
2	0.955	0.945	0.908	0.891	0.906	-0.049
3	0.907	0.950	0.915	0.909	0.870	-0.037
4	0.928	0.918	0.932	0.914	0.914	-0.014
Large	0.924	0.948	0.870	0.859	0.864	-0.061
Diff.	-0.005	-0.006	0.076	0.057	0.049	

**Table 5. Cash-flow and discount-rate betas for 30 size, BE/ME and D/P sorted portfolios**

The table reports sample estimates of cash-flow and discount rate betas for the 30 size, BE/ME and D/P portfolios defined in (4) and calculated according to (18). Estimates for the “bad” and “good” betas are defined as  $\hat{\beta}_{i,C} = \hat{\beta}_{i,CC} + \hat{\beta}_{i,DC}$  and  $\hat{\beta}_{i,D} = \hat{\beta}_{i,CD} + \hat{\beta}_{i,DD}$  and the estimates for the full market beta defined as the sum  $\hat{\beta}_{i,m} = \hat{\beta}_{i,CC} + \hat{\beta}_{i,DC} + \hat{\beta}_{i,CD} + \hat{\beta}_{i,DD}$ . “Diff.” refers to the difference between the extreme cell and “St. dev.” refers to the sample estimate of the standard deviation of the estimated betas.

Panel A. 10 book-to-market portfolios

	$\hat{\beta}_{i,m}$	$\hat{\beta}_{i,C}$	$\hat{\beta}_{i,D}$	$\hat{\beta}_{i,CC}$	$\hat{\beta}_{i,CD}$	$\hat{\beta}_{i,DC}$	$\hat{\beta}_{i,DD}$
Growth	0.906	-0.113	1.019	0.145	-0.076	-0.258	1.095
2	0.953	-0.104	1.056	0.178	-0.222	-0.282	1.278
3	0.984	-0.089	1.074	0.254	-0.293	-0.343	1.366
4	0.887	-0.069	0.956	0.329	-0.565	-0.397	1.521
5	0.920	-0.058	0.978	0.349	-0.453	-0.406	1.431
6	0.886	-0.036	0.923	0.397	-0.508	-0.434	1.430
7	0.894	-0.030	0.924	0.384	-0.704	-0.414	1.628
8	0.893	-0.029	0.922	0.428	-0.777	-0.457	1.699
9	0.896	-0.025	0.920	0.542	-1.243	-0.567	2.163
Value	0.901	-0.024	0.925	0.768	-1.703	-0.792	2.628
Diff.	-0.006	0.088	-0.094	0.623	-1.626	-0.534	1.532
St.dev.	0.032	0.034	0.060	0.181	0.493	0.153	0.454

Panel B. 10 dividend-price portfolios

	$\hat{\beta}_{i,m}$	$\hat{\beta}_{i,C}$	$\hat{\beta}_{i,D}$	$\hat{\beta}_{i,CC}$	$\hat{\beta}_{i,CD}$	$\hat{\beta}_{i,DC}$	$\hat{\beta}_{i,DD}$
Low	0.954	-0.104	1.059	0.223	-0.096	-0.328	1.155
2	0.949	-0.109	1.058	0.118	-0.084	-0.228	1.143
3	0.939	-0.084	1.023	0.182	0.095	-0.266	0.928
4	0.905	-0.063	0.968	0.223	-0.200	-0.286	1.168
5	0.881	-0.053	0.934	0.259	-0.252	-0.312	1.186
6	0.881	-0.047	0.928	0.386	-0.767	-0.433	1.695
7	0.924	-0.043	0.967	0.388	-0.777	-0.431	1.745
8	0.896	-0.043	0.939	0.438	-1.015	-0.481	1.954
9	0.823	-0.028	0.851	0.387	-0.873	-0.415	1.724
High	0.740	-0.024	0.764	0.411	-1.027	-0.435	1.791
Diff.	-0.214	0.080	-0.295	0.187	-0.931	-0.107	0.636
St.dev.	0.065	0.030	0.091	0.113	0.431	0.088	0.364

Panel C. 10 size portfolios

	$\hat{\beta}_{i,m}$	$\hat{\beta}_{i,C}$	$\hat{\beta}_{i,D}$	$\hat{\beta}_{i,CC}$	$\hat{\beta}_{i,CD}$	$\hat{\beta}_{i,DC}$	$\hat{\beta}_{i,DD}$
1	0.918	-0.045	0.963	0.528	-1.611	-0.573	2.574
2	0.945	-0.055	1.000	0.509	-1.330	-0.564	2.331
3	0.923	-0.066	0.989	0.497	-1.249	-0.563	2.238
4	0.937	-0.069	1.006	0.446	-1.099	-0.515	2.104
5	0.948	-0.069	1.017	0.435	-1.020	-0.504	2.036
6	0.940	-0.067	1.007	0.397	-0.907	-0.464	1.913
7	0.969	-0.064	1.033	0.380	-0.746	-0.444	1.779
8	0.961	-0.073	1.034	0.359	-0.700	-0.432	1.734
9	0.962	-0.071	1.033	0.328	-0.554	-0.399	1.586
10	0.973	-0.091	1.064	0.205	-0.118	-0.296	1.181
Diff.	-0.055	0.046	-0.101	0.323	-1.493	-0.277	1.392
St.dev.	0.019	0.012	0.028	0.098	0.428	0.087	0.401

**Table 6. Cross-sectional regressions of average premia on cash-flow and discount-rate betas**

The table reports results from cross-sectional regressions of average portfolio excess returns on estimated cash-flow and discount-rate betas. "CAPM" refers to equation (20), "Two-factor I-CAPM" refers to equation (21), "Two-factor I-CAPM\*" refers to the previous model when the constant is removed, "Four-factor I-CAPM" refers to equation (19) and "Four-factor I-CAPM\*" refers to the previous model when the constant is removed. Also, the last column reports estimates for the CRRA ( $\gamma$ ) and the risk premium  $\lambda$  estimated from the restricted model in (22). Robust standard errors are reported in parentheses and the corresponding t-statistics are reported in square brackets below coefficient estimates. \*(\*\*, \*\*\*) denotes significance at the 10% (5%, 1%) level.

Panel A. 25 BE/ME sorted portfolios					
	CAPM	Two-factor I-CAPM	Two-factor I-CAPM*	Four-factor I-CAPM	Four-factor I-CAPM*
$\lambda_0$	0.029* (0.016) [1.901]	-0.003 (0.015) [-0.229]		-0.002 (0.013) [-0.121]	
$\lambda_m$	-0.023 (0.017) [-1.355]				
$\lambda_C$		0.083*** (0.031) [2.717]	0.077*** (0.014) [5.296]		
$\lambda_D$		0.017 (0.017) [1.009]	0.013*** (0.001) [14.233]		
$\lambda_{CC}$				0.076** (0.029) [2.627]	0.073*** (0.015) [4.942]
$\lambda_{CD}$				0.011 (0.016) [0.729]	0.009*** (0.002) [4.399]
$\lambda_{DC}$				0.087*** (0.029) [2.909]	0.084*** (0.016) [5.118]
$\lambda_{DD}$				0.016 (0.015) [1.030]	0.014*** (0.002) [7.476]
adj.- $R^2$	3.4%	39.9%	42.4%	52.7%	54.9%
$F$ -test (all zero) ( $p$ -value)		8.983 (0.001)		7.688 (0.001)	
$\chi^2$ - test ( $p$ -value)		$\lambda_0 = \lambda_D = 0$ 194.294 (0.000)		$\lambda_0 = \lambda_{CD} = 0$ 18.465 (0.000)	$\lambda_{CC} = \lambda_{DC}$ 5.447 (0.019)
$\chi^2$ - test ( $p$ -value)					$\lambda_{CD} = \lambda_{DD}$ 8.277 (0.004)
CRRA ( $\gamma$ ) ( $t$ -stat)					5.775*** (7.878)
$\lambda$					0.013*** (0.001) [14.233]

Panel B. 30 portfolios: 10 BE/ME, 10 D/P and 10 size sorted portfolios

	CAPM	Two-factor I-CAPM	Two-factor I-CAPM*	Four-factor I-CAPM	Four-factor I-CAPM*
$\lambda_0$	0.010* (0.006) [1.799]	-0.004 (0.005) [-0.766]		-0.002 (0.003) [-0.566]	
$\lambda_m$	-0.003 (0.006) [-0.470]				
$\lambda_C$		0.071*** (0.015) [4.668]	0.0061*** (0.009) [6.519]		
$\lambda_D$		0.016** (0.006) [2.747]	0.012*** (0.001) [18.042]		
$\lambda_{CC}$				0.024* (0.029) [2.627]	0.019*** (0.009) [2.305]
$\lambda_{CD}$				0.006 (0.004) [1.690]	0.004*** (0.001) [3.806]
$\lambda_{DC}$				0.028* (0.014) [2.030]	0.023*** (0.011) [2.179]
$\lambda_{DD}$				0.009** (0.004) [2.549]	0.008*** (0.001) [5.606]
adj. $-R^2$	-2.7%	45.6%	46.4%	82.7%	83.1%
$F$ -test (all zero) ( $p$ -value)		13.178 (0.000)		35.733 (0.000)	
$\chi^2$ -test ( $p$ -value)		$\lambda_0 = \lambda_D = 0$ 321.302 (0.000)		$\lambda_0 = \lambda_{CD} = 0$ 14.429 (0.001)	$\lambda_{CC} = \lambda_{DC}$ 0.846 (0.357)
$\chi^2$ -test ( $p$ -value)					$\lambda_{CD} = \lambda_{DD}$ 17.017 (0.000)
CRRA ( $\gamma$ ) ( $t$ -stat)					5.304*** (9.713)
$\lambda$					0.012*** (0.001) [18.042]

Panel C. 45 portfolios: 25 BE/ME and 20 risk sorted portfolios

	CAPM	Two-factor I-CAPM	Two-factor I-CAPM*	Four-factor I-CAPM	Four-factor I-CAPM*
$\lambda_0$	0.014 (0.009) [1.665]	0.001 (0.007) [0.152]		0.004 (0.006) [0.647]	
$\lambda_m$	-0.007 (0.009) [-0.769]				
$\lambda_C$		0.071*** (0.015) [4.663]	0.073*** (0.010) [7.111]		
$\lambda_D$		0.011 (0.008) [1.459]	0.012*** (0.001) [17.959]		
$\lambda_{CC}$				0.056*** (0.014) [3.962]	0.062*** (0.010) [6.149]
$\lambda_{CD}$				0.004 (0.007) [0.595]	0.008*** (0.001) [6.230]
$\lambda_{DC}$				0.065** (0.015) [4.378]	0.072*** (0.011) [6.283]
$\lambda_{DD}$				0.008 (0.007) [1.277]	0.013*** (0.001) [9.540]
adj.- $R^2$	-0.9%	42.7%	44.0%	60.4%	61.0%
$F$ -test (all zero) ( $p$ -value)		17.411 (0.000)		17.790 (0.000)	
$\chi^2$ -test ( $p$ -value)		$\lambda_0 = \lambda_D = 0$ 315.247 (0.000)		$\lambda_0 = \lambda_{CD} = 0$ 38.683 (0.000)	$\lambda_{CC} = \lambda_{DC}$ 7.699 (0.005)
$\chi^2$ -test ( $p$ -value)					$\lambda_{CD} = \lambda_{DD}$ 16.904 (0.000)
CRRA ( $\gamma$ ) ( $t$ -stat)					5.788*** [11.130]
$\lambda$					0.012*** (0.001) [17.959]

Panel D. 75 portfolios: 25 Size-BE/ME, 20 risk, 10 BE/ME, 10 D/P and 10 size sorted portfolios

	CAPM	Two-factor I-CAPM	Two-factor I-CAPM*	Four-factor I-CAPM	Four-factor I-CAPM*
$\lambda_0$	0.012** (0.005) [2.339]	-0.002 (0.004) [-0.553]		-0.001 (0.003) [-0.413]	
$\lambda_m$	-0.004 (0.005) [-0.809]				
$\lambda_C$		0.073*** (0.011) [6.750]	0.068*** (0.007) [9.544]		
$\lambda_D$		0.015*** (0.005) [3.103]	0.012*** (0.000) [24.981]		
$\lambda_{CC}$				0.052*** (0.009) [5.262]	0.049*** (0.007) [6.824]
$\lambda_{CD}$				0.009** (0.004) [2.272]	0.007*** (0.001) [7.812]
$\lambda_{DC}$				0.060*** (0.010) [5.601]	0.058*** (0.008) [6.943]
$\lambda_{DD}$				0.013*** (0.004) [3.294]	0.011*** (0.001) [11.383]
adj. $-R^2$	-0.4%	44.4%	44.9%	64.8%	65.3%
$F$ -test (all zero) ( $p$ -value)		30.543 (0.000)		35.193 (0.000)	
$\chi^2$ -test ( $p$ -value)		$\lambda_0 = \lambda_D = 0$ 618.399 (0.000)		$\lambda_0 = \lambda_{CD} = 0$ 60.483 (0.000)	$\lambda_{CC} = \lambda_{DC}$ 9.889 (0.002)
$\chi^2$ -test ( $p$ -value)					$\lambda_{CD} = \lambda_{DD}$ 29.874 (0.000)
CRRA ( $\gamma$ ) ( $t$ -stat) $\lambda$					5.594*** [14.640] 0.012*** (0.000) [24.981]