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A Competing Risk Decomposition of Average Duration Effects[1]

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Abstract

In 2009 the Irish government cut the unemployment benefit paid to 18 year olds from €204 to €100. Previous work has shown that this benefit cut resulted in substantially shorter unemployment durations for those subject to the cut. However, in addition to knowing the effect of a benefit cut on duration it is also important to know the relative importance of alternative exit states in explaining these shorter durations. In this paper I propose a competing risks decomposition which is valid irrespective of the dependence structure across exit states or the shape of the underlying hazards. The decomposition distinguishes between incidence and duration effects and also provides an intuitive visual representation of these effects. Although the aggregate effect of the Irish benefit cut is similar across genders, the decomposition shows that the channels through which the benefit cut affects labour supply differs substantially across genders. While the incidence effect is relatively unimportant for men, this is not the case for women, with the benefit cut resulting in a substantial shift away from exits to inactivity and into exists to work for young female claimants.

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1. Introduction

Ireland was one of the countries worst affected by the Great Recession, with the unemployment rate rising from 4.5% in 2007 to 15% in 2012. As in many countries, younger workers were hardest hit (van Ours 2015), with youth unemployment in Ireland rising from 9.1% to 30.4% over the same period. In response to increasing youth unemployment, in 2009 the Irish government substantially reduced unemployment benefit for those aged 18 and 19. For those affected by the cut, weekly benefits were cut from €204.30 to €100. Doris et al. (2017) use a regression discontinuity approach to evaluate the impact of this cut and find that those subject to the cut had significantly shorter unemployment durations. In this paper I provide a detailed analysis of the relative importance of different exit states in explaining this aggregate effect.

To examine the importance of alternative exit states one must consider how to deal with competing risks; a competing risk is an event whose occurrence prevents the occurrence of another outcome. Such competing risks may be important when analysing unemployment durations. Unemployment spells can end for many reasons, including exits from the labour force, exits to a job (full/part-time) or exits to training and modelling competing risks may have important policy implications. Unemployment durations that end because the claimant moves from unemployment assistance to an alternative welfare payment or leaves the labour force, may be less desirable than unemployment spells that end because the claimant finds a job (van den Berg et al. 2017).

When spells can end for a variety of reasons, researchers and policy makers may wish to understand the impact of competing exits on the overall effect. The typical approach in labour economics is to estimate cause-specific competing risk hazard functions (Han and Hausman 1990, Narendranathan and Stewart 1993). However, estimation of these models typically imposes strong assumptions on the dependence structure across exit states. Furthermore, I will show that cause-specific hazards are not the appropriate approach for decomposing differences in average duration, even when the maintained assumptions are valid. This follows from the fact that cause-specific hazards, while useful in examine the rate of occurrence of an outcome in the subset of people who are event free, are less useful in examining the absolute risk of a cause-specific outcomes over time (Austin et al. 2016).

In this paper, I extend previous work on competing risks by proposing a new decomposition that focuses on the differences in durations between two groups. The new approach decomposes the overall difference in durations into the contributions of distinct exit states, distinguishing between duration and incidence effects, and is valid irrespective of the dependence structure across exit states. The decomposition has an intuitive visual representation and builds on the Cumulative Incidence Function approach previously used in competing risk analysis. I use this decomposition to re-examine the labour market response of 18 year olds jobseekers in Ireland to the 50% cut in unemployment benefits introduced in in 2009. Although the aggregate effect of the benefit cut is similar across genders, the decomposition shows that the channels through which the benefit cut affects labour supply differs substantially for males and females.

The outline of the paper is as follows. Section 2 provides a short overview of previous work on the relationship between unemployment benefit and duration, before discussing in detail the competing risks approach to duration analysis. In doing so I distinguish between the cause-specific hazard approach and cause-specific cumulative incidence approach. Section 3 derives the new decomposition and discusses how this decomposition relates to existing approaches. Section 4 discusses the benefit cut analysed in this paper, namely the 50% cut in unemployment assistance for young claimants introduced in Ireland in 2009. Section 5 outlines the administrative data used to evaluate this cut and Section 6 provides the results of the decomposition focusing on gender differences in response to the cut. Section 7 concludes.

2. Literature Review

Doris et al. (2017) and Tatsiramos and Van Ours (2014) provide recent summaries of empirical research examining the impact of unemployment benefit rates on unemployment duration. In a basic job search model a cut in unemployment benefit is expected to reduce unemployment duration by reducing reservation wages and/or increasing search intensity. Many of the empirical papers find evidence consistent with this prediction with estimated benefit duration elasticities ranging between 0.5 and 1.0. However, while these overall effects are of interest, they tells us nothing about the relative contribution of different exit states to the shorter durations.

When unemployment spells can end for multiple reasons (e.g. a job, a training course, an education course) a competing risks framework is required. A competing risk is an event whose occurrence precludes the occurrence of another event. One cannot observe people who exit unemployment to training also terminating the same spell with an exit to work. However, such a distinction be may be crucial for policy makers. Shorter durations that are driven by exits out of the labour market are likely to be valued differently than shorter durations driven by exits to work (van den Berg et al 2017).

When fitting models in the presence of competing risk data, researchers can choose to model either the cause-specific hazard or on the cause-specific cumulative incidence function (CIF). The use of cause-specific hazards allows one to estimate the effect of covariates on the rate of occurrence of a specific outcome in the subset of the population who are event free at the specified time. Specifically, the exit-k cause-specific hazard, λ_k, is given by

$$\lambda_k(\mathrm{t}) = \lim_{\Delta t \downarrow 0} \frac{\Pr(t \leq Y \leq t + \Delta, K = k | Y \geq t)}{\Delta t}$$

where Y denotes duration and k denotes a given exit state. The typical approach to dealing with competing risks in labour economics is to estimate $\lambda_k(\mathrm{t})$, either parametrically or non-parametrically (Heckman and Honore 1989, Edin 1989, Katz and Meyer 1990, Han and Hausman 1990, Narendranathan and Stewart (1991, 1993), Pudney and Thomas 1995, Dolton and O'Neill 1996, McVicar 2008 and van den Berg et al. 2017). In this approach competing risks are typically modelled in terms of a set of latent random variables, with each latent variable representing the time to failure from a given cause. The observed data includes the smallest of the latent variables and the cause of failure. While the cause-specific hazards are identified in this approach, neither the joint distribution of the latent variables or the marginal survival functions are identified. Cox (1962) shows that for any joint distribution of latent failure times across states, there exists a corresponding joint distribution with independent failure times, which gives rise to the same cause-specific hazards. Therefore, in the absence of regressors, the latent variable competing risk hazard approach is not identified without parametric restrictions. Heckman and Honore (1989) show that in a model with regressors, it is sometimes possible to identify the joint distribution of failure times in a continuous time model without distributional assumptions; the main identifying assumption with single spell data requires that the effect of regressors differ across exit states. However, this restriction cannot be tested using observable data.[2]

[2] For a discussion of some of some of these identification issues see Heckman and Honore (1989), Han and Hausman (1990), Omori (1998) and Kalblfeisch and Prentice (2002).

In the absence of exclusion restrictions, recovery of the joint and marginal distributions of the latent variables in a competing risks framework requires distributional assumptions. Typically this involves assuming that the competing states are independent. The independence assumption means that at a given time, *t*, subjects who remain at risk of exit to state *k*, have the same future risk for the occurrence of event *k* as those who have exited to other states by time *t*; exits to competing risks are non-informative about exits to the risk of interest. Given independence, one can recover the marginal distributions and the joint distributions from the estimated cause-specific hazards.

However, a number of issues arise with this approach. Firstly, in the biomedical literature, independent risks are considered unlikely (Austin et al. 2016). The same is likely to be true when analysing unemployment durations. In his analysis of the Hartz labour market reforms in Germany, Price (2016) distinguishes between exits to regular jobs and exits to mini-jobs, a legally distinct class of low paid, part-time jobs and argues that exits to these states are unlikely to be independent. He argues that exits to these states may be either complements (if some claimants receive more job offers across the board) or substitutes (if searching for a regular job crowds out search effort for mini-jobs).

Secondly, even in situations where independent competing risks may be reasonable, naïve constructs based on the single cause-specific hazards are unlikely to have useful probabilistic interpretations (Putter et al. 2007, Austin et al. 2016, Varadhan et al. 2010). To see this, consider estimating the incidence of failures from cause *k* by time *t*; $\Pr(Y \leq t, K=k)$. This is known as the cause-specific Cumulative Incidence Function ($CIF_k(t)$).[3] Under independence of competing risks,

[3] It is also referred to as marginal or crude failure probability.

the marginal survival distribution, $S_k(t) = \exp(-\int_0^t \lambda_k(s)\,ds)$ is identified. One possibility therefore is to use estimates of 1-$S_k(t)$ to estimate $\mathrm{CIF}_k(t)$. However, even if the maintained independence assumption is correct, one can show that 1-$S_k(t)$ is an upward biased estimator of the probability of having failed from cause *k* by time *t*. To see this note that 1-$S_k(t)$ can be rewritten as $\int_0^t \lambda_k(s) S_k(s) ds$. Furthermore note that the probability of having failed by cause *k* by time *t*, is given by

$$\Pr(\mathrm{Y} \leq \mathrm{t}, \mathrm{K} = \mathrm{k}) = CIF_k(t) = \int_0^t \lambda_k(s) S(s) ds$$

where $S(t) = \exp\left(-\sum_{k=1}^{K}\left[\int_0^t \lambda_k(s)\,ds\right]\right)$ is the probability of not having failed from *any* event as of time t. Since $S(t) \leq S_k(t)$, it is clear that the naïve estimator overestimates the cumulative incidence of an event in the presence of competing risks, even when the risks are independent.[4] For the naïve estimator to have a meaningful probabilistic interpretation one must think of a world in which the competing events do not occur (Austin et al. 2016), a world which is of limited relevance for policy evaluation.

From this it is also clear that the effect of a covariate on the cause-specific hazard function may be very different to its effect on the likelihood of exiting due to that cause. As noted above, the likelihood of having left due to cause *k*, by time *t*, depends on *all* the cause specific hazards. Consequently, the effect of any given covariate on the likelihood of exiting due to cause *k*, will be affected indirectly by the effect of the covariate on all competing causes. In general, it is difficult to deduce the sign of a covariate effect on the CIF directly from the cause-specific hazard

[4] For further discussion of this and other issues that arise with competing risks see Andersen et al. (2002, 2012).

parameters (Kyyra 2009). A covariate may have a strong effect on the cause-specific hazard but no effect on the cumulative incidence.

For these reasons Coviello and Bogges (2004) and Austin and Fine (2017) recommend, that that when competing risks are present, the appropriate estimate of the probability of failure from a specific cause (cause-specific incidence) requires the specification and estimation of the CIF or the associated sub-distribution hazard. Fine and Gray (1999) discuss how estimates of the sub-distribution hazard can be used to determine the effect of covariates on the cause-specific CIF under the assumption of a proportional hazard model for the sub-distribution.

The objective of the current paper is to decompose the overall treatment effect of the unemployment benefit reduction into competing risks components. A small number of recent papers in biostatics have proposed competing risk decompositions of life-expectancy, distinguishing between causes of death (e.g. Beltran-Sanchez et al. (2008) and Vaupel and Canudas-Romo (2003). However, as noted in Andersen et al. (2013) none of these proposed measures are additive, while some require independence of causes. Andersen (2013) proposes an alternative decomposition in which the cause-specific components can be interpreted as the expected number of life-years lost due to each cause before a specified date. The Andersen decomposition is additive, does not require independence and the individual terms can be represented in terms of the area underneath the CIF. However, as I will show in the next section, analysis based solely on the areas under the CIF can be confounded by differences in prevalence rates of the competing events.[5] To overcome this I develop an extended decomposition which

[5] For a related discussion of this issue see Huang et al. (2016)

isolates the separate cause-specific duration and incidence effects. I show that these separate components can be written in terms of re-weighted CIFs.

Although the use of CIFs is common in the biostatistics literature (Varadhan et al. 2010) it has rarely featured when modelling unemployment durations. Exceptions include Lo and Wilke (2010) and Lo et al. (2017), who combine estimated CIFs with a parametric copula, to estimate the marginal distribution of latent durations in the presence of competing risks; durations that would be observed in the absence of competing risks. However, as noted above, the interpretation of these marginal distributions is not straightforward. Price (2016) uses both cause-specific hazards and the cumulative incidence functions to model the effect of the Hartz labour market reform on unemployment durations in Germany. He argues that to assess how shifts between full-time and part-time jobs impact estimated wages post-reform, what matters are not the hazard rates, but rather how these rates translate into the share of claimants who eventually end up in each state.

In the remainder of this paper, I extend the previous work on competing risks by proposing a new competing risks decomposition of differences in average duration between groups. The new approach is valid irrespective of the dependence structure across exit states or the shape of the underlying CIFs. Furthermore the new approach permits an extended decomposition that distinguishes between differences in durations, conditional on an exit state, and differences in the prevalence of exits to particular states. When prevalence rates are equal across groups my decomposition is equivalent to the decomposition proposed by Andersen (2013). However, when incidence rates differ, the decomposition proposed in this paper captures both a duration and an incidence effect. This extension is particularly useful if some exit states are seen as more desirable than others are, as is often the case when modelling unemployment duration.

3. Competing Risk Decomposition

In this section, I examine the contribution of competing exit states to the overall difference in mean durations between two groups. I decompose the overall duration effect into separate state specific components, which can be written as simple functions of the CIF. The decomposition is easy to implement and does not require independence across exit states. I also present a visual representation of the decomposition that helps understand the contribution of differences in the timing of exits across states to the overall duration effect.

I define the treatment effect as the difference in average duration between the treated group and the control group, as would be the case with random assignment (LaLonde 1986, Bloom et al. 1997, Dolton and O'Neill, 2002, van den Berg et al. 2006). Specifically, the treatment effect is

$$\Delta Y = E[Y_T] - \quad E[Y_C]$$

where Yi is the random variable representing the duration of unemployment for group $i=T,C$, with upper bound Y_{Mi}. Let $Y_M=\max\{Y_{Mi}\}$ and for exposition assume that all spells are completed and identify Y_M by the maximum duration observed.[6] The competing risk model has K possible exit states, and denote the overall proportion of group i exiting into each of the K states by f_{ki} for $k=1....K$. In my analysis the groups consist of a treatment group, $i=T$, subject to the treatment and a control group, $i=C$, who do not receive the treatment. By definition of the

[6] I will discuss extensions to censored durations in the next section.

cumulative incidence function for state *k* and group *i*, (CIF_{ki}), it follows that $f_{ki} \equiv CIF_{ki}(Y_M)$ and in the absence of censoring $\sum_k \{f_{ki}\} = 1$, for *i*=*T*,*C*.

The starting point of our analysis is a cause-specific decomposition similar to that proposed by Andersen (2013). Following his exposition it can be shown that that the difference in average durations between the control and treatment group can be written as

$$\Delta Y = \sum_k \left[\left\{ Y_M f_{kT} - \int_0^{Y_M} [(CIF_{kT}(y))] dy \right\} - \left\{ Y_M f_{kC} - \int_0^{Y_M} [(CIF_{kC}(y))] dy \right\} \right] \quad (1)$$

where the terms inside the square brackets represent the contribution of state k to the overall difference in durations. Assuming all spells are completed, so that $\sum_k \{f_{ki}\} = 1$, we can further simplify this decomposition, and rewrite $\Delta Y = \sum_k \left\{ \int_0^{Y_M} \{CIF_{kC}(y) - CIF_{kT}(y)\} \, dy \right\}$; this is the decomposition proposed by Andersen (2013). Both the decomposition in (1) and the related Andersen decomposition, measure the cause-specific contribution to differences in durations by the differences in the areas under the cause specific CIFs. However, as noted by Huang et al. (2016), analysis based solely on the areas under the CIFs can be confounded by differences in the prevalence rates of the competing events across groups. This is explicit in decomposition (1) but is also true of the related Andersen decomposition. For instance using just the difference in areas under the CIFs it would not be possible to distinguish between a programme that resulted in fewer people moving to work (incidence effect) but resulted in those who did move moving more quickly (duration effect), and a programme that had no effect at all on either the duration or incidence of people moving to work.

When analysing the contributions of exits to work to changes in unemployment durations it will often be important to distinguish between the impact of a programme on the duration of time before an exit to a specific state (duration effect) and its impact of the proportion of people moving to different states (incidence effect). This is not possible with the aggregate decomposition, which in turn limits its usefulness for policy makers.

I propose an extension of the aggregate decomposition given in (1) that tackles this issue by further decomposing the aggregate effect into separate duration effect and incidence effects.

To do this reconsider the overall decomposition

$$\Delta Y = \sum_k \left\{ \left\{ Y_M f_{kT} - \int_0^{Y_M} [(CIF_{kT}(y))] dy \right\} - \left\{ Y_M f_{kC} - \int_0^{Y_M} [(CIF_{kC}(y))] dy \right\} \right\}$$

This can be rewritten as

$$\Delta Y = \sum_k \left[\left\{ f_{kT} \left\{ \left[Y_M - \int_0^{Y_M} \left[\left[\frac{(CIF_{kT}(y))}{f_{kT}} \right] dy \right] \right] - \left[Y_M - \int_0^{Y_M} \left[\left[\frac{(CIF_{kT}(y))}{f_{kT}} \right] dy \right] \right] \right\} \right\} + \{f_{kT} - f_{kC}\} \left[Y_M - \int_0^{Y_M} \left[\frac{(CIF_{kT}(y))}{f_{kT}} \right] dy \right] \right] \quad (2)$$

Denoting $\left[\frac{(CIF_{kT}(y))}{f_{kT}}\right]$ as a cause-specific conditional cumulative incidence function ($CCIF_k$; see Huang et al. 2016) it can be shown that $\left\{ Y_M - \int_0^{Y_M} [CCIF_{kT}(y)] dy \right\} = E(Y_T | K_T = k)$ or equivalently $\int_0^{Y_M} [CCIF_{kT}(y)] dy = Y_M - E(Y_T | K_T = k)$.[7] In the same way that Andersen (2013) shows that the area under the unconditional CIF can be interpreted as the days lost due to

[7] For proof see Appendix 1.

cause *k*, we see that the area under the *conditional* cumulative incidence function as the days lost *conditional on exiting to state k.*

Combining the fact that $E(Y_i|K_i = k) = \left\{Y_M - \int_0^{Y_M}[CCIF_{kT}(y)]dy\right\}$ with equation (2) allows us to write the extended decomposition as

$$\Delta Y = \sum_k \left[\langle \underbrace{f_{kT}\{[E(Y_T|K_T = k)] - [E(Y_C|K_C = k)]\}}_{duration\ \ effect}\rangle + \langle \underbrace{\{f_{kT} - f_{kC})\}[E(Y_C|K_C = k)]}_{incidence\ \ effect}\rangle\right]$$

(3)

Written in this way it is clear that the first term of the extended decomposition measures the duration effect of state *k*, weighted by the incidence of *k* in the treatment group. If the cause specific incidences are the same for the control and treatment groups, $f_{kT} = f_{kC}$, then all of the state *k* effect, will be captured by the conditional duration term, with a negative contribution denoting quicker exits to a given state. The second term measures the incidence effect, weighted by average duration for group *C*.[8] Clearly if the average duration, conditional on being in state *k*, is the same for both groups, but more of the treatment group enter state *k*, then the first term will be zero; all of the state *k* effect will be picked up as a *positive* contribution in the incidence term. How one interprets this component depends on the desirability of the exit state. When exit states are seen as desirable (e.g. work) then a positive incidence effect reflects well on a programme. The opposite is true for exist states that are considered undesirable (e.g. exits into inactivity).

Some further manipulation allows us to rewrite the duration effect as

[8] As with all decompositions of this type, one can obtain an alternative decomposition by reversing the weightings.

$$f_{kT}\{[E(Y_T|K_T = k)] - [E(Y_C|K_C = k)]\} = \left\{\int_0^{Y_M} \omega_k[CIF_{kC}(y)]dy - \int_0^{Y_M}[CIF_{kT}(y)]dy\right\} \quad (4)$$

where $\omega_k = \frac{f_{kT}}{f_{kC}}$. Thus in the same way that Andersen (2013) shows that the unconditional difference in durations due to cause *k*, can be represented as the difference in the areas underneath the cause specific CIFs, rewriting the duration effect in this way shows that, in a similar fashion, the conditional duration effect due to cause *k*, can be represented as the difference in *rescaled* CIFs, where the scaling factor for the control group is simply ω_k.[9]

4. Application: Changes to Unemployment Benefit

I now use the above decomposition to further understand the impact of a substantial cut to youth unemployment benefits in Ireland in 2009. The social welfare system in Ireland is divided into three main types of payments; social insurance payments, means-tested payments, and universal payments. Jobseeker's Allowance (JA) is a means tested payment that is paid indefinitely to those who are unemployed provided the claimant continues to meet conditions of eligibility. To receive JA a claimant must satisfying the means test, be aged between 18 and 66, be unemployed and actively seeking work. In response to the fiscal crisis that occurred in Ireland with the onset of the Great Recession, the Irish government imposed substantial cuts on JA payments for young claimants. Prior to 2009, all claimants were entitled to €204 a week. In May 2009, claimants aged 18 had their weekly rate cut to €100. The fact that the cut only applied to new claimants allows me

[9] In practice the components of the decomposition in (4) can be easily estimated in Stata using the ***stplost*** command Overgaard et al. (2015).

to use those who entered unemployment just before and just after the legislation to identify the treatment effect.

In addition to exemptions for existing claimants, new claimants were exempted from the cut if they had a dependent child, if they had had a spell of unemployment in the previous 12 months or if they were transferring from Disability Allowance. We will take these exemptions into account in the analysis below, by using month of entry as an instrumental variable for treatment status.

Doris et al. (2017) analysed this benefit cut in detail using a Regression Discontinuity design and found a substantial reduction in unemployment durations as a result of the benefit cut. Claimants subject to the cut had unemployment durations that were 50% lower than the control group. However, given that the stated motivation for these cuts was to "ensure that young people are better off in education, employment or training than claiming,"[10] it seems appropriate to consider in detail the relative role of alternative exit states in explaining these aggregate effects. In the remainder of this paper, I apply the decomposition outlined in Section 3 to further evaluate this benefit cut, distinguishing between incidence and duration effects. To allow for possible differences in the response of the benefit cut by gender, I further extend the work of Doris et al. (2017) and conduct the analysis separately for males and females.

[10] http://www.welfare.ie/en/pressoffice/Pages/pr231013.aspx

5. Data

To carry out the analysis I use the Longitudinal Jobseekers Database provided by the Department of Social Protection. This is an administrative data set covering every claimant who has received a jobseekers or one parent family payment since 2007. The data provide administrative records for the start and end date of every new claim since January 2007, allowing me to establish both the exact start date and duration for the entire population of new JA claims initiated between 2007 and 2014. Throughout the analysis, I measure duration in weeks. When considering the competing risks model, I consider five different exit states; training, work, education, inactivity and “other”.

Table 1 provides summary statistics separately for those entering unemployment in April 2009 and for those entering in May 2009. The first row shows the proportion of both groups subject to the benefit cut. As expected those entering unemployment in the month following the legislation were much more likely to be affected by the cut. For both men and women, over 80% of those entering in May were subject to the cut. The exemptions noted earlier for those with dependent children or for those transferring from Disability Allowance account for the remaining 20% of claimants in this group. On the other hand less than 20% of those entering in April were subject to the benefit cut. In theory no one entering unemployment prior to the legislated date should have been subject to the cut. The fact that some of those entering unemployment in April were subject to the cut appears to be due to short delays in processing claims.

The second row of Table 1 reports the average unemployment duration, by month of entry and gender. For all four groups average unemployment duration exceeded one year, highlighting the poor labour market prospects faced by youths in Ireland during the Great Recession. However, those who entered in May 2009, and were covered by the 50% cut in JA, had unemployment durations that were on average 30 weeks shorter than those entering in April.[11] Since the date of legislation does not provide a sharp discontinuity in treatment status, I use the month of entry as an instrument for treatment status to identify a causal effect of the benefit cut. In this case the IV estimator is simply the Wald estimator, with the difference in durations by month of entry rescaled by difference in treatment status. Using this IV approach I estimate a causal effect of the benefit cut of -51.80 weeks for males and -43.46 weeks for females. These reductions in unemployment duration imply a benefit duration elasticity of 1.02 for males and 0.93 for females and are in keeping with previous international work (Tatsiramos and Van Ours (2014)). They are also similar to effects reported in Doris et al. (2017). They report an aggregate elasticity of 1.08 for this benefit cut using a regression discontinuity design, which estimates the effect using observations in the days before and after the legislation.

Of particular interest in my study is the labour market states to which claimants exited. The next five rows of Table 1 provide the proportion of each group exiting to each of the alternative states. Looking at men, we see that three exit states dominated; namely training, work and “other”. Furthermore, the proportions exiting to each state are similar for those entering in April and in May, suggesting that the aggregate duration effect for men is driven by differences in duration conditional on a given exit state. The third column of Table 1 shows, that for women subject to the benefit cut, the proportion leaving to each state is similar to those of men and are dominated by the same three exit states. However, for women in receipt of the higher benefit the pattern is

[11] In our application approximately 4% of observations are censored. In the results discussed here censored observations are omitted from the analysis. It is relatively straightforward to extend the decomposition to account for censoring (see Appendix 2). In this case the average duration effect can be interpreted in terms of restricted mean differences (see for example Royston and Parmar (2013)). Because of the small number of cases that are censored, controlling for censoring makes little difference in our application. For instance, the raw mean duration effect for men is -30 weeks when censored observations are omitted, -34.8 when censored observations are treated as complete and -34.88 when censoring is accounted for.

notably different. For this group, there is a uniform distribution across exit states, driven by a bigger proportion exiting to inactivity and education, at the expense of those exiting to work. While the results in column 2 and 3 suggest that differences in state specific durations are the driving force behind the male effect, the results in columns 4 and 5 suggest that differences in incidences may play a larger role for women. In the remainder of the paper, I use the decomposition outlined in Section 3 to examine these issues more formally.

6. Results

We being our analysis by estimating cause specific hazard functions. As noted earlier this is the typical approach used in labour economics when estimating competing-risks models. The results are given in Table 2. The estimates are obtained from a Cox proportional hazard model, treating exits to competing states as censored. The results are similar for both men and women, with significant a treatment effects evident in exits to training, work and "other", but not in exits to education or inactivity. However, as noted in Section 2, the cause-specific hazard approach, while useful for looking at exit rates, is less useful for understanding differences in absolute risk across states.

To obtain further insight into the relative importance of alternative exits states in explaining differences in average durations I carry out decomposition derived in Section 3. The results of the decomposition are given in Table 3. The first row reproduces the IV estimate of the overall treatment effect. As noted earlier I estimate a large and statistically significant duration response to the benefit cut for both men and women. The aggregate effect is similar across genders; males subject to the benefit cut have unemployment durations that are over 51 weeks shorter than those in receipt of the higher benefit, while the effect for females is approximately 44 weeks.

The results in columns one and four show the results for the basic decomposition, which makes no distinction between incidence and duration effects. Looking at the results for males, we see that no single exit state dominates the overall effect. While the contribution of exits to education and inactivity are small, the other three exit states, training, work and “other”, all contribute a substantial component to the overall effect. The shorter overall duration for 18 year old males is therefore a result of shorter durations to each of these three states. These findings are in keeping with the cause-specific hazard results reported in Table 2.

Although the overall aggregate effect for women is very similar to the male effect, the basic decomposition results show that this effect is generated through very different channels. While exits to work is the dominant factor for men, it appears to contribute the least to the female duration effect. In contrast, exits to inactivity, while not important for men, is the dominant channel for women. The fact that very similar aggregate effects for men and women, operate through very different channels, highlights the value of the competing risk decomposition developed in this paper. Furthermore, the states identified as important for women using the decomposition differ from those identified using the cause-specific hazard approach. While exits to inactivity are identified as the most important channel in the competing-risks decomposition, the importance of this exit state was not apparent using the cases specific hazard-approach.

The results of the extended decomposition are given in columns 2-3 (men) and 5-6 (women) and Figures 1 and 2. The results in Table 3 provides estimates for both the duration and incidence effects, while Figures 1 and 2 provide the graphical representation of the duration effect.

Looking at the duration effect for men, we see that, conditional on exiting to a given state, those in receipt of the lower benefit exited quicker than those in the control group. This is particularly true for the three main effects identified earlier; training, work and “other”. This is illustrated graphically in Figure 2, where for each of these three exit states the rescaled CIF for the control group lies below the CIF of the Treatment group.

The incidence effect on the other hand, shows that for men differences in the likelihood of exiting to a given state are less important. The fact that the work incidence effect for men is negative implies that members of the treatment group are *less likely* to exit to this state. Since work is likely to be considered a desirable state, this negative incidence effect would be viewed as an undesirable outcome. However, it is smaller than the duration effect, which sees treated men exit more quickly to work. Competing duration and incidence effects, such as these, are not possible under the proportional hazard assumption typically assumed in the literature. If members of the treatment group are more likely to exit to a state during the early part of their spell, then the proportional hazards assumption requires that this must be true at every duration. Proportional hazard models therefore restricts the duration and incidence effect to operate in the same way, a restriction that is violated in our example.

Overall, the male results suggest that the incidence effect is relatively unimportant for men; what matters for men is the timing of the exits to each of the states. However, the same is not true for women. Looking at the duration effects in Table 3, we see that the results are quite similar to those reported for men. Conditional on exiting to a given state, those in receipt of the lower benefit exit quicker than those in the control group. This is particularly true for work where the duration effect is larger for women than for men. This is also evident in the CIOFs presented in Figure 3.

However, in addition to the conditional duration effect, the fact that the work incidence effect is *positive* and large for women implies that women who are subject to the benefit cut are *more* likely to enter work. Both of these work effects would be considered desirable but they offset each other in the aggregate decomposition.

The importance of distinguishing between duration and incidence effects for women is also evident when we consider exits to inactivity. Although there is little difference in the timing of exits to inactivity for women in the treatment and control groups, the large negative overall effect of inactivity is driven entirely by the incidence effect; women subject to the benefit cut are much *less* likely to exit to inactivity.

Exits to inactivity for women can also be used to further illustrate the value of the extended decomposition. The left-hand panel of Figure 3 compares the unconditional CIF functions for the treatment and control groups for this state, as suggested by Andersen (2013). Such a comparison clearly yields substantial differences between the two groups. While the differences in incidences are obvious in this graph, it is not possible from this comparison alone to distinguish the relative importance of duration effects and incidence effects. The right hand panel reproduces the graph for the *conditional duration* effect using the decomposition developed in this paper. Since these two curves are very similar, we can conclude that the overall effect observed in the right hand panel is driven entirely by the incidence component, as reported in Table 3.

In summary, while the overall initial response to the benefit cut is very similar for men and women, the decomposition reveals differences in the channels through which men and women respond to the benefit cut. For men the response primarily took the form of quicker exits to all of the destination states, with little effect on the overall incidence across states. For women however, we observe both a conditional duration effect and a substantial incidence effect. The benefit cut resulted not only in women moving into work more quickly, but also in more women choosing to move into work rather than inactivity. Although the focus of this paper is on the initial duration of unemployment, the estimated differences in the relative importance of incidence effects by gender suggest that the long-run effects of the benefit cuts may also differ by gender. The benefit cut may have an additional long-run effect for men, in so far as the shorter unemployment durations reduce the potential scarring effects associated with unemployment. However, the resulting change in the distribution of claimants across exist states for women may result in additional long run effects over and above those normally associated with reduced scarring.

7. Conclusion

When dealing with duration data researchers must confront how to deal with competing risks. If the primary outcome of interest is the exit rate to a given state then one could estimate a cause-specific hazard. For those interested in the overall incidence of exits to different states then cause-specific hazards are insufficient and by themselves may potentially give misleading results. In such cases, researchers rely on estimating Cumulative Incidence Functions. In this paper, I consider how to deal with competing risk when the outcome of interest is the difference in average duration between two groups. To examine this question, I propose a competing risk decomposition, which identifies the contribution of each exit state to the overall difference in duration between the two groups, distinguishing between differences in duration conditional on an exit state and differences in the incidence of exits to each state.

I use the decomposition to examine the impact of a 50% cut to unemployment benefit for 18 year-old claimants in Ireland. I consider the effect separately for men and women. For both genders, the aggregate effect of the benefit cut was substantial, with unemployment duration falling by 52 weeks for men and 44 weeks for women. However, the competing risk decomposition reveals substantial gender differences in the channels through which this effect operates. For men, the treatment has little impact of the relative proportions exiting to each state. Instead, those subject to the benefit cut exit to all states quicker. This is particularly true of the exits to training, to work and to “other” states. I find similar duration effects for women, particularly for exits to work and to “other” states. However, for women, the benefit cut also has a significant impact on the relative incidences across states. Those subject to the benefit cut are much more likely to exit to work and less likely to exit to an inactive state. The additional incidence effects apparent for women indicate another channel through which the benefit cut effects behaviour for young women. This additional channel may have important policy implications, implications that are only evident following the competing risks decomposition.

Table 1: Summary Statistics by Control and Treatment Status state

	Males		Females	
	May	April	May	April
Average unemployment Duration (weeks)	66.53	99.98	61.20	92.00
Treatment Status	0.83	0.18	0.90	0.19
Proportion exit to Training	0.35	0.31	0.31	0.30
Proportion exit to Work	0.31	0.36	0.36	0.21
Proportion exit to Education	0.05	0.06	0.06	0.11
Proportion exit to Other	0.26	0.25	0.20	0.19
Proportion exit to Inactive	0.03	0.03	0.06	0.18
N	428	377	297	210

Table 2: Cause-Specific Hazard Functions

Cause-Specific Hazard	Males Treatment Effect	Females Treatment Effect
Training	0.47*** (0.13)	0.37*** .16
Work	0.26** (0.13)	0.89*** .18
Education	0.40 0.30	-0.13 .31
Other	0.41*** (0.14)	0.40* .21
Inactive	0.69 (0.44)	-0.43 .29

Table 3: Competing Risk Decomposition (Duration in Weeks)

	Males			Females		
Treatment Effect	-51.50** (6.00)			-43.46** (9.72)		
Decomposition	Overall Decomposition (1)	Duration Effect (2)	Incidence Effect (3)	Overall Decomposition (4)	Duration Effect (5)	Incidence Effect (6)
Training	-21.71	-15.76	-5.95	-4.32	-5.14	0.82
Work	-22.89	-15.70	-7.19	-2.07	-23.50	21.43
Education	-3.69	-2.30	-1.39	-7.38	-1.38	-6.00
Other	-10.76	-12.48	1.72	-10.00	-11.02	1.02
Inactive	-3.98	-2.85	-1.13	-19.72	0.07	-19.79

Figure 1: Duration Decomposition Profiles
18 year old Males

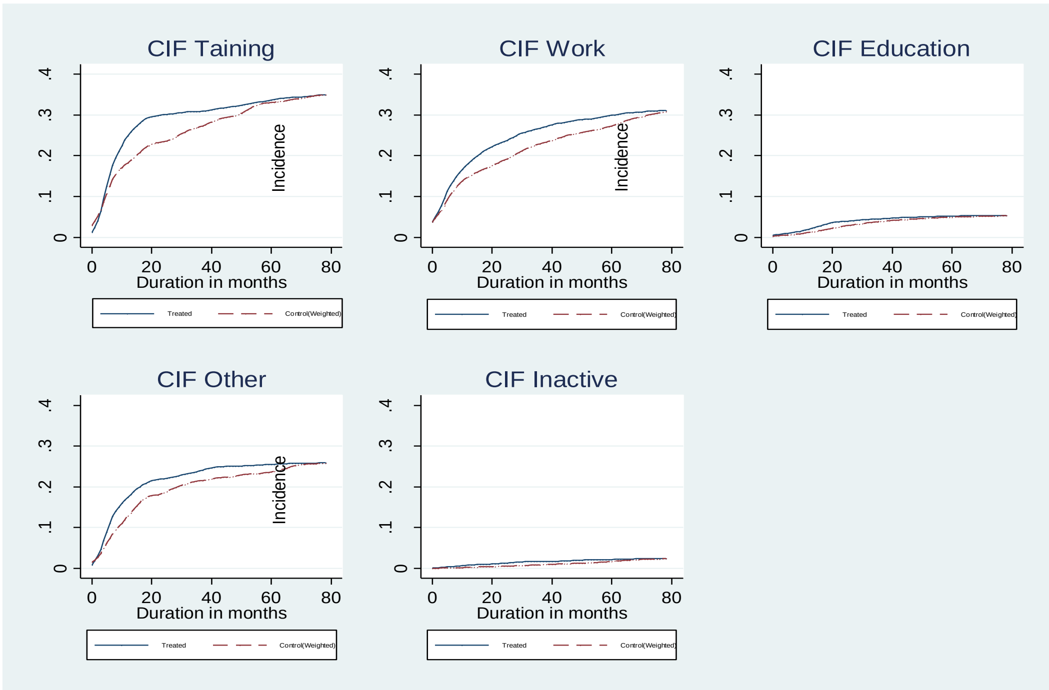


Figure 2: Duration Decomposition Profiles
18 year old Females

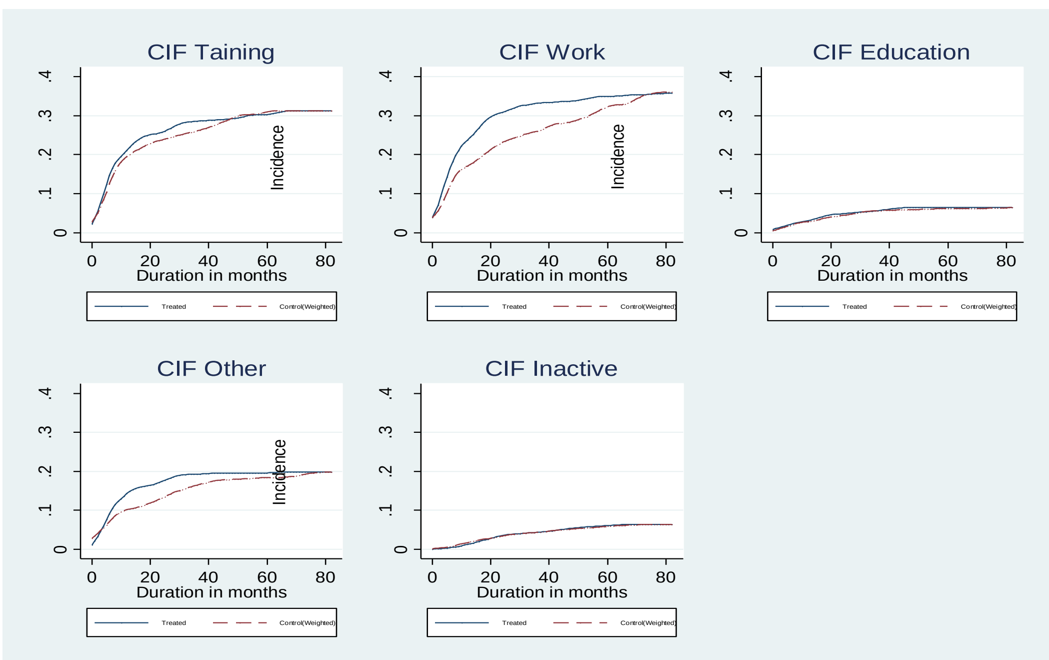
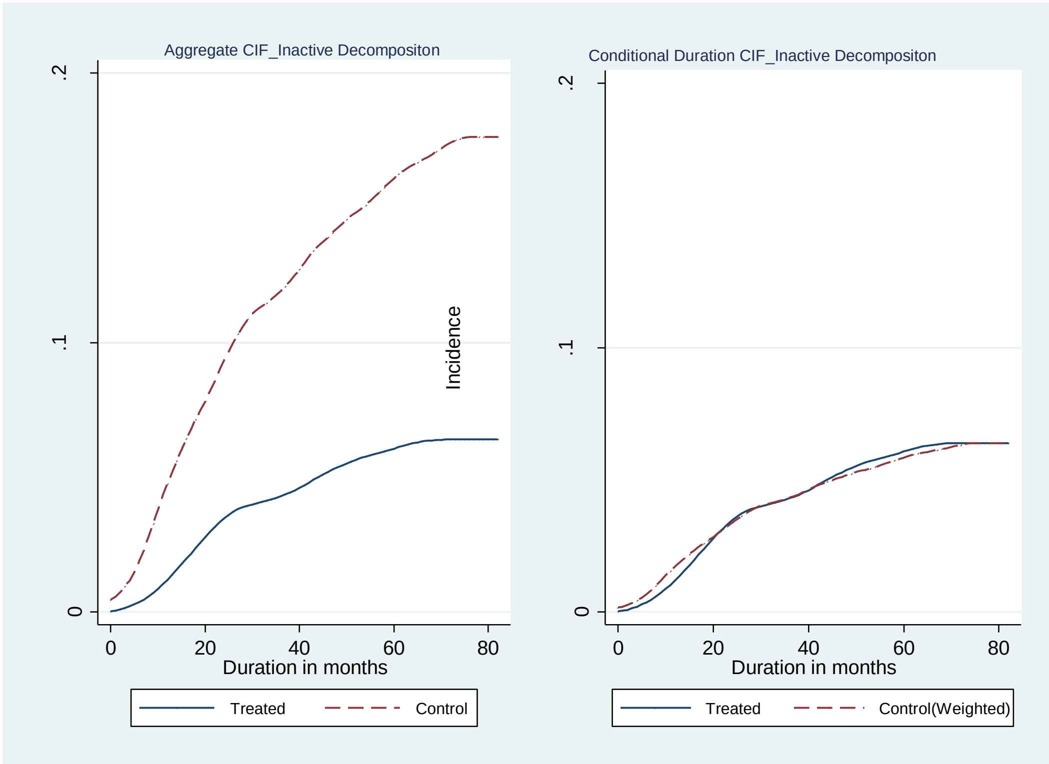


Figure 3: Duration Decomposition Profiles
18 year old Females



Appendix 1:

Theorem: Let Y be a random variable representing the duration of unemployment for group with upper bound T_M then

$$E(Y_T|K_T = k) = \left[Y_M - \int_0^{Y_M} \left[\left[\frac{(CIF_{kT}(y))}{f_{kT}}\right] dy\right]\right]$$

Proof:

$$E(Y_i|K_i = k) = \int_0^{Y_M} y. f_i(y|K_i = k) \ \ dy$$

$$= \frac{1}{f_{kT}} \int_0^{Y_M} y. f_{kT}. f_T(y|K_T = k) \ \ dy \ = \frac{1}{f_{kT}} \int_0^{Y_M} y \ \ f_T(y, k) \ \ dy$$

$$= \frac{1}{f_{kT}} \int_0^{Y_M} y \ \ \frac{dCIF_{kT}(y)}{dy} dy \ = \frac{1}{f_{kT}} \int_0^{Y_M} Pr_T(Y > y, D = k) \ \ dy$$

$$= \frac{1}{f_{kT}} \left\{ \int_0^{Y_M} (1 - Pr_T(Y \leq y, D = k) - (1 - Pr_T(D = k)) \ \ dy \right\}$$

$$= \frac{1}{f_{kT}} \left\{ \int_0^{Y_M} \left((Pr_T(D = k) - Pr_T(Y \leq y, D = k)) \right) \ \ dy \right\}$$

$$\frac{1}{f_{kT}} \left\{ Y_M f_{kT} - \int_0^{Y_M} [(CIF_{kT}(y))] dy \right\}$$

$$= \left\{ Y_M - \int_0^{Y_M} \left[\frac{(CIF_{kT}(y))}{f_{kT}}\right] dy \right\}$$

Appendix 2:Dealing with Censored Observations

Let Y be a random variable representing the duration of unemployment for group with undefined upper bound and consider an arbitrary cut duration τ. Andersen (2013) showed that the difference in the τ-restricted mean lifetimes can be written as

$$e_T(0,\tau) - e_C(0,\tau) = \sum_k \left[\left\{\int_0^\tau \{CIF_{kC}(y) - CIF_{kT}(y)\}\, dy\right\}\right]$$

To derive the extended decomposition in this instance define $\theta_k = \frac{Pr_T(Y\le\tau, D=k)}{Pr_C(Y\le\tau, D=k)} = \frac{CIF_{kT}(\tau)}{CIF_{kC}(\tau)}$ and rewrite

$$e_T(0,\tau) - e_C(0,\tau)$$

$$= \sum_k \left[\langle \underbrace{\left\{\left\{\left(\int_0^\tau \theta_k [CIF_{kC}(y)]dy - \int_0^\tau [CIF_{kT}(y)]dy\right\}\right\}\right)}_{Duration\ \ Effect} \rangle \right.$$

$$\left. + \underbrace{\langle\{1-\theta_k\}\left[\left[\int_0^\tau [CIF_{kC}(y)]dy\right]\right\rangle}_{Incidence\ \ Effect} \right]$$

$$= \sum\nolimits_k \left[\left[\left\{\left\{\left(\underbrace{CIF_{kT}(\tau)\left\{\left[\tau - \int_0^\tau \left[\left[\frac{(CIF_{kT}(y))}{CIF_{kT}(\tau)}\right] dy\right]\right] - \left[\tau - \int_0^\tau \left[\left[\frac{(CIF_{kC}(y))}{CIF_{kT}(\tau)}\right] dy\right]\right]\right\}}_{duration\ \ effect}\right)\right\}\right\} + \right.$$

$$\left. \underbrace{\{CIF_{kT}(\tau) - CIF_{kC}(\tau)\}\left[\tau - \int_0^\tau \left[\frac{(CIF_{kC}(y))}{CIF_{kT}(\tau)}\right] dy\right]}_{incidence\ \ effect}\right]\right] + \underbrace{R_\tau}_{Residual} \quad \text{(A1)}$$

where $R_\tau = \tau\{S_T(\tau) - S_C(\tau)\}$ is a residual term capturing differences in survival rates at time τ. It is straightforward to show that $\lim_{\tau\to\infty} R_\tau = 0$ provided $\lim_{\tau\to\infty} f_T(\tau) = \lim_{\tau\to\infty} f_C(\tau)$, so that the density of durations do not differ in the extreme tails.

Following the same approach to that used in Appendix 1, we can show that

$$E(Y_T|K_T = k, Y_T \leq \tau) = \left[\tau - \int_0^\tau \left[\left[\frac{(CIF_{kT}(y))}{CIF_{kT}(\tau)}\right] dy\right]\right].$$

Thus we can interpret the first component in the extended decomposition (A1) as the duration effect up to time τ and the second component as the contribution of differences in incidences as of time τ on differences in restricted means, with an additional residual effect that, under reasonable conditions, goes to zero as τ becomes large.

To proceed in the presence of censoring, simply recognise that the key components of this decomposition are $\int_0^\tau [CIF_k(y)]dy$, $CIF_k(\tau)$ and $S(\tau)$ and furthermore that unbiased estimators exist for these terms even in the presence of independent right censoring (see Andersen 2013). Implementing the decomposition simply requires replacing the unknown $\int_0^\tau [CIF_k(y)]dy$, $CIF_k(\tau)$ and $S_i(\tau)$ with their unbiased estimators.

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