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Irrational Market Makers^{*}

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Abstract

We analyze a model where irrational and rational informed traders exchange a risky asset with irrational market makers. Irrational traders misperceive the mean of prior information (optimistic/pessimistic bias) and the variance of the noise in their private signal (overconfidence/underconfidence bias). Irrational market makers misperceive both the mean and the variance of the prior information. We show that moderately underconfident traders can outperform rational ones and that irrational market makers can fare better than rational ones. Lastly, we find that extreme level of confidence implies high trading volume.

JEL Classification: G00; G10; G14.

Keywords: Irrational Traders; Irrational Market makers; Overconfidence; Optimism.

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1 Introduction

Economic and financial theories have widely used the assumption that agents behave rationally. Such an assumption has failed to explain some properties observed in financial markets such as *(i)* the excessive volume traded [see Odean (1998b)], *(ii)* underreaction or overreaction of market participants [see Debondt and Thaler (1985)], and *(iii)* the excessive volatility observed in financial markets [see Shiller (1981, 1989)]. In order to explain these properties, financial economists have assumed that investors have psychological traits that lead them to behave irrationally. However, the possibility of behavioral biases for market makers has been mostly ignored. Subrahmanyam (2007) suggests in the conclusion of his paper that this research avenue needs to be developed. Recent evidences in the literature show that market makers, despite being experienced experts, are also subject to psychological biases. Corwin and Coughenour (2008) show that market makers suffer from limited attention leading to an increase in transaction cost for less active stocks in their portfolio as they focus on the more actives ones. Oberlechner and Osler (2012) find that currency dealers are overconfident on average as they underestimate uncertainty and overestimate their own abilities. Goetzmann and Zhu (2005) find evidences of participants mood influencing the stock market. However, they find that it is not due to individual investors trading patterns. This leaves the possibility of the market makers' moods affecting it.¹

In this paper we define a theoretical framework where rational and irrational investors trade a risky asset with irrational market makers. Introducing irrational market makers contrasts with the existing literature and allows us in particular to have a better understanding of how financial markets perform. The novelty of this article is twofold. First we consider the possibility for market makers to be irrational, and second we study the interactions between irrational traders and irrational market makers.

Different forms of irrationality have been documented. However, some are more prevalent than others. Among those are overconfidence and optimism. Generally speaking overconfidence can be defined as the tendency of the subjective confidence in judgments to be greater than the objective accuracy. This has led people to overestimate their knowledge (miscalibration) and exaggerate their ability to control events (illusion of control). Overconfidence also takes the form of the better than average effect and finally unrealistic optimism where people believe that they are less likely to experience a negative event than others. According to Fabre and François-Heude (2009) optimism is the tendency to perceive a situation as more likely to result in a favorable outcome, irrespective of the objective probability of that outcome actually occurring.

We define an irrational investor as a trader suffering from overestimation/underestimation of his knowledge (miscalibration) and optimism/pessimism. This is then translated into a trader having erroneous beliefs about *(i)* the mean

¹Papers, such as Tetlock (2011) Baker and Stein (2004) and Linnainmaa (2007) to name but a few, have shown that liquidity providers using limit orders are not always rational and therefore do not always obtain zero expected profits.

of prior information (returns of the risky asset) and, *(ii)* the variance of the noise in their private information. The former refers to the optimistic/pessimistic bias and the latter refers to the underconfident/overconfident bias.² Equivalently, an irrational market maker has erroneous beliefs about the mean (optimistic bias) and the variance of prior information (underconfidence/overconfidence bias). We believe that modelling the optimistic bias as having only an effect on the mean reflects the definition given by Fabre and François-Heude (2009).³ Using this theoretical framework, we develop a model of financial markets where irrational traders along with rational traders trade a risky asset with a market maker. This new setting combining rational and irrational traders with irrational market makers allows us to derive interesting new features.

The market maker's irrationality has different impact on the market. We show that the variance misperception has an effect on price through its effect on the level of liquidity. A variance optimistic or overconfident market maker believes that prior information is more precise than it is and therefore believes that private information is less substantial than it actually is. As a consequence, she adjusts her price less aggressively and increases market depth. As market depth increases, the orders submitted by traders have less impact on the price and informed traders whether rational or irrational respond by trading more intensely. The exact opposite effects take place for a variance pessimistic or underconfident market maker. We now turn to the effect of the mean misperception. An optimistic (pessimistic) market maker increases (decreases) the overall level of price as she wrongly believes that the expectation of the risky asset is higher (lower) than it actually is. Again, whether rational or irrational, traders adjust the quantity they trade to the increased or decreased price. Given those basic forces the following results are obtained.

- First of all, we show that, in contrast to most of the literature in Finance [see Odean (1998b) or Kyle and Wang (1997)], irrational market makers can obtain non-zero expected profit even if behaving competitively. This is clearly the case when the market maker is variance pessimistic but does not misperceive the mean of prior information (when the market maker misperceives the mean, the results are not as clear cut). In that case the market depth is decreased which increases the price. As the price is increased, informed traders reduce the quantity they trade. However, as the liquidity trading is inelastic the market markers' expected profits are positive. The opposite is true for a variance optimistic market maker with correct beliefs of the mean of prior information.
- Second, in accordance with some of the previous literature [see Benos (1998), Odean (1998b) and Kyle and Wang (1997)] we show that the presence of irrational traders may lead to the nonexistence of an equilibrium.

²It is only recently that researchers have shown that overconfidence and optimism are empirically distinguishable [See Régner et al. (2004) and Glaser et al. (2010)]. Hilton (2007) shows the simultaneous presence of these two biases and their impact on decision making.

³Modelling in such a way the optimistic bias follows their model.

This happens when traders would like to trade an infinite quantity and market makers would like to supply infinite liquidity. Furthermore, we find that a variance optimistic market maker exacerbates the nonexistence of equilibrium whereas a variance pessimistic market maker alleviates it. A variance optimistic market maker worsens the excessive trading whereas a variance pessimistic has the opposite effect.

- Third, under the presence of irrational market makers, we find that an underconfident trader can outperform a rational trader. This striking new result contradicts Wang (2001) and was not put forward in the literature so far. It relies on two important points: (i) the market maker's irrationality and (ii) the disagreement about the mean of prior information between the underconfident trader and the irrational market maker. Indeed, contrary beliefs, if sufficiently different, lead the underconfident trader to buy (or sell) when prices are too low (or too high) on average. The first point, (i), is documented in recent papers: Oberlechner and Osler (2012), Glaser and Weber. (2007), Hilton (2001), and Glaser et al. (2007). The second point, (ii), is illustrated by Krichene (2004), whose analysis recovers the euro-dollar rate from option prices for June 2004 as expected by market participants on May 5, 2004. He finds that the market was constituted with two distinct groups of traders. One was expecting an appreciation of the dollar with respect to the euro and one was anticipating a depreciation of the dollar against the euro. Such a situation with the presence of two groups having distinct beliefs can occur during a transition period for the market when some market participants change their beliefs regarding the asset while others keep their beliefs.
- Fourth, our model predicts high level of traded volume for extreme level of confidence including the case where traders are underconfident. In the previous literature this excessive volume traded has been explained by the presence of overconfident traders [see Odean (1998b)]. In our setting, we show that the volume traded might not be a monotonic function of the traders' level of confidence. Indeed it can display a *U*-shape form as a function of the level of overconfidence. Our model also predicts high level of volatility. We find that the traders' responsiveness to private information is greatly affected by the market makers' confidence. For the case where irrational traders are underconfident, an underconfident market maker leads to too much trading from rational traders. It is their large trading on private information that leads to the non-existence of the equilibrium.

The paper unfolds as follows. The next section reviews related work. Then, the general model is presented along with the definition of an equilibrium for our model. In section 4, we derive the equilibrium for the general case and we also analyze two benchmark cases i.e. the case where no market participants are irrational and the case where only a subset of traders are irrational. In section 5, we provide comparative statics concerning the expected volume and stress

the impact of irrationality on financial markets for the general case. Finally, in section 6 we summarize our results and conclude. All proofs are gathered in the appendix.

2 Related work

There is a large body of evidence in the psychology literature suggesting that people do not always have an accurate perception of themselves and their surrounding world. Such misperceptions impact people's decision making. Evidences suggest that people may display overconfidence such as miscalibration, positive illusions, better than average effect and illusion of control.⁴ Miscalibration is defined as the tendency for people to overestimate the precision of their knowledge. Ito (1990) demonstrates the existence of miscalibration in the foreign exchange market. Further evidence is provided by Odean (1998a, 1998b, 1999) and Hilton (2001). Positive illusions has been documented in Taylor and Brown (1988, 1994) and Weinstein (1980). Taylor and Brown (1988, 1994) analyze the "better than average" effect whereas Weinstein (1980) looks at unrealistic optimism. Financial practitioners are also well aware of the existence of such psychological traits for investors trading in financial markets. For example, the Union des Banques Suisses together with Gallup Organization have launched in October 1996 the *Index of Investor Optimism*.⁵

Although underconfidence has received less attention than overconfidence, recent evidence has cast doubt on the generality of overconfidence. People tend to be overconfident on easy tasks where absolute performance is high and tend to be underconfident on difficult tasks where absolute performance is low (Hoelzl and Rustichini (2005), and Moore and Cain (2007) among others). It has also been found that underconfidence in judgements of learning can increase with practice. This phenomenon is named Underconfidence With Practice (UWP) [See Koriat et al. (2002) and Serra and Dunlosky (2005) for instance]. In an experimental setting, Glaser et al. (2007) compare the trend recognition and forecasting ability of novices and financial participants. A third of the financial participants report working for the market making industry. Two types of trend recognition are analyzed: probability estimates and confidence intervals. Underconfidence occurs for probability estimates for both groups, though to a lesser extent for financial professionals.

Most of the literature in Finance predicts that overconfident investors trade to their disadvantage and fare worse than their rational counterpart [see Odean (1998b), Gervais and Odean (2001), Caballé and Sákovics (2003), Biais et al. (2005) among others]. However, Kyle and Wang (1997) and Benos (1998) find

⁴See Hilton (2007).

⁵A detailed methodology used to compute the index can be found at www.ubs.com/investoroptimism. The monthly level of the index is also given from its launch date up to now. Other financial institutions also try to assess the market participants' sentiment. For that reason, in September 2006, a survey denominated "Fund Manager Survey" was conducted on behalf of Merrill Lynch.

that moderately overconfident traders may earn larger expected profit than rational ones. Moreover, a common finding to all these papers except Caballé and Sákovics (2003) is that trading volume, and price volatility increase with the level of overconfidence. All these papers differ from ours as none of them consider the possibility of irrational price setters. Odean (1998b) is the only other paper considering irrational liquidity suppliers, this is done in a Grossman-Stiglitz setting whereas we consider strategic market making. Irrational risk averse liquidity suppliers buy costly information and overestimate the precision of that information. These overconfident traders are found to fare worse than uninformed traders. Market depth is shown to increase with the level of overconfidence. The misperception of the mean of prior information extends both Kyle and Wang (1997) and Odean (1998b) and enables us to have a more complete parameterization of irrationality. Therefore, the presence of irrational market makers combined with the misperception in mean and variance of prior information in an oligopoly framework constitute one contribution of our paper.

3 Model

We study a financial market where a market maker and several traders exchange a risky asset whose future value \tilde{v} follows a Gaussian distribution with zero mean and variance σ_v^2 . Traders can be either informed or uninformed (noise traders). Uninformed traders submit a market order that is the realization of a normally distributed random variable \tilde{u} with zero mean and variance σ_u^2 . Informed traders are risk neutral and can be one of two types: rational or irrational. N traders are rational whereas M are irrational. Both types of traders have access to private information, i.e. they observe a noisy signal of the future value of the risky asset

$$\tilde{s}_k = \tilde{v} + \tilde{\varepsilon}_k, \text{ with } \tilde{\varepsilon}_k \sim N(0, \sigma_\varepsilon^2) \quad \forall k = 1, \dots, N + M \text{ and } \tilde{v} \sim N(0, \sigma_v^2).$$

These two types of traders differ in the beliefs they hold about both the distribution of the risky asset value (prior information) and the noise in the signal received.

As introduced in the previous sections, the irrational traders may display several psychological traits: an optimism/pessimism bias and an overconfident/underconfident bias. We define an optimistic (pessimistic) trader as a trader who has an erroneous belief about the mean of prior information with $\tilde{v} \sim N(a, \sigma_v^2)$. An optimistic (pessimistic) trader mistakenly believes that the mean has a value of $a > 0$ (< 0). The next psychological trait concerns the beliefs of the variances of both prior information and signal noise. In our work, an irrational trader j behaves as if his signal \tilde{s}_j were drawn according to the following noise distribution $\tilde{\varepsilon}_j \sim N(0, \frac{1}{\kappa} \sigma_\varepsilon^2)$. The parameter $\frac{1}{\kappa}$ denotes the level of confidence. The case $\kappa > 1$ (respectively $\kappa < 1$) represents the case of an overconfident (respectively underconfident) trader.⁶ To simplify the notations,

⁶Note that the parameter κ takes into account both the overconfident/underconfident bias

all irrational traders participating in the market are of the same type, i.e. they misperceive the mean and the variance in the same way. We present in the Appendix a generalization with different types of irrational traders, including the case where some irrational traders are optimist only and others are overconfident/underconfident only. The results confirm the ones obtained with only one type of irrational traders. Notice also that irrational traders rationally anticipate the behavior of both the market maker and the remaining informed traders.

The main contribution of the paper is to consider that market makers are also irrational. Our paper provides an understanding of the effect of irrational market makers on the market. We assume that all market makers have homogeneous irrational beliefs, contrary to the traders, and behave competitively.⁷ Therefore, we can aggregate the behavior of the market makers and treat them as one player in the game. We assume that each market maker has no access to any private signal and that she misperceives the expectation and variance of the distribution of prior information. Each market maker believes that the distribution of the asset is such that

$$\tilde{v} \rightarrow N(\bar{a}, \bar{\kappa}\sigma_v^2).$$

We interpret the parameter $\bar{\kappa}$ as the “variance optimistic/pessimistic” effect or the “overconfidence/underconfidence” effect: “variance optimistic” market maker for $\bar{\kappa} < 1$ and “variance pessimistic” market maker for $\bar{\kappa} > 1$. At last, an optimistic market maker believes that $\bar{a} > 0$.

The trading protocol is identical to Kyle (1985). The strategy of each rational trader i is a Lebesgue measurable function, $X_i^r : \mathfrak{R} \rightarrow \mathfrak{R}$, such that $\tilde{x}_i^r = X_i^r(\tilde{s}_i)$ for $i = 1, \dots, N$.⁸ The strategy of each irrational trader j is identically defined: $X_j^{ir} : \mathfrak{R} \rightarrow \mathfrak{R}$ such that $\tilde{x}_j^{ir} = X_j^{ir}(\tilde{s}_j)$ for $j = 1, \dots, M$. Finally, the market maker is risk neutral and behaves competitively. She observes the aggregate order flow $\tilde{y} = \sum_{i=1}^N \tilde{x}_i^r + \sum_{j=1}^M \tilde{x}_j^{ir} + \tilde{u}$ before setting the price \tilde{p} . Let $P : \mathfrak{R} \rightarrow \mathfrak{R}$ denote a measurable function such that $\tilde{p} = P(\tilde{y})$.

We now give the definition of an equilibrium for our model.

Definition ($X_1^r, \dots, X_N^r, X_1^{ir}, \dots, X_M^{ir}, P$) *is an equilibrium if the price set by*

and the “better (worse)-than average” bias. Indeed, when deriving the equilibrium with two parameters κ_1 and κ_2 for the variances of the prior information and the noise respectively, it appears that only the ratio $\frac{\kappa_1}{\kappa_2}$ matters and so both effects vary jointly (see the detail of proof in Appendix under Comments on a Different Parameterization of the Model after the proof of Proposition 1). The results are then identical whether we choose one parameterization or the other. However the results are easier to present with only one parameter incorporating the misperception of all variances. Note that we have symmetrically defined the behavior of both the market makers and the traders. In particular both of them misperceive prior information.

⁷Considering a model where market makers have different parameters defining their irrationality possibly including that some market makers are rational is left for further research. Indeed, in this case the model would require market makers to behave strategically. Each of them would set a price taking into account the type of other market makers and so on. This type of framework where traders can split orders with different markers has been studied by Bernhardt and Hughson (1997).

⁸See Stokey and Lucas (2001) for a definition of measurable functions.

the market maker is such that

$$\tilde{p} = \overline{E} [\tilde{v} | \tilde{y}],$$

where \overline{E} denotes the fact the market maker's expectation is computed given her erroneous beliefs and, given that price, the market orders maximize the traders' expected profit conditional on the information received

$$\begin{aligned} X_i^r &\in \arg \max_{x_i^r \in \mathfrak{R}} E[(\tilde{v} - P(\tilde{y})) x_i^r | s = s_i] \quad \forall i = 1, \dots, N, \\ X_j^{ir} &\in \arg \max_{x_j^{ir} \in \mathfrak{R}} E^{ir}[(\tilde{v} - P(\tilde{y})) x_j^{ir} | s = s_j] \quad \forall j = N + 1, \dots, N + M, \end{aligned}$$

where E^{ir} denotes the fact that the expectation for the irrational trader is computed given his erroneous beliefs.

All agents know the number of rational and irrational traders as well as the type of the irrational traders. Traders behave strategically meaning that they take into account the impact of their orders on the price.

4 The equilibrium

In this section we solve for the general case where two types of investors (irrational and rational) and irrational market makers participate in the market. After presenting the general proposition, we analyze two special cases of that proposition: the case where all market participants are rational and the case where only some traders are irrational but not the market makers. This presentation will help us understand the effect of irrational traders and irrational market makers.

We now give the proposition establishing the form of the linear equilibrium.

Let us define τ as the noise-to-signal ratio i.e. $\tau = \frac{\sigma_v^2}{\sigma_y^2}$.

Proposition 1 *There exists a unique linear equilibrium if and only if the following condition is satisfied*

$$M\kappa(1 + 2\tau)^2 [\kappa(\bar{\kappa} - \tau) + 2\tau\bar{\kappa}] + N(\kappa + 2\tau)^2 [(2\tau + 1)\bar{\kappa} - \tau] > 0. \quad (1)$$

The form of the equilibrium is given by

$$\begin{aligned} x_i^r &= \alpha^{r*} + \beta^{r*} s_i, \quad \forall i = 1, \dots, N, \\ x_j^{ir} &= \alpha^{ir*} + \beta^{ir*} s_j, \quad \forall j = 1, \dots, M, \end{aligned}$$

$$p = \mu^* + \lambda^* y = \mu^* + \lambda^* \left(\sum_{i=1}^N x_i^r + \sum_{j=1}^M x_j^{ir} + u \right),$$

where the coefficients are given in the appendix. The parameters β^* 's represent the traders' responsiveness to private information whereas the α^* 's represent the part of their order not based on private information. The parameter λ^* represents the inverse of liquidity and μ^* the part of the price not depending on y .

Proof. See Appendix. ■

Impact of market maker’s over/under-confidence.

We first comment on condition (1). This condition identifies the situation where both the liquidity for the market and the quantity submitted by traders are finite. From the expression, one can see that a variance optimistic market maker exacerbates the occurrence of the non-existence of an equilibrium whereas a pessimistic one alleviates it. This can be explained as follows. A variance optimistic market maker thinks that prior information is more precise than it is and therefore believes that private information is less substantial than it actually is. As a consequence, she adjusts her price less aggressively and increases market depth, i.e. decreases λ^* . As a consequence, informed traders whether rational or irrational respond by trading more intensely, implying that the non-existence of equilibrium is more likely to occur. In other words a variance optimistic market maker leads to too much trading as it sets a high level of market depth leading to traders to trade more as their orders have less impact on the price. The exact opposite effects take place for a variance pessimistic market maker and therefore explains the fact that the non-existence of equilibrium is less likely to occur in that case. However, an equilibrium may not exist with variance pessimistic market makers if the overconfident traders are too overconfident. Note that this condition does not depend on the optimism parameters of the market makers or the traders.⁹

We now provide a more detailed analysis of the existence condition. It is useful to look at condition (1) as providing a lower bound for $\bar{\kappa}$ in order to have an equilibrium. In that case the condition can be rewritten as

$$\bar{\kappa} > \frac{\left(M\kappa^2(1+2\tau)^2 + N(\kappa+2\tau)^2\right)\tau}{(1+2\tau)(\kappa+2\tau)\left(M\kappa(1+2\tau)^2 + N(\kappa+2\tau)^2\tau\right)}.$$

A market breakdown can occur with only one type of traders present in the market provided that the market makers misperceive the variance. Indeed when $M = 0$, $\bar{\kappa}$ must be greater than $\frac{\tau}{2\tau+1}$ to obtain an equilibrium whereas when $N = 0$, we need $\bar{\kappa} > \frac{\tau\kappa}{2\tau+\kappa}$. Figure 1, below, presents the equilibrium existence condition for three different cases i.e. when $N = 0$, when $M = 0$ and when both M and N are strictly positive. The existence of an equilibrium is guaranteed when $\bar{\kappa}$ is above the curve corresponding to the particular situation. When both types of traders participate in the market, whether the existence condition is more restrictive or not than with only one type of traders depends on the level of overconfidence/underconfidence of the irrational traders and on how rational traders react to the presence of irrational traders. When irrational traders are overconfident ($\kappa > 1$), rational traders reduce their trading intensity to limit the impact of their order on the price as overconfident traders trade “too much”. This leads overall to a less restrictive existence condition. However, when irrational traders have a large level of underconfidence, rational traders

⁹This is mainly due to the structure of the model: linearity of the equilibrium and normal distribution functions.

react by increasing their trading intensity leading to a more restrictive existence condition. For a lower level of underconfidence, the existence condition can be more restrictive with the two types of traders or less restrictive than if one type of trader participates in the market.

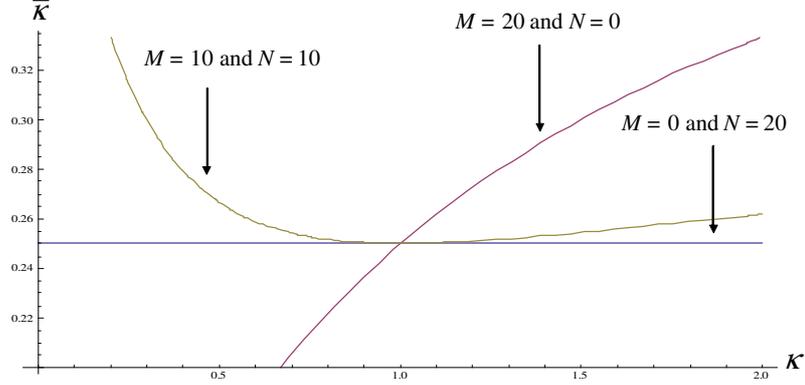


Figure 1: Lower bound for $\bar{\kappa}$ defined by the existence condition as a function of κ . The graph is done for $\sigma_v^2 = \sigma_u^2 = 1$ and $\sigma_\varepsilon^2 = 0.5$.

Impact of market maker's optimism/pessimism.

We now turn to the effect of the market maker's mean misperception onto the level of price. On the one hand, an optimistic (pessimistic) market maker increases (decreases) the overall level of price (through μ^*) as she wrongly believes that the expectation of the risky asset is higher (lower) than it actually is. On the other hand, she also corrects for the inflated or decreased order flow due to the misperception of the mean, a , by irrational traders. The presence of optimistic (pessimistic) traders induces an inflated (smaller) order flow which is corrected by the market maker by setting a negative (positive) intercept exactly equal to that inflated (smaller) order flow. The combination of the two effects determines the size and the sign of the intercept of the price function. When the market maker and the irrational traders hold opposite beliefs about the mean of prior information, the effect on the intercept is unambiguous.

The following Corollary establishes the form of the equilibrium for the case where all market participants are rational (Benchmark 1) and for the case where some traders are irrational whereas the market makers are rational (Benchmark 2).

Corollary 1

Benchmark 1: "All Rational Case" *The linear equilibrium has the following form*

$$x_i = \beta s_i, \text{ with } i = 1, \dots, M + N, \text{ with } \beta = \frac{\sigma_u}{\sigma_v \sqrt{(M+N)(1+\tau)}},$$

$$p = \lambda y, \text{ with } y = \sum_{i=1}^{M+N} x_i + u, \text{ with } \lambda = \frac{\sigma_v \sqrt{(M+N)(\tau+1)}}{[M+N+1+2\tau]\sigma_u}.$$

Benchmark 2: *There exists a unique linear equilibrium if and only if the following condition is satisfied*

$$M\kappa(1+2\tau)^2[\kappa(1-\tau)+2\tau]+N(\kappa+2\tau)^2(1+\tau)>0. \quad (2)$$

The form of the equilibrium is given by

$$\begin{aligned} x_i^r &= \hat{\beta}^r s_i, \quad \forall i = 1, \dots, N, \\ x_j^{ir} &= \hat{\alpha}^{ir} + \hat{\beta}^{ir} s_j, \quad \forall j = 1, \dots, M, \\ p &= \hat{\mu} + \hat{\lambda} = \hat{\mu} + \hat{\lambda} \left(\sum_{i=1}^N x_i^r + \sum_{j=1}^M x_j^{ir} + u \right), \end{aligned}$$

where the coefficients are given in the appendix.

Proof. The first benchmark is proved by setting in Proposition 1 $\kappa = 1$, $a = 0$, $\bar{\kappa} = 1$ and $\bar{a} = 0$.

The second benchmark is proved by setting in Proposition 1 $\bar{\kappa} = 1$ and $\bar{a} = 0$ while letting $\kappa \neq 1$, and $a \neq 0$. ■

The first benchmark corresponds to the case analyzed by Admati and Pfleiderer (1988) whereas the second one is qualitatively identical to Kyle and Wang (1997). As in Proposition 1, due to the presence of optimistic/pessimistic traders, the price function has a non-zero intercept, $\hat{\mu}$, as $\hat{\alpha}^{ir} \neq 0$. The market maker correctly anticipates that part of the order flow and the rational traders do not react to the irrational traders misperception of the mean i.e. $\hat{\alpha}^r = 0$.

The equilibrium may not exist in Benchmark 2 whereas it always exists in Benchmark 1. Comparing the two benchmarks we can see that this happens due to the presence of overconfident traders. More precisely this depends on the level of their overconfidence and on their number. We obtain that the equilibrium fails to exist when irrational traders are too overconfident ($\kappa > \frac{2\tau}{\tau-1}$) and when irrational traders are too numerous leading to “too much” trading.

We now compare the trading intensities, i.e. the reaction to private information β , of the informed market participants for the two benchmarks and the general case under study. Different effects are at work independently of the case. When only rational traders are present, a noisier information leads those traders to trade less intensely on their private information. When irrational traders are present, the previous effect is also at work for both irrational and rational traders. However, more noise implies that the more overconfident are irrational traders the more they trade on their private information. As explained before, rational traders respond to the presence of irrational traders. For very underconfident (overconfident) traders, rational traders react less (more) the noisier the private information. For intermediate values of confidence the effect is ambiguous. These effects are also affected by the number of traders but overall remain true for any numbers of rational and irrational traders. A variance optimistic (pessimistic) market maker amplifies (reduces) the previous effects. In Figure 2 and Figure 3, we draw the trading intensities, i.e. the responsiveness to private information as defined by the parameter β , as a function of κ where

the total number of traders is 20 from which 10 are rational. This is done for both types of traders rational and irrational. The values of a and \bar{a} do not affect trading intensities and we do not need to set them to any particular values.

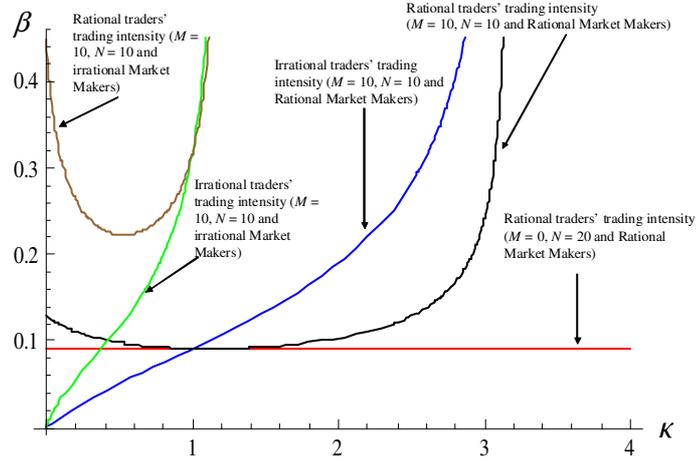


Figure 2: Trading Intensities, β 's, for both types of traders as a function of κ . When the market maker is assumed irrational, her irrationality is $\bar{\kappa} = 0.5$. The graph is done for $\sigma_v^2 = \sigma_u^2 = 1$, $\sigma_\varepsilon^2 = 5$ and $M + N = 20$.

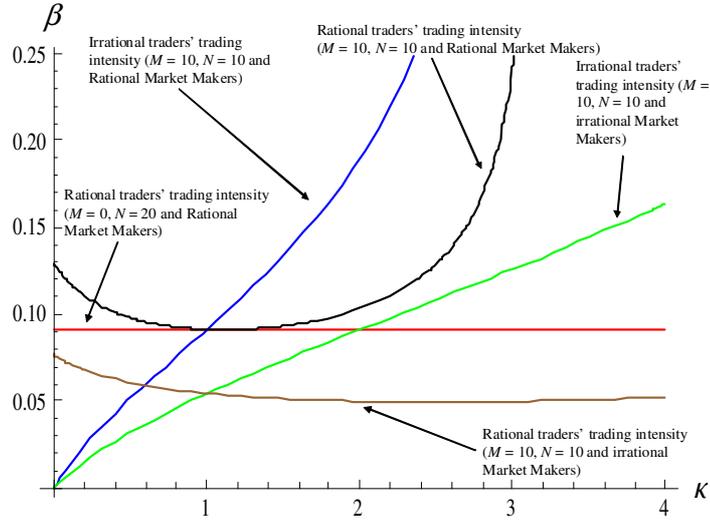


Figure 3: Trading Intensities, β 's, for both types of traders as a function of κ . When the market maker is assumed irrational, her irrationality is $\bar{\kappa} = 2$. The graph is done for $\sigma_v^2 = \sigma_u^2 = 1$, $\sigma_\varepsilon^2 = 5$ and $M + N = 20$.

We now compare the liquidity parameter, λ , across the different models. This is done in Figures 4 and 5. When both types of traders are present, overall, traders trade more on their private information leading to more liquidity with rational market makers and variance optimistic ones. In that case and as explained before variance optimistic market makers provide more liquidity than rational ones. Variance pessimistic market makers lead to less trading on private information which in turn leads to less liquidity in the market. As can be seen in the second graph however when irrational traders become more overconfident they increase their trading intensity which can lead to more liquidity than in the “all rational case”. In Figures 4 and 5, the number of traders is fixed equal to 20. When the two types of traders are present 10 are irrational traders whereas 10 are rational traders. The values of a and \bar{a} do not affect trading intensities and we do not need to set them to any particular values.

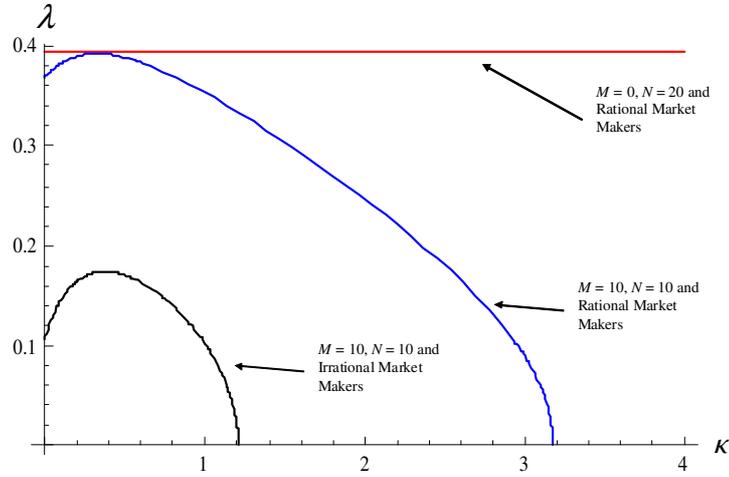


Figure 4: Inverse of Liquidity, λ , as a function of κ . When the market marker is assumed irrational, her irrationality $\bar{\kappa} = 0.5$. The graph is done for $\sigma_v^2 = \sigma_u^2 = 1$, $\sigma_\varepsilon^2 = 5$ and $M + N = 20$.

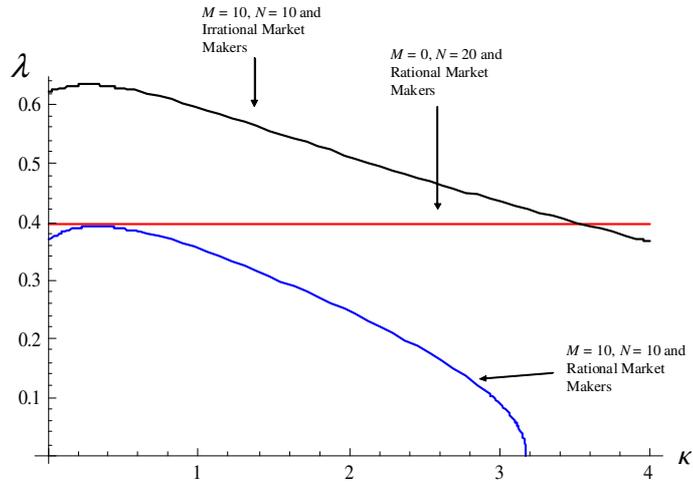


Figure 5: Inverse of Liquidity, λ , as a function of κ . When the market marker is assumed irrational, her irrationality $\bar{\kappa} = 2$. The graph is done for $\sigma_v^2 = \sigma_u^2 = 1$, $\sigma_\varepsilon^2 = 5$ and $M + N = 20$.

A question of interest is whether irrational traders can outperform rational traders. It should be pointed out that condition (3) is derived under the existence condition for the equilibrium. This is answered in the following propo-

sition.

Proposition 2 *The expected profits for the rational traders and irrational traders are given, respectively, by*

$$E[\Pi^{r*}] = \frac{(\kappa+2\tau)^2}{\lambda^* d^2} \left[\sigma_v^2 (\tau + 1) + \bar{a}^2 (2\tau + 1)^2 \right],$$

$$E[\Pi^{ir*}] = \frac{(2\tau+1)^2}{\lambda^* d^2} \left[\sigma_v^2 \kappa (\kappa (1 - \tau) + 2\tau) - \bar{a} (\kappa + 2\tau) \times (2\tau a - (\kappa + 2\tau) \bar{a}) \right],$$

where $d = (2\tau + N + 1)(\kappa + 2\tau) + M\kappa(2\tau + 1)$ and λ^* is given in the Appendix of Proposition 1.

An irrational trader outperforms a rational trader if and only if

$$\sigma_v^2 (\kappa - 1) (\kappa (1 - 2\tau^2) + 2\tau (1 + \tau)) > \bar{a} a (\kappa + 2\tau) (2\tau + 1)^2. \quad (3)$$

In particular, whenever the irrational traders and the market makers hold sufficiently different opposite beliefs about the mean of prior information (either $a > 0$ and $\bar{a} < 0$ or $a < 0$ and $\bar{a} > 0$) irrational traders can outperform rational traders. Note that this is also true for underconfident traders.

Proof. See Appendix. ■

The expected profits are computed under the true distributions of \tilde{v} and $\tilde{\varepsilon}$. The level of optimism or pessimism, a , does not impact the expected profit. This is due to two reasons. The first one being that a is independent of any relevant information for the market maker. The second one comes from the fact that the market maker perfectly evaluates the part of the order flow coming from the misperception of the mean and therefore rightly correct for it when setting the price. From the expressions of the expected profits, one can see that following intuition rational traders always obtain positive expected profits. Irrational traders may obtain positive or negative expected profits. This depends on their level of confidence, on the noise-to-signal ratio and on whether and to what extent the traders' beliefs about the mean are different to the market makers'.

Condition (3) is necessary and sufficient for the irrational traders to outperform the rational traders. This condition can hold only if the equilibrium exists, that is if condition (1) holds true. Under the equilibrium either irrational traders can outperform in expected terms the rational traders or the opposite. It should be pointed out that it does not depend on the number of rational traders M and the number of irrational traders N . This is due to the fact that M and N affect the expected profit of both the irrational and rational traders in exactly the same way, and when comparing expected profits their effect cancel each other. It is also independent of $\bar{\kappa}$. Indeed $\bar{\kappa}$ affects the market depth only and the rational and irrational traders react in exactly the same way to an increase or decrease of market depth due to $\bar{\kappa}$. Given that the market maker is also irrational, this condition depends on both the market maker's and the irrational trader's misperception of the mean, and on the irrational trader's beliefs about the variances.

As in the second benchmark case, an irrational trader can outperform a rational trader if his irrationality can act like a commitment device to trade large quantity. However, in the general case two components α^{irr*} and β^{irr*} are available to the irrational trader for this commitment device. Given the market maker's mean misperception, α^{irr*} is now a key element to analyze this commitment device. Beliefs about the mean from both the irrational traders and the market maker affect the coefficient α^{irr*} . Contrary beliefs, if sufficiently different, lead the irrational trader to buy when prices are too low on average or sell when prices are too high on average. For instance, if market makers are pessimistic ($\bar{a} < 0$), they decrease the overall level of price and if traders are optimistic ($a > 0$), they want to buy the asset. Having this combination of beliefs and sufficiently different beliefs implies that traders buy when prices are too low. This obviously increases the overall irrational traders' aggressiveness. As a consequence, compared to the second benchmark, traders with a higher level of overconfidence can outperform rational traders. Moreover, underconfident traders who trade less aggressively on their private information ($\beta^{irr*} < \beta^{r*}$) than their rational counterpart can fare better than rational traders. Finally that condition states that an irrational trader who is neither overconfident nor underconfident ($\kappa = 1$) but misperceives the mean only can outperform a rational trader as long as the mean misperception of that trader and of the market maker are opposite. In that case, we have $\beta^{irr*} = \beta^{r*}$, and α^{irr*} is used as the commitment device as $|\alpha^{irr*}| > |\alpha^{r*}|$.

We now look at the expected profit of the market maker. Market makers are competitive but irrational, therefore they compute the price such that $E[y(p - v) | y] = 0$ leading to $p = E(v | y)$. As they do not use the right beliefs as in Kyle (1985), we show that their profits can be positive or negative.

Lemma 1 (Market maker's expected profit)

The expected profit of the market maker is given by:

$$E[\Pi^{MM*}] = \frac{(2\tau+1)(\kappa+2\tau)}{\lambda^* d^2} [\sigma_v^2 (\bar{\kappa} - 1) (M\kappa(2\tau + 1) + N(\kappa + 2\tau)) + \bar{a}(2\tau + 1)(2M\tau a - \bar{a}(\kappa + 2\tau)(M + N))],$$

where $d = (2\tau + N + 1)(\kappa + 2\tau) + M\kappa(2\tau + 1)$ and λ^* is given in the Appendix of Proposition 1.

Proof: See Appendix.

The following table summarizes, when clear, how an irrational market maker performs overall.

$E[\Pi^{MM*}]$	$\bar{\kappa} < 1$			$\bar{\kappa} > 1$		
	$\bar{a} > 0$	$\bar{a} = 0$	$\bar{a} < 0$	$\bar{a} > 0$	$\bar{a} = 0$	$\bar{a} < 0$
$a > 0$	> 0 or < 0	< 0	< 0	> 0 or < 0	> 0	> 0 or < 0
$a = 0$	< 0	< 0	< 0	> 0 or < 0	> 0	> 0 or < 0
$a < 0$	< 0	< 0	> 0 or < 0	> 0 or < 0	> 0	> 0 or < 0

Table 1: Irrational Market Maker's Expected Profit ($E[\Pi^{MM*}]$)

The market maker, when rational ($\bar{\kappa} = 1$, $\bar{a} = 0$), obtains zero expected profit. However, when she is irrational, she may obtain an expected profit different from zero.

- The parameter $\bar{\kappa}$ affects the level of liquidity. An increase in $\bar{\kappa}$ increases λ . This reduces the informed traders' sensitivity to their private information. However, as noise trading is price inelastic and other things being equal, the market maker's expected profit increases when $\bar{\kappa}$ increases. This alone explains the results obtained when \bar{a} is equal to zero independently of the value of a .
- Other things being equal, the misperception of the mean by market makers has a negative effect on their expected profits. Indeed, an increase in price will lead traders to sell more whereas a decrease in price will lead them to buy more.
- Moreover, when the price increases (decreases) and if the irrational traders are pessimistic (optimistic) they are even more incline to sell (buy) leading to even more negative expected profits for the market maker. If the price increases and the irrational traders are optimistic, the market maker's expected profit is either positive or negative depending on the magnitude of both misperceptions of the mean. This explains why most of the cells when $\bar{\kappa} < 1$ have a determinate sign and why when $\bar{\kappa} > 1$ most of the cells have an indeterminate sign.

5 Impacts on financial markets

We now turn to some important measures of market performance such as price efficiency, ex-ante volatility and trading volume.

Lemma 2 (Price Efficiency and Ex-ante Volatility)

- The ex-ante volatility is equal to

$$\text{var}(\tilde{p}) = \frac{\sigma_v^2(N(\kappa+2\tau)+M\kappa(2\tau+1))((\kappa+2\tau)(N+\bar{\kappa}(2\tau+1))+M\kappa(2\tau+1))}{((N+2\tau+1)(\kappa+2\tau)+M\kappa(2\tau+1))^2}.$$

It increases with $\bar{\kappa}$. The effect of κ is ambiguous.

- The price efficiency is given by

$$[\text{var}(\tilde{v}|\tilde{p})]^{-1} = \frac{(N+\bar{\kappa}(2\tau+1))(\kappa+2\tau)+M\kappa(2\tau+1)}{\sigma_v^2\bar{\kappa}(\kappa+2\tau)(1+2\tau)}.$$

It decreases with $\bar{\kappa}$ and increases with κ .

Proof. See appendix. ■

As described before, an optimistic (pessimistic) market maker sets prices less (more) aggressively by increasing (decreasing) liquidity, traders respond to it by increasing (decreasing) their trading intensity. For the ex-ante volatility,

the effect on the market depth dominates the effects on the trading intensity. Regarding now the price efficiency, on the one hand an increase of $\bar{\kappa}$ decreases the traders' trading intensity and on the other hand it increases volatility.

We now explore the effect of each of the parameters defining the irrationality on the trading volume.

Lemma 3 (Trading Volume)

Let us define $\tilde{z} = \sum_{i=1}^N |\tilde{x}_i^r| + \sum_{j=1}^M |\tilde{x}_j^{ir}| + |\tilde{u}|$. The trading volume is given by

$$E[\tilde{z}] = N\alpha^r \operatorname{erf}\left(\frac{\bar{a}(2\tau+1)}{\sigma_v \sqrt{2(1+\tau)}}\right) + M\alpha^{ir} \operatorname{erf}\left(\frac{2\tau a - \bar{a}(\kappa+2\tau)}{\kappa\sigma_v \sqrt{2(1+\tau)}}\right) \\ + \sqrt{\frac{2}{\pi}} \left\{ \sigma_u + N\sigma_r \exp\left(-\frac{(\bar{a}(2\tau+1))^2}{\sigma_v^2 2(1+\tau)}\right) + M\sigma_{ir} \exp\left(-\frac{(2\tau a - \bar{a}(\kappa+2\tau))^2}{2\sigma_v^2(\kappa)^2(1+\tau)}\right) \right\},$$

where $\operatorname{erf}(x) = \frac{2}{\pi} \int_0^x \exp(-t^2) dt$, $\sigma_r = \sigma_v^2 (\beta^r)^2 (1 + \tau)$ and $\sigma_{ir} = \sigma_v^2 (\beta^{ir})^2 (1 + \tau)$.

Proof. See Appendix. ■

It is a well documented fact that overconfidence leads to greater volume traded [see Odean (1998b), Gervais and Odean (2001), Kyle and Wang (1997) and Benos (1998) among others]. We also find that result if we limit ourselves to the case of overconfident traders with a rational market maker and with irrational traders who do not misperceive the mean ($a = 0$). This is shown in Figure 6 below.

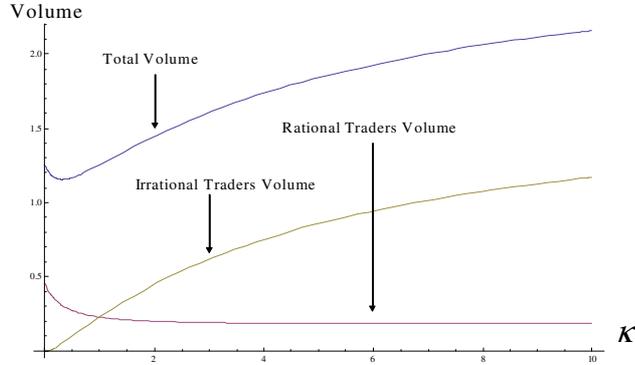


Figure 6: Total volume ($E[\tilde{z}]$), rational traders volume ($E\left(\sum_{i=1}^N |\tilde{x}_i^r|\right)$) and irrational traders volume ($E\left(\sum_{i=1}^N |\tilde{x}_i^{ir}|\right)$) as a function of the level of overconfidence when the market maker is rational ($\bar{a} = 0$ and $\bar{\kappa} = 1$) and $a = 0$. The simulations are done with $\sigma_v^2 = \sigma_\varepsilon^2 = \sigma_u^2 = 1$ and $M = N = 10$.

In Figure 6, we can see that increasing the level of overconfidence leads to two contradicting effects on the volume. Higher level of overconfidence leads rational traders to decrease the quantity they trade as they try to reduce the overall effect of the volume on price. The quantity traded by irrational traders increases with the level of overconfidence. This leads to the total volume depicted in the graph above.

However, when either $\bar{a} \neq 0$, or $a \neq 0$ or $\bar{\kappa} \neq 1$, this result is dramatically changed. Our model predicts high level of volume traded for extreme level of confidence (for the informed trader). We obtain that the trading volume can be a non-monotonic function (*U-shaped*) of the level of confidence. When $\bar{a} \neq 0$, traders increase their trading if the price is lower on average, or trade on the misperception of the mean ($a \neq 0$). The two parameters \bar{a} and a have the same effect on the traded volume. A larger mean misperception whether positive or negative implies greater volume traded. If this increase is greater than the reduction in volume due to the fact that traders trade less intensely on private information, we obtain the result highlighted in Figures 7 and 8 below.

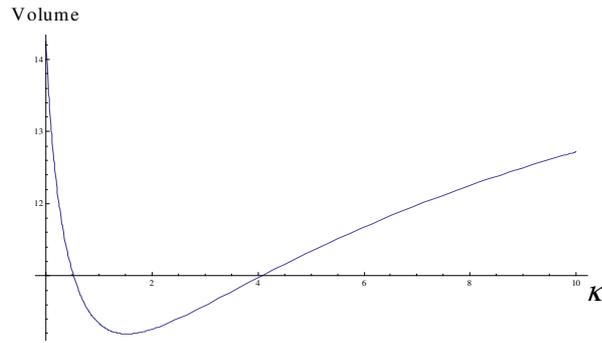


Figure 7: Total volume ($E[\tilde{z}]$) as a function of the level of overconfidence when $\bar{\kappa} = 1$, $\bar{a} = 1$ and $a = 0$. The simulations are done with $\sigma_v^2 = \sigma_\varepsilon^2 = \sigma_u^2 = 1$ and $M = N = 10$.

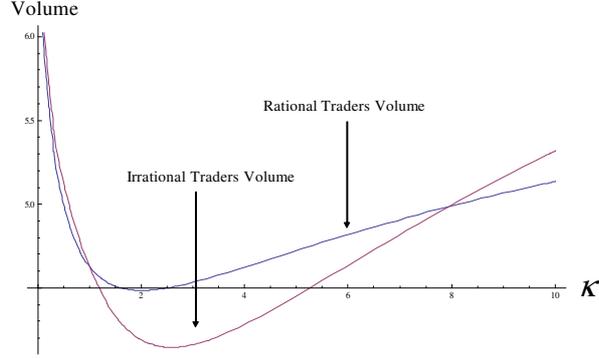


Figure 8: Rational traders volume ($E\left(\sum_{i=1}^N |\tilde{x}_i^r|\right)$) and irrational traders volume ($E\left(\sum_{i=1}^N |\tilde{x}_i^{ir}|\right)$) as a function of the level of overconfidence when $\bar{\kappa} = 1$, $\bar{a} = 1$ and $a = 0$. The simulations are done with $\sigma_v^2 = \sigma_\varepsilon^2 = \sigma_u^2 = 1$ and $M = N = 10$.

As explained before a lower $\bar{\kappa}$ implies more trading. In Figure 9, we show that if this over-trading is large enough it can result in large volume traded by underconfident traders as well as rational traders.

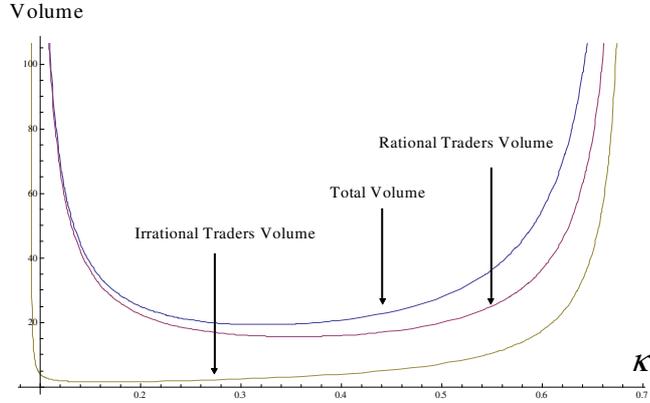


Figure 9: Total volume ($E[\tilde{z}]$), rational traders volume ($E\left(\sum_{i=1}^N |\tilde{x}_i^r|\right)$) and irrational traders volume ($E\left(\sum_{i=1}^N |\tilde{x}_i^{ir}|\right)$) as a function of the level of overconfidence when $\bar{\kappa} = 0.3$ and $a = \bar{a} = 0$. The simulations are done with $\sigma_v^2 = \sigma_\varepsilon^2 = \sigma_u^2 = 1$ and $M = N = 10$.

In Figure 10, we show the effect of the market maker's misperception of the

mean on the volume traded by the different traders. The effect of \bar{a} on the rational traders' and irrational traders' volume is symmetric. Extreme beliefs imply larger volume being traded, though the volume traded by irrational traders is also affected by their own misperception of the mean.

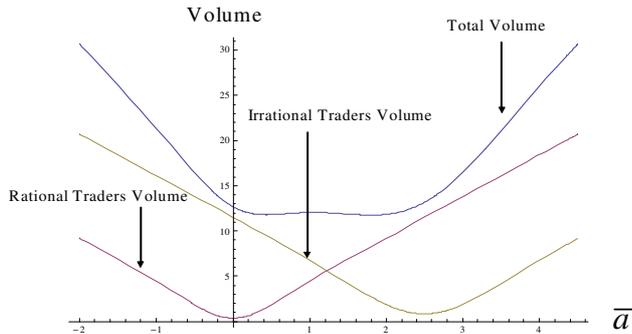


Figure 10: Total volume ($E[\tilde{z}]$), rational traders volume ($E\left(\sum_{i=1}^N |\tilde{x}_i^r|\right)$) and irrational traders volume ($E\left(\sum_{i=1}^N |\tilde{x}_i^{ir}|\right)$) as a function of the market maker's mean misperception when $\bar{\kappa} = 1$ and $a = 5$. The simulations are done with $\sigma_v^2 = \sigma_\varepsilon^2 = \sigma_u^2 = 1$ and $M = N = 10$.

To the best of our knowledge, our model is the first one to show that excessive volume can be a result of underconfidence in the market, and that irrationality in mean can impact the trading volume.

6 Conclusion

We develop a model of financial markets where irrational traders along with rational traders trade a risky asset with an irrational market maker. We model irrational traders as traders who, as well as misperceiving the expected returns of the asset, misperceive the variance of the volatility of the noise in the private signal. Those traders display different psychological traits: a pessimistic/optimistic one (misperception of the mean) and an underconfident/overconfident one (misperception of the variance of the noise in the private information).

We compare our general model to two benchmarks: an “all rational” case where all market participants are rational and the case where only a subset of traders are irrational with rational market makers. When comparing the two benchmarks we find that the presence of irrational traders may lead to the non-existence of the equilibrium due to “too much” trading taking place. We show that a variance optimistic market maker exacerbates the non-existence of equilibrium whereas a variance pessimistic market maker alleviates it. The

introduction of an irrational market maker affects differently the traders. Rational traders have larger expected profits when the market maker is irrational. The impact on the irrational traders' expected profit is not as clear. Irrational traders can have lower or greater expected profits due to the introduction of an irrational market maker. We show that a moderately underconfident trader can outperform a rational trader and it is true for an optimistic or pessimistic trader. The necessary condition to obtain that result is that the irrational traders and the market maker must hold opposite beliefs about the mean of prior information. This is a striking and new result. Moreover, we also show that an irrational market maker can, in expected terms, have positive profits. In addition, we show that the volume traded might be non-monotonic function of the traders' level of confidence.

Our model predicts high level of volume traded for extreme level of confidence including the case where traders are underconfident. This result raises the question whether the observed high volume is due to overconfidence or to underconfidence. This could be tested empirically as some indices such as the Index of Investor Optimism or the "Fund Manager Survey" by Merrill Lynch measure the level of confidence in the market. An experimental approach such as the one of Bloomfield et al. (2000) could be also be used to test our model. Our model also predicts high level of volatility.

An interesting extension of the model would be to look at how the results obtained in the present model would be modified in a dynamic setting by allowing traders to learn from the past. This is left for future research. The model could also be extended by assuming that market makers are heterogeneous with respect to their level of irrationality. The price competition in that context could be analyzed. This extension is also left for future research.

7 Appendix

7.1 Proofs

Proof of Proposition 1 (Irrational Market Makers)

After maximizing the traders expected utility we get for the different parameters

$$\alpha^{ir*} = \frac{1}{\lambda(M+1)} \left[a \frac{\tau}{\kappa + \tau} (1 - \lambda(M-1)\beta^{ir} - \lambda N\beta^r) - \mu - \lambda N\alpha^r \right],$$

$$\beta^{ir*} = \frac{\kappa(1 - \lambda N\beta^r)}{\lambda((M+1)\kappa + 2\tau)},$$

$$\alpha^{r*} = -\frac{1}{\lambda(N+1)} [\mu + \lambda M\alpha^{ir}],$$

$$\beta^{r*} = \frac{1 - \lambda M\beta^{ir}}{\lambda(N+1 + 2\tau)}.$$

The market maker sets a price, p , such that

$$p = \bar{E}[\tilde{v}|y] = \bar{E}[\tilde{v}] + \frac{\overline{cov}(\tilde{v}, y)}{\overline{var}(y)} (y - \bar{E}(y)), \quad (4)$$

where the upper bar denotes that the expectation, covariance and variance are computed given the wrong beliefs of the market maker.

Given the market maker's additive misperception we obtain

$$\lambda^* = \frac{(M\beta^{ir} + N\beta^r) \bar{\kappa}}{(M\beta^{ir} + N\beta^r)^2 \bar{\kappa} + (M\beta^{ir^2} + N\beta^{r^2}) \tau + \frac{\sigma_u^2}{\sigma_v^2}}, \quad (5)$$

$$\mu^* = (1 - \lambda M\beta^{ir} - \lambda N\beta^r) \bar{a} - \lambda M\alpha^{ir} - \lambda N\alpha^r. \quad (6)$$

Solving the above system of six equations with six unknowns leads if and only if $M\kappa(1+2\tau)^2[\kappa(\bar{\kappa}-\tau)+2\tau\bar{\kappa}] + N(\kappa+2\tau)^2((2\tau+1)\bar{\kappa}-\tau) > 0$ for the irrational traders

$$\alpha^{ir*} = \frac{(2\tau+1)(2\tau a - \bar{a}(\kappa+2\tau))\sigma_u}{\sigma_v \sqrt{M\kappa(1+2\tau)^2[\kappa(\bar{\kappa}-\tau)+2\tau\bar{\kappa}] + N(\kappa+2\tau)^2((2\tau+1)\bar{\kappa}-\tau)}},$$

$$\beta^{ir*} = \frac{\kappa(2\tau+1)\sigma_u}{\sigma_v \sqrt{M\kappa(1+2\tau)^2[\kappa(\bar{\kappa}-\tau)+2\tau\bar{\kappa}] + N(\kappa+2\tau)^2((2\tau+1)\bar{\kappa}-\tau)}},$$

for the rational traders

$$\alpha^{r*} = -\frac{\bar{a}(\kappa+2\tau)(2\tau+1)\sigma_u}{\sigma_v \sqrt{M\kappa(1+2\tau)^2[\kappa(\bar{\kappa}-\tau)+2\tau\bar{\kappa}] + N(\kappa+2\tau)^2((2\tau+1)\bar{\kappa}-\tau)}},$$

$$\beta^{r*} = \frac{(\kappa+2\tau)\sigma_u}{\sigma_v \sqrt{M\kappa(1+2\tau)^2[\kappa(\bar{\kappa}-\tau)+2\tau\bar{\kappa}] + N(\kappa+2\tau)^2((2\tau+1)\bar{\kappa}-\tau)}},$$

for the market maker

$$\mu^* = \frac{(2\tau+1)[\bar{a}(\kappa+2\tau)(M+N+1) - 2M\tau a]}{M\kappa(2\tau+1) + (2\tau+N+1)(\kappa+2\tau)},$$

$$\lambda^* = \frac{\sigma_v \sqrt{M\kappa(1+2\tau)^2[\kappa(\bar{\kappa}-\tau)+2\tau\bar{\kappa}] + N(\kappa+2\tau)^2((2\tau+1)\bar{\kappa}-\tau)}}{[M\kappa(2\tau+1) + N(\kappa+2\tau) + (2\tau+1)(\kappa+2\tau)]\sigma_u}.$$

Comments on a Different Parameterization of the Model.

An alternative model would have been to consider that irrational traders behave as if their signals, $\tilde{s}_j = \tilde{v} + \tilde{\varepsilon}_j$ for $j = 1, \dots, M$, were drawn according to the two following distributions

$$\tilde{v} \sim N(a, \kappa_1 \sigma_v^2),$$

$$\tilde{\varepsilon}_j \sim N(0, \kappa_2 \sigma_\varepsilon^2).$$

The market makers' irrationality remains the same. In that case solving for the linear equilibrium by following the same steps as in Proposition 1, would lead to the following result.

Result If and only if

$$M\kappa_1(1+2\tau)^2[\kappa_1(\bar{\kappa}-\tau)+2\kappa_2\tau] + N(\kappa_1+2\kappa_2\tau)^2[(2\tau+1)\bar{\kappa}-\tau] \geq 0,$$

there exists a unique linear equilibrium. It is characterized by the following parameters for the irrational traders

$$\begin{aligned}\check{\alpha}^{ir} &= \frac{(2\tau+1)(2\kappa_2\tau a - \bar{a}(\kappa_1+2\kappa_2\tau))\sigma_u}{\sigma_v \sqrt{M\kappa_1(1+2\tau)^2[\kappa_1(\bar{\kappa}-\tau)+2\kappa_2\tau]+N(\kappa_1+2\kappa_2\tau)^2((2\tau+1)\bar{\kappa}-\tau)}}, \\ \check{\beta}^{ir} &= \frac{\kappa_1(2\tau+1)\sigma_u}{\sigma_v \sqrt{M\kappa_1(1+2\tau)^2[\kappa_1(\bar{\kappa}-\tau)+2\kappa_2\tau]+N(\kappa_1+2\kappa_2\tau)^2((2\tau+1)\bar{\kappa}-\tau)}},\end{aligned}$$

for the rational traders

$$\begin{aligned}\check{\alpha}^r &= -\frac{\bar{a}(\kappa_1+2\kappa_2\tau)(2\tau+1)\sigma_u}{\sigma_v \sqrt{M\kappa_1(1+2\tau)^2[\kappa_1(\bar{\kappa}-\tau)+2\kappa_2\tau]+N(\kappa_1+2\kappa_2\tau)^2((2\tau+1)\bar{\kappa}-\tau)}}, \\ \check{\beta}^r &= \frac{(\kappa_1+2\kappa_2\tau)\sigma_u}{\sigma_v \sqrt{M\kappa_1(1+2\tau)^2[\kappa_1(\bar{\kappa}-\tau)+2\kappa_2\tau]+N(\kappa_1+2\kappa_2\tau)^2((2\tau+1)\bar{\kappa}-\tau)}},\end{aligned}$$

for the market maker

$$\begin{aligned}\check{\mu} &= \frac{(2\tau+1)[\bar{a}(\kappa_1+2\kappa_2\tau)(M+N+1)-2M\kappa_2\tau a]}{M\kappa_1(2\tau+1)+N(\kappa_1+2\kappa_2\tau)+(2\tau+1)(\kappa_1+2\kappa_2\tau)}, \\ \check{\lambda} &= \frac{\sigma_v \sqrt{M\kappa_1(1+2\tau)^2[\kappa_1(\bar{\kappa}-\tau)+2\kappa_2\tau]+N(\kappa_1+2\kappa_2\tau)^2((2\tau+1)\bar{\kappa}-\tau)}}{[M\kappa_1(2\tau+1)+N(\kappa_1+2\kappa_2\tau)+(2\tau+1)(\kappa_1+2\kappa_2\tau)]\sigma_u}.\end{aligned}$$

Comparing those results with the ones obtained in Proposition 1, it can be seen that by setting $\kappa = \frac{\kappa_1}{\kappa_2}$ in the model developed in the paper leads to the equilibrium described here.

Proof of Proposition 2 It is straightforward to show that the irrational traders' expected profit are equal to

$$\begin{aligned}E[\Pi^{ir*}] &= \frac{(2\tau+1)^2}{\lambda^* d^2} [\sigma_v^2 \kappa (\kappa(1-\tau) + 2\tau) \\ &\quad - \bar{a}(\kappa+2\tau) \times (2\tau a - (\kappa+2\tau)\bar{a})],\end{aligned}\tag{7}$$

for the rational traders

$$E[\Pi^{r*}] = \frac{(\kappa+2\tau)^2}{\lambda^* d^2} [\sigma_v^2 (\tau+1) + \bar{a}^2 (2\tau+1)^2],\tag{8}$$

where $d = (2\tau+N+1)(\kappa+2\tau) + M\kappa(2\tau+1)$ and λ^* is given in the proof of Proposition 1.

Given the above expressions, finding the sign of $E(\Pi^{ir*}) - E(\Pi^{r*})$ is equivalent to finding the sign of

$$\sigma_v^2 (\kappa-1) (\kappa(1-2\tau^2) + 2\tau(1+\tau)) - \bar{a}a(\kappa+2\tau)(2\tau+1)^2.$$

The comparison of the expected profits is then straightforward.

Proof of Lemma 1:

The market maker's expected profit are equal to

$$E[\Pi^{MM*}] = -NE(\Pi^r) - ME(\Pi^{ir}) + E(\Pi^{Liq}).$$

It is straightforward to show that the expected profit of the liquidity traders, $E(\Pi^{Liq})$, are equal to $-\lambda^* \sigma_u^2$. Plug the expressions found for the two types of

traders (7) and (8) and for the liquidity traders into the expression above and after some manipulations, one can get

$$E [\Pi^{MM}] = \frac{(2\tau+1)(\kappa+2\tau)}{\lambda^* d^2} [\sigma_v^2 (\bar{\kappa} - 1) (M\kappa(2\tau+1) + N(\kappa+2\tau)) + \bar{a}(2\tau+1)(2M\tau a - \bar{a}(\kappa+2\tau)(M+N))],$$

where $d = (2\tau + N + 1)(\kappa + 2\tau) + M\kappa(2\tau + 1)$.

Proof of Lemma 2: Straightforward.

Proof of Lemma 3:

The volume is given by

$$y = \sum_{i=1}^N |x_i^{r*}| + \sum_{j=1}^M |x_j^{ir*}| + |u|.$$

Given the form of both x_i^{r*} and x_j^{ir*} , we have that they both follow a normal distribution such that

$$x_i^r \sim N\left(\alpha^{r*}, \sigma_v^2 (\beta^{r*})^2 (1 + \tau)\right) = N\left(\alpha^{r*}, \sigma_r^{2*}\right)$$

$$x_j^{ir} \sim N\left(\alpha^{ir*}, \sigma_v^2 (\beta^{ir*})^2 (1 + \tau)\right) = N\left(\alpha^{ir*}, \sigma_{ir}^{2*}\right).$$

From Leone et al. (1961) we obtain

$$E[|x_i^{r*}|] = \sigma_r^* \sqrt{\frac{2}{\pi}} \exp\left(-\frac{(\alpha^{r*})^2}{2\sigma_r^{2*}}\right) + \alpha^{r*} \operatorname{erf}\left(\frac{\alpha^{r*}}{\sqrt{2\sigma_r^{2*}}}\right),$$

$$E[|x_j^{ir*}|] = \sigma_{ir}^* \sqrt{\frac{2}{\pi}} \exp\left(-\frac{(\alpha^{ir*})^2}{2\sigma_{ir}^{2*}}\right) + \alpha^{ir*} \operatorname{erf}\left(\frac{\alpha^{ir*}}{\sqrt{2\sigma_{ir}^{2*}}}\right),$$

$$E[|u|] = \sigma_u \sqrt{\frac{2}{\pi}},$$

where the function $\operatorname{erf}(x) = \frac{2}{\pi} \int_0^x e^{-t^2} dt$.

The expected volume or trading volume is then defined as

$$E\left[\sum_{i=1}^N |x_i^{r*}| + \sum_{j=1}^M |x_j^{ir*}| + |u|\right] = E[|u|] + NE[|x_i^{r*}|] + ME[|x_j^{ir*}|].$$

This leads to the expected volume equal to

$$\begin{aligned} & \sqrt{\frac{2}{\pi}} \left\{ \sigma_u + N\sigma_r^* \exp\left(-\frac{(\alpha^{r*})^2}{2\sigma_r^{2*}}\right) + M\sigma_{ir}^* \exp\left(-\frac{(\alpha^{ir*})^2}{2\sigma_{ir}^{2*}}\right) \right\} \\ & + N\alpha^{r*} \operatorname{erf}\left(\frac{\alpha^{r*}}{\sqrt{2\sigma_r^{2*}}}\right) + M\alpha^{ir*} \operatorname{erf}\left(\frac{\alpha^{ir*}}{\sqrt{2\sigma_{ir}^{2*}}}\right). \end{aligned}$$

Given the expressions of α^{r*} , α^{ir*} , σ_r^{2*} , and σ_{ir}^{2*} we obtain for the expected volume

$$\begin{aligned} & \sqrt{\frac{2}{\pi}} \left\{ \sigma_u + N\sigma_r^* \exp\left(-\frac{(\bar{a}(2\tau+1))^2}{2\sigma_v^2(1+\tau)}\right) + M\sigma_{ir}^* \exp\left(-\frac{(2\tau a - \bar{a}(\kappa+2\tau))^2}{2\sigma_v^2(\kappa)^2(1+\tau)}\right) \right\} \\ & + N\alpha^{r*} \operatorname{erf}\left(\frac{\bar{a}(2\tau+1)}{\sigma_v \sqrt{2(1+\tau)}}\right) + M\alpha^{ir*} \operatorname{erf}\left(\frac{2\tau a - \bar{a}(\kappa+2\tau)}{\kappa \sigma_v \sqrt{2(1+\tau)}}\right). \end{aligned}$$

7.2 Extension: Two types of Irrational Traders

In this subsection we only change the assumption concerning the homogeneity of irrational traders. We now assume that there are two groups of irrational traders: among the M irrational traders M_1 traders are of type 1 and M_2 traders are of type 2. A type 1 irrational trader behaves as if his signal, $\tilde{s}_j^1 = \tilde{v} + \tilde{\varepsilon}_j$ for $j = 1, \dots, M_1$, were drawn according to the two following distributions

$$\begin{aligned}\tilde{v} &\sim N(a^1, \sigma_v^2), \\ \tilde{\varepsilon}_j &\sim N\left(0, \frac{1}{\kappa^1} \sigma_\varepsilon^2\right).\end{aligned}$$

A type 2 irrational trader behaves as if his signal, $\tilde{s}_j^2 = \tilde{v} + \tilde{\varepsilon}_j$ for $j = 1, \dots, M_2$, were drawn according to the two following distributions

$$\begin{aligned}\tilde{v} &\sim N(a^2, \sigma_v^2), \\ \tilde{\varepsilon}_j &\sim N\left(0, \frac{1}{\kappa^2} \sigma_\varepsilon^2\right).\end{aligned}$$

The N remaining traders are assumed to be rational and are defined as before. The market makers are irrational.

Given the two types of irrational trader, the market maker observes the following aggregate order flow $\tilde{y} = \sum_{i=1}^N \tilde{x}_i + \sum_{j=1}^{M_1} \tilde{x}_j^{ir1} + \sum_{j=1}^{M_2} \tilde{x}_j^{ir2} + \tilde{u}$.

The following proposition shows the form of the equilibrium for this new setting.

Proposition 3 *There exists a unique linear equilibrium if and only if the following condition is satisfied*

$$\begin{aligned}M_1 \kappa^1 (1 + 2\tau)^2 (\kappa^2 + 2\tau)^2 (\kappa^1 (\bar{\kappa} - \tau) + 2\tau \bar{\kappa}) \\ + M_2 \kappa^2 (1 + 2\tau)^2 (\kappa^1 + 2\tau)^2 (\kappa^2 (\bar{\kappa} - \tau) + 2\tau \bar{\kappa}) + N (\kappa^2 + 2\tau)^2 (\kappa^1 + 2\tau)^2 ((\bar{\kappa} - \tau) + 2\tau \bar{\kappa}) \geq 0.\end{aligned}$$

The form of the equilibrium is given by

$$\begin{aligned}x_i^r &= \alpha^r + \beta^r s_i, \quad \forall i = 1, \dots, N, \\ x_j^{ir1} &= \alpha^{ir1} + \beta^{ir1} s_j, \quad \forall j = 1, \dots, M_1 \\ x_j^{ir2} &= \alpha^{ir2} + \beta^{ir2} s_j, \quad \forall j = 1, \dots, M_2\end{aligned}$$

$$p = \mu + \lambda y = \mu + \lambda \left(\sum_{i=1}^N x_i^r + \sum_{j=1}^{M_1} x_j^{ir1} + \sum_{j=1}^{M_2} x_j^{ir2} + u \right).$$

The expressions of the parameters are given in the appendix.

Proof:

After maximizing the traders' expected utility, as in the proof of proposition 2, we get the following system of equations

$$\begin{aligned}
\alpha^{ir1} &= \frac{1}{\lambda(M_1+1)} \left[a^1 \frac{\tau}{\kappa^1+2\tau} (1 - \lambda(M_1 - 1) \beta^{ir1} - \lambda N \beta^r - \lambda M_2 \beta^{ir2}) - \mu - \lambda N \alpha^r - \lambda M_2 \alpha^{ir2} \right], \\
\beta^{ir1} &= \frac{\kappa^1 (1 - \lambda N \beta^r - \lambda M_2 \beta^{ir2})}{\lambda((M_1+1)\kappa^1+2\tau)}, \\
\alpha^{ir2} &= \frac{1}{\lambda(M_2+1)} \left[a^2 \frac{\tau}{\kappa^2+2\tau} (1 - \lambda(M_2 - 1) \beta^{ir2} - \lambda N \beta^r - \lambda M_1 \beta^{ir1}) - \mu - \lambda N \alpha^r - \lambda M_1 \alpha^{ir1} \right], \\
\beta^{ir2} &= \frac{\kappa^2 (1 - \lambda N \beta^r - \lambda M_1 \beta^{ir1})}{\lambda((M_2+1)\kappa^2+2\tau)}, \\
\alpha^r &= -\frac{1}{\lambda(N+1)} [\mu + \lambda M_1 \alpha^{ir1} + \lambda M_2 \alpha^{ir2}], \\
\beta^r &= \frac{1 - \lambda M_1 \beta^{ir1} - \lambda M_2 \beta^{ir2}}{\lambda(N+1+2\tau)}.
\end{aligned}$$

The market maker sets a price, p , as in (4). This leads to the following two extra equations

$$\begin{aligned}
\lambda &= \frac{(M_1 \beta^{ir1} + M_2 \beta^{ir2} + N \beta^r) \bar{\kappa}}{(M_1 \beta^{ir1} + M_2 \beta^{ir2} + N \beta^r)^2 \bar{\kappa} + (M_1 \beta^{ir1^2} + M_2 \beta^{ir2^2} + N \beta^{r^2}) \tau + \frac{\sigma_u^2}{\sigma_v^2}}, \\
\mu &= (1 - \lambda M_1 \beta^{ir1} - \lambda M_2 \beta^{ir2} - \lambda N \beta^r) \bar{a} - \lambda M_1 \alpha^{ir1} - \lambda M_2 \alpha^{ir2} - \lambda N \alpha^r.
\end{aligned}$$

Solving the above system of eight equations with eight unknowns leads to the following result:

There exists an equilibrium iff the following condition is satisfied

$$\begin{aligned}
&M_1 \kappa^1 (1 + 2\tau)^2 (\kappa^2 + 2\tau)^2 (\kappa^1 (\bar{\kappa} - \tau) + 2\tau \bar{\kappa}) \\
&+ M_2 \kappa^2 (1 + 2\tau)^2 (\kappa^1 + 2\tau)^2 (\kappa^2 (\bar{\kappa} - \tau) + 2\tau \bar{\kappa}) + N (\kappa^2 + 2\tau)^2 (\kappa^1 + 2\tau)^2 ((\bar{\kappa} - \tau) + 2\tau \bar{\kappa}) \geq 0.
\end{aligned}$$

The equilibrium parameters are given by

$$\begin{aligned}
\beta^{ir1} &= \frac{\kappa^1 (1 + 2\tau) (\kappa^2 + 2\tau)}{\lambda d}, \\
\beta^{ir2} &= \frac{\kappa^2 (1 + 2\tau) (\kappa^1 + 2\tau)}{\lambda d}, \\
\beta^r &= \frac{(\kappa^1 + 2\tau) (\kappa^2 + 2\tau)}{\lambda d}, \\
\alpha^{ir1} &= \frac{(1 + 2\tau) (\kappa^2 + 2\tau) (2a^1 \tau - \bar{a} (\kappa^1 + 2\tau))}{\lambda d}, \\
\alpha^{ir2} &= \frac{(1 + 2\tau) (\kappa^1 + 2\tau) (2a^2 \tau - \bar{a} (\kappa^2 + 2\tau))}{\lambda d}, \\
\alpha^r &= -\frac{\bar{a} (1 + 2\tau) (\kappa^2 + 2\tau) (\kappa^1 + 2\tau)}{\lambda d},
\end{aligned}$$

$$\lambda = \frac{\sigma_v}{\sigma_u d} \left[M_1 \kappa^1 (1 + 2\tau)^2 (\kappa^2 + 2\tau)^2 (\kappa^1 (\bar{\kappa} - \tau) + 2\tau \bar{\kappa}) \right. \\ \left. + M_2 \kappa^2 (1 + 2\tau)^2 (\kappa^1 + 2\tau)^2 (\kappa^2 (\bar{\kappa} - \tau) + 2\tau \bar{\kappa}) + N (\kappa^2 + 2\tau)^2 (\kappa^1 + 2\tau)^2 ((\bar{\kappa} - \tau) + 2\tau \bar{\kappa}) \right]^{\frac{1}{2}}, \\ \mu = \frac{(1+2\tau)(\bar{\alpha}(1+M_1+M_2+N)(\kappa^2+2\tau)(\kappa^1+2\tau)-2\tau(a^2M_2(\kappa^1+2\tau)+a^1M_1(\kappa^2+2\tau)))}{d},$$

where

$$d = (\kappa^1 + 2\tau) (\kappa^2 + 2\tau) (1 + N + 2\tau) + N \kappa^1 \kappa^2 \\ + (1 + 2\tau) (2\tau (\kappa^1 M_1 + \kappa^2 M_2) + \kappa^1 \kappa^2 (1 + M_1 + M_2)).$$

■.

The form of the equilibrium is similar to the equilibrium found in proposition 1 with the presence of two types of irrational traders. The equilibrium condition can be interpreted in the same way as before that is when traders want to trade unbounded quantity and the market want to supply infinite liquidity the equilibrium fails to exist.

We do not show the condition for which irrational traders can fare better than rational traders as the condition is more cumbersome than before. However, the intuition of the condition is the same as for the previous case.¹⁰

A particular case of interest would be to consider the case where one group is optimist (type 1) whereas the other one is overconfident/underconfident (type 2). The following lemma gives the form of the equilibrium.

Lemma A1: There exists a unique linear equilibrium if and only if the following condition is satisfied

$$(M_1 + N) (\kappa^2 + 2\tau)^2 (\bar{\kappa} (1 + 2\tau) - \tau) + M_2 \kappa^2 (1 + 2\tau)^2 (\kappa^2 (\bar{\kappa} - \tau) + 2\tau \bar{\kappa}) \geq 0.$$

The form of the equilibrium is given by

$$x_i^r = \alpha^r + \beta^r s_i, \quad \forall i = 1, \dots, N, \\ x_j^{ir1} = \alpha^{ir1} + \beta^{ir1} s_j, \quad \forall j = 1, \dots, M_1 \\ x_j^{ir2} = \alpha^{ir2} + \beta^{ir2} s_j, \quad \forall j = 1, \dots, M_2$$

$$p = \mu + \lambda y = \mu + \lambda \left(\sum_{i=1}^N x_i^r + \sum_{j=1}^{M_1} x_j^{ir1} + \sum_{j=1}^{M_2} x_j^{ir2} + u \right).$$

The expressions of the parameters are given by

$$\beta^{ir1} = \beta^r = \frac{(1 + 2\tau) (\kappa^2 + 2\tau)}{\lambda d_1}, \\ \beta^{ir2} = \frac{\kappa^2 (1 + 2\tau) (\kappa^1 + 2\tau)}{\lambda d_1},$$

¹⁰All these computations all available upon request from the authors.

$$\alpha^{ir1} = \frac{(1+2\tau)(\kappa^2+2\tau)(2a^1\tau - \bar{a}(1+2\tau))}{\lambda d_1},$$

$$\alpha^{ir2} = \alpha^r = -\frac{\bar{a}(1+2\tau)^2(\kappa^2+2\tau)}{\lambda d_1},$$

$$\lambda = \frac{\sigma_v(1+2\tau)}{\sigma_u d_1} \left[(M_1 + N)(\kappa^2 + 2\tau)^2 (\bar{\kappa} - \tau + 2\tau\bar{\kappa}) \right. \\ \left. + M_2 \kappa^2 (1+2\tau)^2 (\kappa^2 (\bar{\kappa} - \tau) + 2\tau\bar{\kappa}) \right]^{\frac{1}{2}},$$

$$\mu = \frac{(1+2\tau)(\bar{a}(1+M_1+M_2+N)(\kappa^2+2\tau)(1+2\tau) - 2\tau a^1 M_1 (\kappa^2+2\tau))}{d_1},$$

where

$$d_1 = (1+2\tau)(\kappa^2+2\tau)(1+N+2\tau) + N\kappa^2 \\ + (1+2\tau)(2\tau(M_1 + \kappa^2 M_2) + \kappa^2(1+M_1+M_2)).$$

Proof. This Lemma is proved by setting $\kappa^1 = 1$ and $a^2 = 0$ in the proof of the previous Proposition. ■

As before the intuition of that configuration is the same as the previous one.

8 Bibliography

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