Monte-Carlo Estimations of the Downside Risk of Derivative Portfolios*

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Abstract

We simulate the performances of a standard derivatives portfolio to evaluate the relevance of benchmarking in terms of downside risk reduction. The simulation shows that benchmarking always leads to significantly more severe losses in average than those generated by letting the portfolio reach the end of a given horizon. Moreover, switching from a 0-correlation across underlyings to a very mild form of correlation significantly increases the probability of reaching the downside benchmark before maturity, whereas adding more correlation does not significantly increase this figure.

Keywords: Derivatives; Portfolio management; Benchmarking; Downside risk; Monte-Carlo simulations.

1 Introduction

Controlling the downside risk of derivative portfolios is a permanent concern in financial practice. The first explanation to this phenomena is the long series of financial disasters associated with derivatives, such as for instance Barings Bank and Long Term Capital Management that resulted in losses ranging in billions of US dollars. Lessons from those experiences led financial regulators, through the Basel II Capital Agreement for instance, to severely monitor the risk taken by financial institutions. The second explanation has psychological roots, mainly associated with the largely-observed aversion to losses as formalized by Kahneman and Tversky [10], and to a lesser extent by the fear of mis-assessment of uncertainty as in Du and Budescu [3] (see for instance Jarrow and Zhao [9] and Liu et al. [11] for a discussion).

Practical control of downside risk nearly always involves *benchmarking*; that is, liquidating or entirely reshuffling a position once a pre-determined level of losses is reached. This issue is documented in Basak et al. [1] for equities portfolios, and in Pedersen [12] for derivative portfolios. This practice is typically based on the belief that a trading strategy, leading to certain level of losses **once**, is too risky and must be abandoned to a new one.

The objective of the paper is to evaluate the relevance of benchmarking in terms of loss reduction, and to isolate some factors significantly affecting the riskiness of derivative portfolios. We carry out a Monte-Carlo simulation of the performance of a derivative portfolio consisting of four classes of options (a European, Asian, lookback and cash-or-nothing option) with same maturity, each class consisting of one hundred derivatives written on one hundred different underlyings. The prices of the underlyings all follow a standard geometric Brownian motion, as in the Black-Scholes framework. The initial wealth is equally allocated across all options, and once maturity is reached the proceedings are reinvested in the same portfolio in the same manner. Such reinvestments span roughly six years, and the return of the portfolio after those six years is compared with the potential losses generated by benchmarking at pre-determined levels. The scenario is simulated 2500 times to generate Monte-Carlo estimators on which we base our discussion (see Glasserman [6] for an introduction).

The experiment shows that benchmarking leads to significantly more severe losses in average than those generated by letting the portfolio reach the end of the six years, for every level of benchmark and for every level of correlation across underlyings. The recovery rate, as defined in Section 1.3, is however decreasing with the benchmark level albeit always significantly high. Moreover, switching from a 0-correlation across underlyings to a very mild form of correlation enormously increases the probability of reaching the downside benchmark before maturity, for every benchmark level, whereas adding more correlation does not significantly increase this figure.

The intuition for those results can be derived from the well-known Gambler's Ruin problem, as described in Grimmett and Stirzaker [7] Chapter 3, even if the random process characterizing our portfolio return is far more complex and thus requires simulations. Consider a gambler tossing a fair coin, and winning (resp. loosing) one monetary unit if head (resp. tail) occurs at each toss. The gambler tosses the coin until either her wealth reaches a pre-determined upper-bound or ruin occurs. Standard results claim that the game will end for sure, and the average number of tosses needed to reach one of those two events decreases exponentially as the bound get closer to the initial wealth. A ruin corresponds to reaching a downside benchmark in our setting, and we observe similar results in the portfolio simulation. However, letting the gambler's game continue even ruin occurs (through retaining barriers for instance) leads to a wealth distribution at a given future horizon whose mean is different from zero. In our experiment, letting the portfolio reach the horizon of six years leads to an average return always greater than the considered benchmark levels (up to 30% losses).

The basic insight is that benchmarking is a one-off decision, which rules out any possibility of recovering current losses. The simulation shows that our fairly standard portfolio displays a surprisingly high potential for future recovery, which cannot be exploited when benchmarking is implemented. The paper thus suggests that benchmarking does not control downside risk but rather aggravates it. Moreover, we point out that derivative portfolios are particularly sensitive to correlation across underlyings, and diversifying the underlyings appears as a safer way of controlling downside risk.

The paper is organized as follows: in Section 2 we describe the experiment, in Section 3 we study the case of 0-correlation across underlyings, in Section 4 we study the consequences of adding a mild form of correlation, in Section 5 we study the average time before reaching a benchmark, and Section 6 concludes. Tables and figures are given after the Bibliography.

2 The experiment

In this section, we describe the assumptions used for the numerical simulation. We start by describing the underlying assets on which the options are written. The simulation involves a set of 400 different underlyings whose price processes in a risk-neutral world are described by the equation

$$dS_t = \mu S_t dt + \sigma S_t dW_t, \tag{1}$$

where S_t is the price of the underlying at time t, $\mu > 0$ is the drift of process, $\sigma > 0$ is the variance of the jumps assumed to be constant over time, and W_t is a Brownian motion with law $N(0, t)$ for every time t. Using standard arguments in Stochastic Calculus, the solution to the stochastic differential equation in (1) is given by

$$d\ln(S_t) = (\mu - \frac{\sigma^2}{2})dt + \sigma dW_t. \tag{2}$$

We need a discretized version of the continuous-time process described in Eq. (2) to carry out our numerical simulations. For every sequence of times $0 < t_0 < ... < t_n$, the discretized price process above satisfies

$$S_{t_i+\Delta t_i} = S_{t_i} \exp\left[(\mu - \frac{\sigma^2}{2})\Delta t_i + \sigma\epsilon\sqrt{\Delta t_i}\right] \text{ for every } i = 0, ..., n-1, \tag{3}$$

where ϵ is a random variable generating the jumps with law $N(0, 1)$. We will fix the time horizon to be $T = 3$ months, and within this horizon we will assume that there are 15 jumps occurring at the end of equal time intervals (this would roughly correspond to weekly jumps) for every underlying. We assume that there exists a riskless asset whose return is $r = 5\%$ per annum. By a standard no-arbitrage condition, it follows that $\mu = r$. We assume that the initial price of all the underlyings is $S_{t_0} = 50$, with volatility $\sigma = 45\%$ per annum. The underlyings differ by the nature of the realized jumps ϵ, and the way those jumps are correlated is central to our analysis. We will describe later our assumptions on those correlations, as we present our results.

2.1 The options

We now describe the classes of options constituting our portfolio. The portfolio formation will be described next section, for now we just focus on the assets that constitute it. Those assets are separated into four classes of options, described next.

The first class of options, denoted by the letter e, consists of 100 *European calls*, each of them written on one of the 100 different underlyings whose processes are described in Eq. (3). The maturity of the calls is 3 months, with strike price $K = 49$. For any such calls, the payoff at the end of the 3 months is thus $\max\{0, S_T - K\}$.

The second class of options, denoted by the letter a, consists of 100 *Asian options*, each of then written on one of the 100 different underlyings. For any possible realization of the underlying $S = (S_t)_{t=0,\ldots,T}$, the payoff of the Asian option is $\max\{0, S_T - \bar{S}\}$, where $\bar{S}$ is the mean of S.

The third class of options, denoted by the letter l, consists of 100 *lookback options*, each of then written on one of the 100 available usual underlyings.

For any possible realization of the underlying $S = (S_t)_{t=0,...,T}$, the payoff of the lookback option is $S_T - \min(S)$, where $\min(S)$ is the minimum of S.

The fourth class of options, denoted by the letter c, consists of 100 *cash-or-nothing options* with strike price $K = 49$ and end-payment $Q = 10$, each of then written on one of the 100 available usual underlyings. For any possible realization of the underlying $S = (S_t)_{t=0,...,T}$, the payoff of the cash-or-nothing option is Q if $S_T > K$ and 0 otherwise.

Table 1 gives the theoretical prices of those derivatives, obtained with standard Monte-Carlo simulations independent of the other simulations used for evaluating the portfolio performances (see Boyle et al. [2], Glasserman [2] Chapters 4-5 or Hull [8] Chapter 22 for an introduction to the methods used here, see Detemple et al. [4] for asymptotic properties of our estimators and Joy et al. [5] for alternative methods).

Table 1

Monte-Carlo estimations of the individual risk-neutral valuations of the options. Codes are written in R (see R project [13]). Figures between brackets are the variances of the estimators. Estimators are calculated with N=100,000 simulations.

European call	Asian call	lookback call	cash-or-nothing
5.2745	3.1966	7.3431	5.0668
(0.02542)	(0.01425)	(0.02470)	(0.01560)

2.2 Portfolio formation

We now describe how our portfolio is formed. For a given initial wealth w_0, we allocate one-fourth of this wealth to every class of options. For a given wealth allocated to one particular class, we purchase an equal number of contracts written on the 100 possible options available for trade. That is, we consider 400 different underlyings, the first hundred are used to write European calls, the second hundred is used to write Asian calls and so on. The wealth is equally distributed across all of those assets to purchase a portfolio of options at the prices given in Table 1.

Once the first time horizon is reached and payoffs are realized, the proceeds are reinvested in a similar portfolio in the same manner as above. This operation is repeated at most 24 times, which would roughly correspond to 6 years of trades. The timing and allocations among classes of assets are described in Fig. 1.

We call a *quarter* any of such times where options expire and proceeds are reinvested. The fact that options are kept until expiration instead of being sold before is not restrictive. Indeed, since the current reselling price of the option reflects any loss-gain incurred during the exercise, the reinvestment of the realized gain-loss into similar assets would not affect the portfolio value since the underlyings follow a Lévy process.

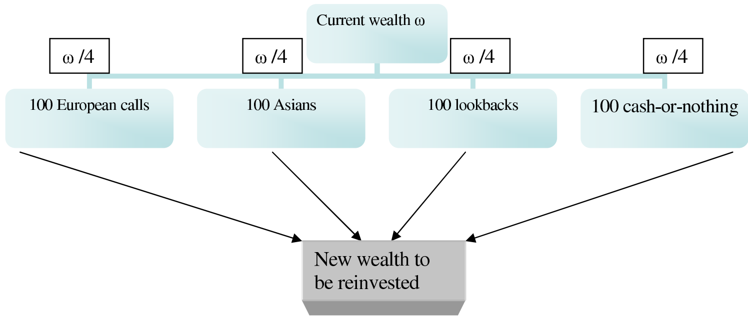

Figure 1: Portfolio formation in any given quarter.

2.3 Benchmarking policy

We now present how the performances of the above portfolio are measured, and under which conditions the decision to cease further trades is made. The initial wealth w_0 is worth one million monetary unit, although this figure does not affect the outcome of the simulation.

We first define *benchmarking* to be the decision to stop trades the first quarter when the current wealth is below a pre-determined fraction of the initial wealth. In contrast, we define *no-benchmarking* to be the decision to carry on the trades until the last $d = 24$. We will consider $N = 2500$ simulations of the investment scenario above, and we define the *failure rate with benchmark* to be the number of simulations where trades are stopped because the benchmark is reached once, divided by N. Similarly, we define the *failure rate without benchmark* to be the number of times during the simulations where the final wealth is below a pre-determined benchmark at quarter $d = 24$, divided by N. We abbreviate failure rate with FR forthwith.

Those two notions allow us to define the *recovery rate* as

$$Recovery\ Rate = \frac{FR\ with\ benchmark - FR\ without\ benchmark}{FR\ with\ benchmark},$$

so that it represents the percentage of simulations that have been wrongfully stopped before the end of the planned exercise.

An interesting alternative to the above definition is first to assume that the proceedings of a liquidation resulting from benchmarking is re-invested at the risk-free rate until the end of the normal exercise, and then the recovery rate can be defined by comparing the compounded proceedings with and the final wealth without benchmarking. Similar qualitative results obtain in this case, and this issue is thus omitted.

3 Benchmarking and recovery rate

We now present the results of our numerical simulations, allowing us to compare the relative efficiency of policies with and without benchmark. In a first step, we present the estimations of the various rates introduced in the previous section, in the case where the underlyings are assumed to be all uncorrelated pairwise. In a second step, we show how a mild pairwise correlation among some underlyings can significantly affect the previous rates, even if the main conclusions and policy recommendations remain the same.

The first set of results involves underlyings whose jumps have 0-pairwise correlations. The experiment estimates the failure rates with and without benchmarking of the portfolio scenario described in Section 1.2. The investment scenario has been simulated $N = 2500$ times.

The experiment unambiguously shows that failure rates are significantly higher when benchmarking is used, regardless of the benchmark level. Moreover, some figures in Table 2 are of practical relevance when evaluating the riskiness of derivatives portfolio in general. A benchmark of .2 is quite common in practice, and thus a well-diversified, fairly-priced portfolio has a 13.08% chance of abruptly reaching this benchmark. That is; without mismanagement and with a good portfolio, there is roughly one chance in eight for a business to go bankrupt when benchmarking. If instead managers show more patience (or possibly stronger nerves), this same business has only a very acceptable 4.44 % chances of reaching this level of losses if the portfolio is run until the end of the scheduled exercise.

The benefits from avoiding benchmarking significantly depends on the benchmark level. This aspect is illustrated in Fig. 6, where the failure rates with benchmarking are contrasted with the corresponding recovery rates. Recovery rates are (approximately) decreasing with benchmark levels, although they remain strikingly high with high benchmarks. For instance, for a benchmark of .2 roughly 65% of terminated trading exercises would have a yielded a lower loss if benchmarking was avoided. That is; instead of stopping trades and thus permanently accepting a 20% loss, in 65% of the cases carrying on trades would have led to a loss (or possibly a profit) strictly less than 20%. Our derivative portfolio thus have a strong recovery potential. This point makes the case against benchmarking: once trades are stopped the loss is permanent and strong recovery possibilities of such portfolios are foregone.

4 Correlated underlyings

We now carry out the same analysis as before with the additional assumption that the underlyings display some form of mild correlation described later. This new assumption will strikingly change the quantitative features of the previous section, in particular the riskiness of the simulated portfolio, whilst not changing the optimality of no-benchmarking. The effect of correlation on the recovery rate is also examined.

We now describe how assets are correlated. We refine the notation in Eq. (3), for exactly the same underlyings and for every $i = 0, ..., n-1$, as

$$S^{\kappa,j}_{t_i+\Delta t_i} = S^{\kappa,j}_{t_i} \exp\left[(\mu - \frac{\sigma^2}{2})\Delta t_i + \sigma \epsilon_{\kappa,j} \sqrt{\Delta t_i}\right], \tag{4}$$

where $\kappa \in \{e, a, l, c\}$ is an index denoting the class of options the underlying is assigned to as in Section 1.1, and $j = 1, ..., 100$ uniquely defines the option within the class of options κ.

So far, we have assumed that the sequence of random variables $(\epsilon_{\kappa,j})_{\kappa,j}$ exhibited 0-pairwise correlation. We now assume that the pairwise correlations are described by variance-covariance matrices

$$\mathrm{cov}(\epsilon_{e,j},\ \epsilon_{a,j},\ \epsilon_{l,j},\ \epsilon_{c,j}) = \begin{pmatrix} 1 & \rho & \rho & \rho \\ \rho & 1 & 0 & 0 \\ \rho & 0 & 1 & 0 \\ \rho & 0 & 0 & 1 \end{pmatrix}$$

for every $j = 1, ..., 100$ and for some $\rho \in (0, 1)$; all the other underlyings exhibiting 0-pairwise correlation. In words, for every j the underlying with index j has a pairwise correlation of ρ with the underlying of the same j where the European call is written on, and is uncorrelated with all the other 397 underlyings. This corresponds to a very mild form of correlation, although this assumption will significantly change the results from the previous section.

Table 3 gives the results for two levels of correlation. Such levels display a clear trend and considering more correlation levels would not add to our point. The failure rates with and without benchmarking display a peak for $\rho = .5$, and figures slightly decrease for $\rho = .7$. The significant variations from the no-correlation case of the previous section show the extreme sensitiveness of option portfolios to even a slight change in correlation.

Figure 3 gives the recovery rates for $\rho = .5$ and $\rho = .7$, derived from Table 3. Such recovery rates are comparable for those correlations, although they are significantly lower than those in the no-correlation case. The main point to notice is that the addition of those mild correlations have reduced the recovery rate to roughly slightly more than 50% (depending on the experiment), which is an intuitive recovery rate as explained in the Introduction. In other words, the addition of correlation across underlyings (even if mild) leads to reduce the recovery rate by roughly 25 %, whereas it remains roughly stationary after values greater than $\rho = .5$.

5 Quarters before failure with benchmarking

We now turn to describing how quickly the simulated portfolio reaches its downside benchmark. In particular, we are interested in observing how sensitive the benchmark occurrence is to the correlation form described in the previous section.

The first fact to notice is that the first quarter to reach benchmark increases with the benchmark level, for every level of correlation. This is a very intuitive aspect of the simulation, since it natural to expect that more time is needed for a portfolio to reach to even lower benchmark.

An important result of the experiment is the significant decrease in first quarters to benchmark between the 0-correlation case and the case with correlation level $\rho = .5$. The point to notice is that the very mild correlation level, as introduced in the previous section, triggers a much earlier cessation of activities caused by benchmarking. Adding more correlation ($\rho = .7$) does not significantly increase the first quarter of occurrence, showing that switching from 0-correlation to some form of correlation is a threshold that significantly affects this event.

In all cases, the average first quarter of occurrence is before the end of our horizon. This reinforces our point that benchmarking is to be avoided when evaluating derivative portfolio, since this practice tends to stop trading activities too early with devastating effects as shown in the previous sections.

6 Conclusions

We have simulated the performance over time of a fictitious derivative portfolio under standard assumptions. The experiment shows that stopping trading activities when reaching a pre-determined downside benchmark leads to severe losses, and those losses are always significantly reduced by allowing trades to continue until a pre-determined horizon.

Moreover, the experiment shows the extreme sensitiveness of the portfolio to the correlation across underlyings. Switching to a 0-correlation case to a mild form of correlation significantly increases the riskiness of the portfolio and the average first quarter of bankruptcy. However, adding more correlation across underlyings does not significantly affect those figures. Regardless of the correlation level, losses are significantly reduced by allowing trades to continue instead of stopping when a downside benchmark is reached.

References

[1] Basak, S., A. Shapiro and L. Tepla (2006) "Risk management with benchmarking," *Management Science* 52, 542-557.

[2] Boyle, P., M. Broadie, and P. Glasserman (1997) "Monte Carlo Methods for Security Pricing," *Journal of Economic Dynamics and Control,* 21, 1267-1322.

[3] Du, N. and D. Budescu (2005) " The effects of imprecise probabilities and outcomes in evaluating investment options," *Management Science* 51, 1791-1803.

[4] Detemple, J., R. Garcia and M. Rindisbacher (2005) "Asymptotic properties of Monte-Carlo estimators of derivatives," *Management Science* 51, 1657-1675.

[5] Joy, C., P. Boyle and K. Tan (1996) "Quasi-Monte Carlo methods in Numerical Finance," *Management Science* 42, 926 - 938.

[6] Glasserman, P. (2004) *Monte-Carlo Methods in Financial Engineering,* New-York: Springer Science.

[7] Grimmett, G. and D. Stirzaker (2006) *Probability and Random Processes* (3rd ed.), Oxford: Oxford University Press.

[8] Hull, J. (2006) *Options, Futures and Other Derivatives* (6th ed.), Upper Saddle River: Prentice Hall.

[9] Jarrow, R. and F. Zhao (2006) "Downside loss aversion and portfolio management," *Management Science* 52, 558-566.

[10] Kahneman, D. and A. Tversky (1979) "Prospect Theory: An analysis of decision under risk," *Econometrica* 47, 263-291.

[11] Liu, J., Longstaff, F. and J. Pan (2003) "Dynamic asset allocation with event risk," *Journal of Finance* 58, 231-259.

[12] Pedersen, C. (2001) "Derivatives and downside risk," *Derivatives Use, Trading and Regulation* 7, 251-268.

[13] The R project, *Statistical Software,* http://www.r-project.org/

Table 2

Monte-Carlo estimations of the failure rates with and without benchmarking (see attached R code). Figures between brackets are the variances of the estimators. Estimators are calculated with N=2500 simulations.

	Benchmark level					
	.5	.1	.15	.2	.25	.3
Benchmarking	0.5556	0.346	0.2316	0.1308	0.0792	0.0472
	(0.0099)	(0.0095)	(0.0084)	(0.0067)	(0.0054)	(0.0042)
No benchmarking	0.1356	0.1076	0.0644	0.0444	0.0328	0.0216
	(0.0068)	(0.0061)	(0.0049)	(0.0041)	(0.0035)	(0.0029)

Table 3

Monte-Carlo estimations of the failure rates with and without benchmarking for various levels of correlation between underlyings. Case I gives the failure rates with benchmarking, Case II gives the failures rates without benchmarking. Figures between brackets are the variances of the estimators. Estimators are calculated with N=2500 simulations.

		Benchmark level					
		.05	.1	.15	.2	.25	.3
Case I	$\rho = .5$	0.764	0.6872	0.6192	0.5136	0.4508	0.3632
		(0.0084)	(0.0092)	(0.0097)	(0.0099)	(0.0099)	(0.0096)
	$\rho = .7$	0.7668	0.6472	0.5824	0.4816	0.3856	0.3448
		(0.0084)	(0.0095)	(0.0098)	(0.0099)	(0.0097)	(0.0095)
Case II	$\rho = .5$	0.3484	0.328	0.296	0.2528	0.2344	0.1824
		(0.0095)	(0.0093)	(0.0091)	(0.0086)	(0.0084)	(0.0077)
	$\rho = .7$	0.3364	0.2952	0.2772	0.2232	0.194	0.164
		(0.0094)	(0.00912)	(0.0089)	(0.0083)	(0.0079)	(0.0074)

Table 4

Monte-Carlo estimations of the average number quarters before failure with benchmarking, for various levels of correlation between underlyings. Figures between brackets are the variances of the estimators. Estimators are calculated with N=2500 simulations.

	Benchmark level					
	.05	.1	.15	.2	.25	.3
$\rho = 0$	13.2548	18.2428	20.5252	22.3592	22.3592	23.5388
	(0.2054)	(0.1727)	(0.1386)	(0.0944)	(0.0679)	(0.0470)
$\rho = .5$	8.9084	11.2352	13.116	15.6196	17.4732	19.1684
	(0.1879)	(0.1950)	(0.1926)	(0.1836)	(0.1662)	(0.1493)
$\rho = .7$	8.7936	12.146	14.0112	16.4144	18.4368	19.504
	(0.1863)	(0.1967)	(0.1906)	(0.1777)	(0.1590)	(0.1434)

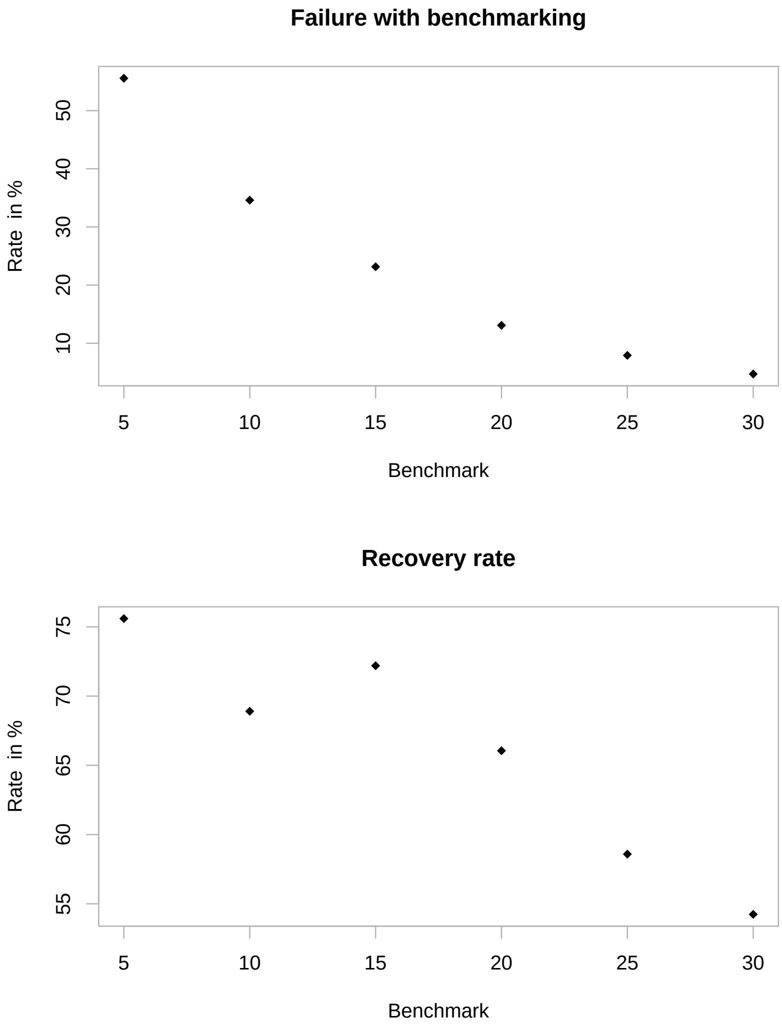

Figure 2: Failure and recovery rates, uncorrelated underlyings.

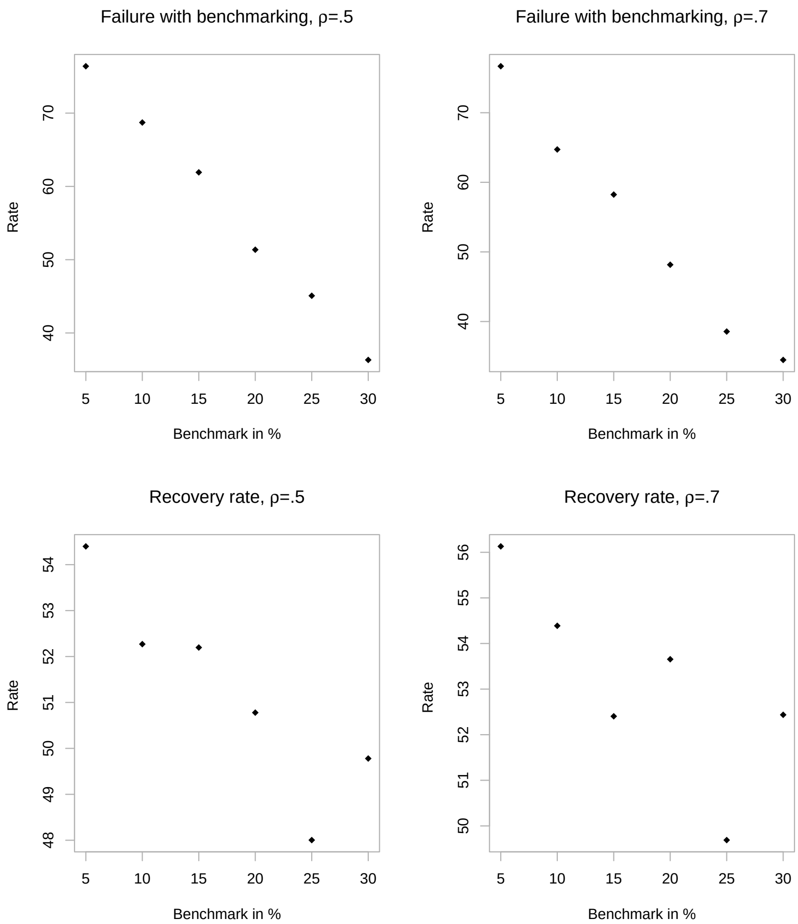

Figure 3: Failure with benchmarking and recovery rates, with correlation.

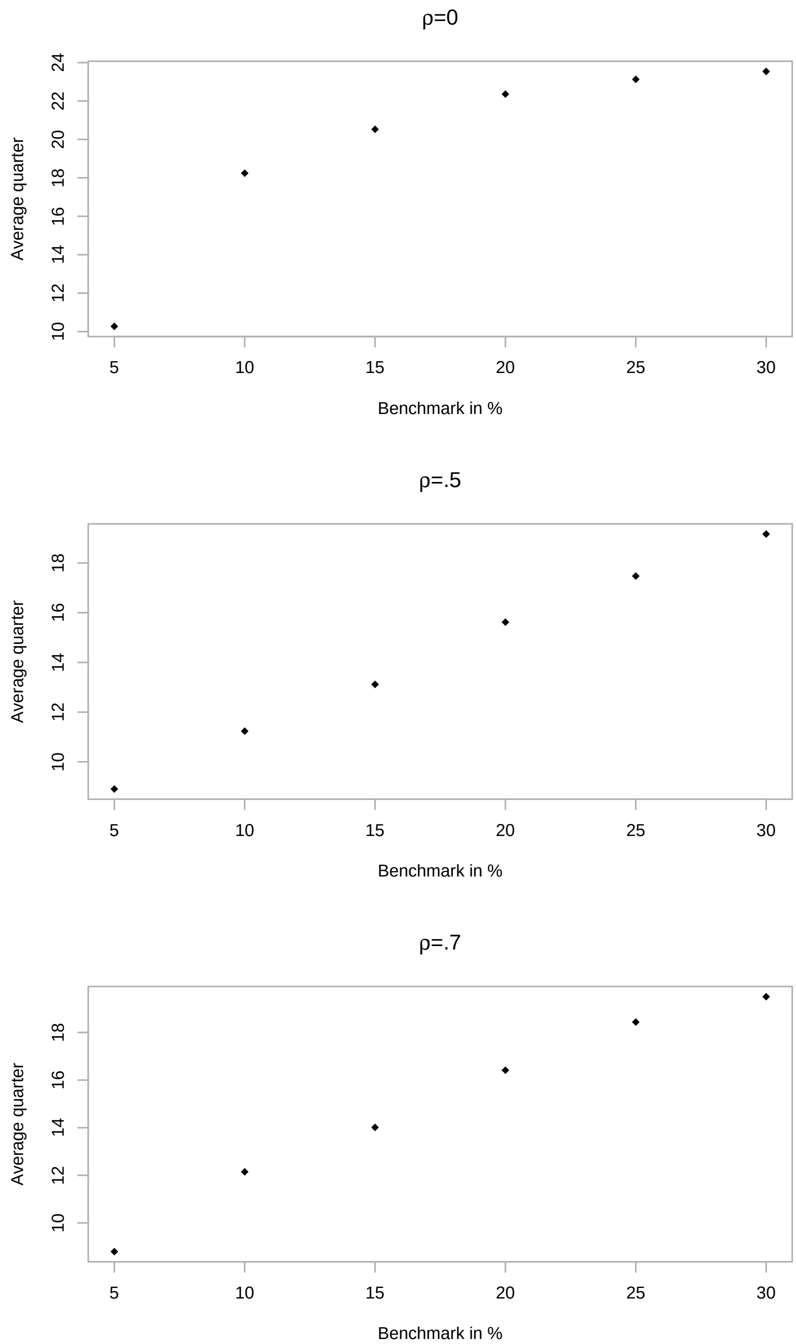

Figure 4: Average quarter before failure for various levels of correlation.