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A Competing Risk Decomposition of the Average Duration Effect of a 50% cut in Unemployment Benefits¹

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Abstract

Competing risks often play an important role in evaluations of treatment effects with a time to event outcome. Approaching this issue in the probability domain, via hazard functions, requires strong assumptions and is not appropriate for studying absolute risks. In this paper I propose a competing risks decomposition in the time domain, which is valid irrespective of the dependence structure across exit states or the shape of the underlying hazards and furthermore distinguishes between risk and duration effects. I use the decomposition to evaluate a 50% cut in unemployment benefits on unemployment durations in Ireland. Although the aggregate effect of the benefit cut is similar across genders, the decomposition reveals that the channels through which the benefit cut operates differs substantially across genders. While the incidence effect is relatively unimportant for men, this is not the case for women, with the benefit cut resulting in a substantial shift away from exits to inactivity and into exits to work for females.

Keywords: Unemployment Duration; Competing Risks; Cumulative Incidence Function; Decomposition

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1. Introduction

Evaluations of treatments with a time-to event outcome often rely on the hazard ratio to estimate the treatment effect. Working in the probability domain typically requires strong assumptions, particularly in the presence of competing risks; a competing risk is an event whose occurrence prevents the occurrence of another outcome. For instance, unemployment spells can end for many reasons, including exits from the labour force, exits to a job (full/part-time) or exits to training and modelling competing risks may have important policy implications. Unemployment durations that end because the claimant moves from unemployment assistance to an alternative welfare payment or leaves the labour force, may be less desirable than unemployment spells that end because the claimant finds a job.

When spells can end for a variety of reasons, researchers and policy makers may wish to understand the impact of competing exits on the overall effect. The typical approach used in economics is to estimate cause-specific competing risks hazard functions (Han and Hausman 1990, Narendranathan and Stewart 1993). However, this approach is not appropriate for decomposing differences in average durations in the time domain; while cause-specific hazards are useful when examining the rate of occurrence of an outcome in the subset of people who are event free, they do not identify the absolute risk of a cause-specific outcome, a key component of any time-domain decomposition.

In this paper I extend previous work on competing risks by proposing a new decomposition that focuses on the differences in restricted mean lifetimes between two groups. In particular I decompose the difference in the restricted mean lifetime into the contributions of distinct exit states, distinguishing between duration and incidence effects. The decomposition builds on the Cumulative Incidence Function approach previously used in competing risks analyses in the

biomedical literature and is valid irrespective of the shape of the underlying hazards or the dependence structure across exit states. The approach is easily implemented using existing software even in the presence of right censored unemployment spells. I use this decomposition to re-examine the labour market response of 18 year olds jobseekers in Ireland to a 50% cut in unemployment benefits introduced in 2009. Although the aggregate effect of the benefit cut is similar for men and women, the decomposition reveals that the channels through which the benefit cut affects labour supply differs substantially across genders.

2. Cause Specific Hazards and Cumulative Incidence Functions

When unemployment spells can end for multiple reasons (e.g. a job, a training course, an education course) a competing risks framework is required. A competing risk is an event whose occurrence precludes the occurrence of another event. One cannot observe people who exit unemployment to training also terminating the same spell with an exit to work. However, when analysing unemployment durations properly identifying the importance of alternative exit states may be crucial for policy makers. Shorter durations that are driven by exits out of the labour market are likely to be valued differently than shorter durations driven by exits to work.

When fitting models in the presence of competing risks, researchers can choose to model either the cause-specific hazard or on the cause-specific cumulative incidence function (CIF). The use of cause-specific hazards allows one to estimate the rate of occurrence of a specific outcome in the subset of the population who are event free at the specified time. Specifically, the exit-k cause-specific hazard, λ_k , is given by

$$\lambda_k(t) = \lim_{\Delta t \downarrow 0} \frac{\Pr(t \leq Y \leq t + \Delta, K = k | Y \geq t)}{\Delta t}$$

where Y denotes duration and k denotes a given exit state. The typical approach to dealing with competing risks in economics is to estimate $\lambda_k(t)$, either parametrically or non-parametrically (Heckman and Honore 1989, Katz and Meyer 1990, Han and Hausman 1990, Narendranathan and Stewart (1993), Dolton and O'Neill 1996, McVicar 2008 and Van den Berg et al. 2014, 2017). In this approach competing risks are typically modelled in terms of a set of latent random variables, with each latent variable representing the time to failure from a given cause. The observed data includes the smallest of the latent variables and the cause of failure. While the cause-specific hazards are identified in this approach, neither the joint distribution of the latent variables nor the marginal survival functions are identified (Cox (1962)). One common approach to overcoming the identification issue is to assume independent competing risks. Recognising the limitations of this assumption, Heckman and Honore (1989) and Abbring and Van den Berg (2003) show that in a model with regressors, it is possible to identify the joint distribution of failure times in a continuous time model without such strong distributional assumptions; the main identifying assumption with single spell data requires that the effect of regressors differ across exit states. However, these approaches require a proportional hazards assumption. This implies that the survival curves for the treated and control groups must have hazard functions that are proportional over time (i.e. constant relative hazard).

While identification and functional form issues pose problems for the hazard approach, there is a more fundamental problem with using this approach to evaluate treatments in the time domain. Even in cases where the competing risk model is identified and the proportional hazard assumption is valid, the cause-specific hazard will still not provide the information needed for a

competing risk decomposition of average durations in the time-domain. To see this, suppose we abstract from treatment effects and consider the case where we are interested in decomposing the expected duration conditional on duration being less than a specified value τ , $E[Y|Y \leq \tau]$.² Assuming K possible exits and using the law of iterated expectations, we can write this as

$$\begin{aligned}
 E[Y|Y \leq \tau] &= \sum_{k=1}^K \Pr(K = k|Y \leq \tau)E[Y|K = k, Y \leq \tau] \\
 &= \sum_{k=1}^K \frac{\Pr(K=k, Y \leq \tau)}{\Pr(Y \leq \tau)} E[Y|K = k, Y \leq \tau].
 \end{aligned}$$

A key component in this expression is the Cumulative Incidence Function, $CIF_k(\tau) = \Pr(K = k, Y \leq \tau)$, which measures the absolute risk or incidence of failures from cause k by time τ ; This probability is given is given by

$$CIF_k(\tau) = \int_0^\tau \lambda_k(s)S(s)ds,$$

where $S(\tau) = \exp(-\sum_{k=1}^K [\int_0^\tau \lambda_k(s) ds])$ is the probability of not having failed from *any* event as of time τ . Clearly, the cause-specific hazard for exit k is not sufficient by itself to identify the contribution of exit k to the overall mean.

From this it is also clear that the effect of a covariate (such as treatment status) on the cause-specific hazard function may be very different to its effect on the likelihood of exiting due to that caus. As noted above, the likelihood of having left due to cause k , by time t , depends on *all* the cause-specific hazards. Consequently, the effect of any given covariate on the likelihood of

² In the next section we will extend this to consider decompositions of the restricted mean life-times, $E[\min(Y, \tau)]$, and show that similar issues arise.

exiting due to cause k , will be affected indirectly by the effect of the covariate on all competing causes. In general, it is difficult to deduce the sign of a covariate effect on the CIF directly from the cause-specific hazard parameters (Kyyra 2009); a covariate may have a strong effect on the cause-specific hazard but no effect on the cumulative incidence.

The objective of the current paper is to decompose the treatment effect of an intervention, measured in the time-domain, into its competing risks components. A small number of recent papers in biostatistics have addressed a related issue, Beltran-Sanchez et al. (2008) and Vaupel and Canudas-Romo (2003) distinguish between causes of death in a decomposition of life-expectancy. However, as noted in Andersen et al. (2013) neither of these proposed measures are additive, while in addition the Beltran-Sanchez et al. (2008) approach requires independence of the competing causes. Andersen (2013) proposes an alternative decomposition in which the cause-specific components are interpreted as the expected number of life-years lost to each cause before a specified date. The Andersen decomposition is additive, does not require independence and the individual terms can be represented in terms of the area underneath the CIFs. However, as I show in the next section, analysis based solely on the area under the CIF can be confounded by differences in prevalence rates of the competing events.³ To overcome this I develop an extended decomposition which isolates the separate cause-specific duration and incidence effects.

Although the use of CIFs is common in the biostatistics literature (Varadhan et al. 2010) it has rarely featured when modelling unemployment durations. Exceptions include Lo and Wilke (2010) and Lo et al. (2017), who combine estimated CIFs with a parametric copula, to estimate the marginal distribution of latent durations in the presence of competing risks; durations that

³ For a related discussion of this issue see Huang et al. (2016)

would be observed in the absence of competing risks. However, the interpretation of these marginal distributions is not straightforward. Price (2016) uses both cause-specific hazards and the cumulative incidence functions to model the effect of the Hartz labour market reform on unemployment durations in Germany. He argues that to assess how shifts between full-time and part-time jobs impact estimated wages post-reform, what matters are not the hazard rates, but rather how these rates translate into the share of claimants who eventually end up in each state, namely the cause specific CIF.

In the remainder of this paper I extend the previous work on competing risks by proposing a new competing risks decomposition of the difference in average duration between two groups. I follow Royston and Parmar (2013) by focusing on the time domain rather than the probability domain. The new approach is valid irrespective of the dependence structure across exit states or the shape of the underlying CIFs and is easily implemented using existing software even in the presence of censored outcomes. Furthermore the new approach permits an extended decomposition that distinguishes between differences in durations, conditional on an exit state, and differences in the prevalence of exits to given states. When prevalence rates are equal across groups my decomposition is equivalent to the decomposition proposed by Andersen (2013). However, when incidence rates differ, the decomposition proposed in this paper captures both a duration and an incidence effect. This extension is particularly useful if some exit states are seen as more desirable than others are, as is often the case when modelling unemployment duration.

3. Competing Risks Decomposition

Suppose we are interested in comparing the difference in the average outcomes of a treated group and a control group at some threshold, τ weeks after an intervention. Let Y be a random variable representing durations. As of time τ the recorded duration for any individual is either the length of the completed spell, if this is less than τ , and τ if the spell is ongoing. To carry out the evaluation in this setting I follow Royston and Parmar (2013) and conduct the analysis in terms of restricted mean survival times. The τ -restricted mean lifetime is defined as

$$e(0, \tau) = \int_0^{\tau} S(y) dy$$

where, as previously, $S(y) = \Pr(Y > y)$. The restricted mean lifetime measures $E(\min(Y, \tau))$; the expected unemployment duration up to time τ . To measure the success of a program as of time τ , I compare the τ -restricted mean lifetimes of a treatment group (T) and a control group (C). The object of interest is therefore $\Delta(\tau) = e_T(0, \tau) - e_C(0, \tau)$. This treatment effect is valid under any distribution of time to event and is readily interpretable in the time domain (Royston and Parmar 2013).

To allow for competing risks assume that the duration can in end in one of K mutually exclusive ways. Andersen (2013) showed that the difference in the τ -restricted mean lifetimes between a treatment group T and a control group C can be written as

$$e_T(0, \tau) - e_C(0, \tau) = \sum_k \left[\int_0^{\tau} \{CIF_{kC}(y) - CIF_{kT}(y)\} dy \right] \quad (1)$$

The terms inside the square brackets provide a measure of the contribution of exit state k to the difference in restricted mean life-times. Non-parametric estimation of the cumulative incidence function in the presence of right-censored survival data is straightforward using Kaplan-

Meier or Aalen-Johansen methods. For example the cause-specific cumulative incidence function is estimated as

$$\widehat{CIF}_k(y) = \sum_{t_j \leq t} \frac{d_{kj}}{Y_j} \hat{S}(t_j-)$$

where d_{kj} is the number of events of cause k at time t_j , Y_j is the number at risk at t_j and $\hat{S}(t_j-)$ is the left hand limit the Kaplan-Meier estimator of the overall survival function at t_j .⁴ In this way the difference in restricted means, $\Delta(\tau)$, is well-defined and decomposable, even when the difference in unrestricted means, $\Delta(\infty)$, may be ill-determined because of censoring.

The Andersen decomposition given in (1) measures the cause-specific contribution to differences in durations by the differences in the areas under the cause-specific CIFs. However, as noted by Huang et al. (2016), analysis based solely on the areas under the CIFs can be confounded by differences in the prevalence rates of the competing events across groups. Using only the difference in areas under the CIFs for instance, it would not be possible to distinguish between a programme that resulted in fewer people moving to work (an incidence effect) but resulted in those who did move moving more quickly (a duration effect), and a programme that had no effect at all on either the duration or incidence of people moving to work.. To account for this I propose an extension of the aggregate decomposition given in (1) that tackles this issue by further decomposing the aggregate effect into a duration effect and an incidence effect.

To derive the extended decomposition in this instance define

⁴ In particular $\hat{S}(t-) = \prod_{t_j \leq t} \frac{Y_j - d_j}{Y_j}$ where d_j is the number of exits at t_j .

$$\theta_k = \frac{Pr_T(Y \leq \tau, D=k)}{Pr_C(Y \leq \tau, D=k)} = \frac{CIF_{kT}(\tau)}{CIF_{kC}(\tau)}$$

and rewrite

$$\begin{aligned} e_T(0, \tau) - e_C(0, \tau) &= \sum_k \left[\left[\int_0^\tau \theta_k [CIF_{kC}(y)] dy - \int_0^\tau [CIF_{kT}(y)] dy \right] \right. \\ &\quad \left. + \left[\{1 - \theta_k\} \left[\int_0^\tau [CIF_{kC}(y)] dy \right] \right] \right] \\ &= \sum_k [CIF_{kT}(\tau) \left[\left(\tau - \int_0^\tau \frac{CIF_{kT}(y)}{CIF_{kT}(\tau)} dy \right) - \left(\tau - \int_0^\tau \frac{CIF_{kC}(y)}{CIF_{kC}(\tau)} dy \right) \right] + \\ &\quad (CIF_{kT}(\tau) - CIF_{kC}(\tau)) \left(\tau - \int_0^\tau \frac{CIF_{kC}(y)}{CIF_{kC}(\tau)} dy \right)] + \underbrace{R_T}_{Residual} \quad (2) \end{aligned}$$

where $R_\tau = \tau \{S_T(\tau) - S_C(\tau)\}$ is a residual term capturing differences in survival rates at time τ .

It is straightforward to show that $\lim_{\tau \rightarrow \infty} R_\tau = 0$ provided $\lim_{\tau \rightarrow \infty} f_T(\tau) = \lim_{\tau \rightarrow \infty} f_C(\tau)$, so that the densities of durations do not differ in the extreme tails.

Denoting $\frac{CIF_{kT}(y)}{CIF_{kT}(\tau)}$ as a cause-specific conditional cumulative incidence function (CCIF_k;

see Huang et al. 2016) it can be shown that $E(Y_T | K_T = k, Y_T \leq \tau) = \left[\tau - \int_0^\tau CCIF_{kT}(y) dy \right]$ or

equivalently $\int_0^\tau CCIF_{kT}(y) dy = \tau - E(Y_T | K_T = k, Y_T \leq \tau)$.⁵ Using this we can also rewrite the

decomposition in (2)

⁵ For proof see Appendix 1.

$$e_T(0, \tau) - e_C(0, \tau) = \sum_k \left[\underbrace{CIF_{kT}(\tau)[E(Y_T|K_T = k, Y_T \leq \tau) - E(Y_C|K_C = k, Y_C \leq \tau)]}_{\text{duration effect}} + \underbrace{(CIF_{kT}(\tau) - CIF_{kC}(\tau))E(Y_C|K_C = k, Y_C \leq \tau)}_{\text{incidence effect}} \right] + \underbrace{R_\tau}_{\text{Residual}} \quad (3)$$

Equation (3) writes the decomposition of the restricted mean difference in terms of simple conditional means. Written in this way it is clear that the first term of the extended decomposition measures the duration effect of state k , weighted by the incidence of k in the treatment group. If the cause-specific incidences are the same for the control and treatment groups, $CIF_{kT}(\tau) = CIF_{kC}(\tau)$ then all of the state k effect, will be captured by the conditional duration term, with a negative contribution denoting quicker exits to a given state. The second term measures the incidence effect, weighted by average duration for group C . Clearly if the average duration, conditional on being in state k , is the same for both groups, but more of the treatment group enter state k , then the first term will be zero; all of the state k effect will be picked up as a *positive* contribution in the incidence term. How one interprets this component depends on the desirability of the exit state. When exit states are seen as desirable (e.g. work) then a positive incidence effect reflects well on a programme. The opposite is true for exits states that are considered undesirable (e.g. exits into inactivity).

In this way the extended decomposition decomposes the differences in restricted mean life times into a component that measures the duration effect of exit state k up to time τ , a component that measures the contribution of differences in incidences as of time τ on differences in restricted means, and a residual effect that, under reasonable conditions, goes to zero as τ becomes large. To proceed it is sufficient to note that unbiased estimators exist for all of the key components of

this extended decomposition, $\int_0^\tau CIF_k(y)dy$, $CIF_k(\tau)$ and $S(\tau)$, even in the presence of independent right censoring (see Andersen 2013).⁶ Implementing the decomposition simply requires replacing the unknown $\int_0^\tau CIF_k(y)dy$, $CIF_k(\tau)$ and $S(\tau)$ with their unbiased estimators. By varying τ the decomposition allows us to examine how the contribution of the alternative states varies over the duration of a spell.

Some further manipulation allows us to rewrite the duration effect as

$$\begin{aligned}
 CIF_{kT}(\tau) & \left[\left(\tau - \int_0^\tau \frac{CIF_{kT}(y)}{CIF_{kT}(\tau)} dy \right) - \left(\tau - \int_0^\tau \frac{CIF_{kC}(y)}{CIF_{kC}(\tau)} dy \right) \right] \\
 & = \int_0^\tau [\theta_k CIF_{kC}(y) - CIF_{kT}(y)] dy \quad (4)
 \end{aligned}$$

where as before $\theta_k = \frac{Pr_T(Y \leq \tau, D=k)}{Pr_C(Y \leq \tau, D=k)} = \frac{CIF_{kT}(\tau)}{CIF_{kC}(\tau)}$. Andersen (2013) shows that the total difference in durations due to cause k , can be represented as the difference in the areas underneath the cause-specific CIFs. Equation (4) shows that, likewise, the duration effect due to cause k , can be represented as the difference in *rescaled* CIFs, where the scaling factor for the control group is simply θ_k . Furthermore if $CIF_{kT}(\tau) = CIF_{kC}(\tau)$ the decomposition in (2) simplifies to that proposed by Anderson (2013); in the absence of any incidence effects both decompositions are identical.

Letting $\tau \rightarrow \infty$ and noting that $e(0, \infty) = E[Y]$ the above decomposition simplifies further so that $\Delta Y = E[Y_T] - E[Y_C]$ can be exactly decomposed as

⁶ In practice these components can be easily estimated in Stata using the *stpci* and *stplost* command (Overgaard et al. (2015))

$$\Delta Y = \sum_k \left[\underbrace{\langle f_{kT} \{E(Y_T | K_T = k) - E(Y_C | K_C = k)\} \rangle}_{\text{duration effect}} + \underbrace{\langle (f_{kT} - f_{kC}) E(Y_C | K_C = k) \rangle}_{\text{incidence effect}} \right] \quad (6)$$

where the overall proportion of group i exiting to a given state k is given by f_{ki} . If all spells are completed, so that censoring is not an issue, the key components of this unrestricted mean decomposition can be estimated directly and non-parametrically from the data. Of course if censoring occurs then ΔY is not identified without strong assumptions and we must rely on the alternative mean restricted decomposition given by (3).

4. Application: Changes to Unemployment Benefit

I now use the above decomposition to further understand the impact of a substantial cut to youth unemployment benefits in Ireland in 2009. The social welfare system in Ireland is divided into three main types of payments; social insurance payments, means-tested payments, and universal payments. Jobseeker's Allowance (JA) is a means tested payment that is paid indefinitely to those who are unemployed provided the claimant continues to meet conditions of eligibility. To receive JA a claimant must satisfying the means test, be aged between 18 and 66, be unemployed and actively seeking work. In response to the fiscal crisis that occurred in Ireland with the onset of the Great Recession, the Irish government imposed substantial cuts on JA payments for young claimants. Prior to 2009, all claimants were entitled to €204 a week. On 29 April 2009, claimants aged 18 had their weekly rate cut to €100. The benefit cut only applied to new claimants, those who started their claim prior to April 29 continued to receive the higher rate. The fact that the cut

only applied to new claimants means that a comparison of those who entered unemployment just before and just after the legislation can be used to identify the treatment effect.⁷

Doris et al. (2017) analysed this benefit cut in detail using a Regression Discontinuity design and found a substantial reduction in unemployment durations as a result of the benefit cut. Claimants subject to the cut had unemployment durations that are 50% lower than the control group. Given that the stated motivation for these cuts was to “ensure that young people are better off in education, employment or training than claiming,”⁸ it seems appropriate to consider in detail the relative role of alternative exit states in explaining these aggregate effects. In the remainder of this paper, I apply the decomposition outlined in Section 3 to further evaluate this benefit cut, distinguishing between incidence and duration effects. To allow for possible differences in the response of the benefit cut by gender, I further extend the work of Doris et al. (2017) and conduct the analysis separately for males and females.

5. Data

To carry out the analysis I use the Longitudinal Jobseekers Database provided by the Department of Social Protection. This is an administrative data set covering every claimant who has received a jobseekers or one parent family payment since 2007. The data provide administrative records for the start and end date of every new claim since January 2007, allowing me to establish both the exact start date and duration of unemployment for the entire population of new JA claims initiated between 2007 and 2014. Throughout the analysis, I measure duration in weeks. When considering

⁷ Doris et al. (2017) find no evidence of anticipation effect prior to the introduction of the legislation.

⁸ <http://www.welfare.ie/en/pressoffice/Pages/pr231013.aspx>

the competing risks model I examine five different exit states; training, work, education, inactivity and “other”.⁹ To identify the treatment effect I compare durations of those beginning an unemployment spell in April 2009 (who received the higher benefit payment) with those beginning an unemployment spell a month later in May 2009 (and who received the substantially reduced payment).

Table 1 provides summary statistics separately for those entering unemployment in April 2009, the month before the legislation came into effect and for those entering in May 2009, the month after. The first row of Table 1 reports the average unemployment duration, by month of entry and gender. For all groups average unemployment duration exceeded one year, highlighting the poor labour market prospects faced by youths in Ireland during the Great Recession. However, those who entered after the benefit cut had unemployment durations that are approximately 30 weeks shorter on average than those entering earlier.

Of particular interest in my study is the labour market states to which claimants exited. The next five rows of Table 1 provide the proportion of each group exiting to each of the alternative states. Looking at men, we see that three exit states dominated; namely training, work and “other”. Furthermore, the proportions exiting to each state are similar for those entering in April and in May, suggesting that the aggregate duration effect for men is driven by differences in duration conditional on a given exit state. The third column of Table 1 shows, that for women subject to the benefit cut, the proportions leaving to each state are similar to those of men and are dominated by the same three exit states. However, for women in receipt of the higher benefit the pattern is notably different. For this group the distribution across exit states is more uniform. The difference

⁹ The “other” category is a residual grouping comprising primarily of durations for which the respondent did not give an exit state for their spell.

relative to those subjected to the lower benefit regime is driven by a bigger proportion of the control group exiting to inactivity and education, and fewer exiting to work. While the results in column 2 and 3 suggest that differences in state specific durations are the driving force behind the male effect, the results in columns 4 and 5 suggest that differences in incidences may play a larger role for women. In the remainder of the paper, I use the decomposition outlined in Section 3 to examine these issues more formally.

6. Results

We begin our analysis by estimating cause-specific hazard functions. As noted earlier this is the typical approach used in economics when estimating competing risks models. The results are given in Table 2. The estimates are obtained from a Cox proportional hazard model, treating exits to competing states as censored. The results are similar for both men and women, with significant treatment effects evident in exits to training, work and “other”, but not in exits to education or inactivity. However, as noted in Section 2, the cause-specific hazard approach, while useful for looking at exit rates, is less useful for understanding differences in absolute risk across states.

To obtain further insight into the relative importance of alternative exits states in explaining differences in average durations I carry out decomposition derived in Section 3.¹⁰ Since all of my sample had completed their unemployment spell 7 years after the intervention, I begin with the exact decomposition provided in equation (6) of Section 3. The results are given in Table 3. The first row reports the overall treatment effects. In keeping with Doris et al. (2017) I estimate a large

¹⁰ The standard errors reported for the decomposition terms are estimated using a bootstrap procedure with 1000 replications.

and statistically significant duration response to the benefit cut for both men and women. The size of the aggregate effect is similar across genders; males subject to the benefit cut have unemployment durations that are over 33 weeks shorter than those in receipt of the higher benefit, while the effect for females is approximately 31 weeks.¹¹

Columns one and four of Table 3 present the total contribution of each exit state to the overall treatment effect, making no distinction between incidence and duration effects. Looking at the results for males, we see that no single exit state dominates the overall effect. While the contribution of exits to education and inactivity are small, the other three exit states, training, work and “other”, all contribute a substantial component to the overall effect. The shorter overall duration for males is therefore a result of exits to each of these three states. These findings are in keeping with the cause-specific hazard results reported in Table 2.

Although the overall aggregate effect for women is very similar to the male effect, the basic decomposition results show that this effect is generated through very different channels. While exits to work is the dominant factor for men, it appears to contribute the least to the female effect. In contrast, exits to inactivity, while not important for men, is the dominant channel for women. The fact that very similar aggregate effects for men and women, operate through very different channels, highlights the value of the competing risks decomposition developed in this paper. Furthermore, the states identified as important for women using the decomposition differ from those identified using the cause-specific hazard approach. While exits to inactivity are identified

¹¹ In practice some of the treatment group were exempted from benefit cut on family or health grounds. I have also estimated the model using month of treatment as an instrument to control for this. While the overall treatment effect increases somewhat there is still little evidence of a significant different aggregate effect for men and women.

as the most important channel in the competing risks decomposition, this was not apparent using the cause-specific hazard approach.

The contributions of the duration and incidence effects to the overall state components are given in columns two and three of Table 3 for men, and columns five and six for women. Looking at the duration effect for men, we see that, conditional on exiting to a given state, those in receipt of the lower benefit exited quicker than those in the control group. This is particularly true for the three main effects identified earlier; training, work and “other” and is consistent with the hazard approach results from Table 2.

The incidence effect on the other hand, shows that for men differences in the likelihood of exiting to a given state are less important. The fact that the work incidence effect for men is negative implies that members of the treatment group are *less likely* to exit to this state. Since work is likely to be considered a desirable state, this negative incidence effect would be viewed as an undesirable outcome. However, it is smaller than the duration effect, which sees treated men exit more quickly to work. Competing duration and incidence effects, such as these, are not possible under the proportional hazard assumption typically assumed in the literature. If members of the treatment group are more likely to exit to a state during the early part of their spell, then the proportional hazards assumption requires that this must be true at every duration. Proportional hazard models therefore restrict the duration and incidence effect to operate in the same way, a restriction that is violated in our example.

Overall, the male results suggest that the incidence effect is relatively unimportant for men; what matters for men is the timing of the exits to each of the states. However, the same is not true for women. Looking at the duration effects in Table 3, we see that the results are quite similar to

those reported for men. Conditional on exiting to a given state, those in receipt of the lower benefit exit quicker than those in the control group. This is particularly true for work where the duration effect is larger for women than for men and this effect is evident in the results from the hazard approach. However, in addition to the conditional duration effect, the fact that the work incidence effect is *positive* and large for women implies that women who are subject to the benefit cut are *more* likely to enter work. Both of these work effects would be considered desirable but they offset each other in the aggregate decomposition.

The importance of distinguishing between duration and incidence effects for women is also evident when we consider exits to inactivity. The estimate in the final row of column five shows that there is little difference in the timing of exits to inactivity for women in the treatment and control groups and this is consistent with the insignificant effect for this state in the cause-specific hazards reported earlier. The large overall effect of inactivity apparent in column four is driven entirely by the incidence effect; women subject to the benefit cut are much *less* likely to exit to inactivity, a feature not captured by the hazard approach. Although it is not possible to identify the causal mechanisms behind this finding it's possible that young women who had been in receipt of the lower benefit for a prolonged spell are required to work to work to support a young, while those in receipt of the higher benefit had a small buffer of income to fall back on without having to work.

Exits to inactivity for women can also be used to graphically illustrate the value of the new decomposition. The left-hand panel of Figure 1 compares the unconditional CIF functions for those entering in April and May for this state, as suggested by Andersen (2013). Such a comparison clearly yields substantial differences between the two groups. While the differences in incidences

can be inferred from this graph, it is not possible from this comparison alone to distinguish between the relative contribution of duration effects and incidence effects to the overall effect. The right hand panel provides the graph for the *conditional duration* effect using the rescaling adjustment suggested by decomposition developed in Section 3. Since these two curves are very similar, we can immediately conclude that the overall effect observed in the left hand panel is driven entirely by the incidence component.

In summary, while the overall initial response to the benefit cut is similar for men and women, the decomposition reveals differences in the channels through which men and women respond to the benefit cut. For men the response primarily takes the form of quicker exits to all of the destination states, with little effect on the overall incidence across states. For women however, we observe both a conditional duration effect and a substantial incidence effect. The benefit cut resulted not only in women moving into work more quickly, but also in more women choosing to move into work rather than inactivity.

The decomposition presented in Section 3, also allows us to examine the evolution of the overall treatment effect by varying the follow-up threshold τ . The results in Tables 4-6 provide the decomposition of the restricted mean differences at three thresholds; two years, four years and six years after the intervention. Looking first at Table 4 we see that for both men and women the restricted mean treatment effect at two years is large and statistically significant different from zero. It is clear from this that the benefit cut had an effect from an early stage of the unemployment spell. Furthermore, in keeping with the overall final effect, there is little difference in the magnitude of the two year treatment effect between men and women. This is also true of the estimated treatment effects at the four year and six year thresholds; although the overall effect

widens as we extend the threshold there is never any substantial difference in the estimated treatment effects by gender. However, in keeping with the findings from the overall decomposition the nature of these effects differ across genders. Looking at Table 4 we see that offsetting incidence effects for women highlighted in Table 3 are already evident at the two year threshold; even in the early stages of the unemployment spell women subjected to the benefit cut are more likely to end the spell by exiting to work and less likely to exit to inactivity. This finding which was the key result of the overall decomposition persists at the four and six year thresholds. For men the dynamics are different. In Table 3 we saw that the overall effect for men was driven primarily by the training and work duration effects. However, looking at Tables 4-6 we see that these patterns take some time to emerge; the significant training duration effect is only evident at the 4 year threshold, while the large and significant work duration effect is not evident until the 6 year threshold. There is no evidence of these effects at the two year threshold. In the early stages of the spell it appears that males subjected to the benefit cut reacted by increasing participation in training schemes, although this initial effect is dominated by the duration effects as we extend the threshold. In this way the ability to conduct the decomposition at different follow-up thresholds clearly provides additional valuable information on the timing of the competing risks effects, information that may have important policy implications.

7. Conclusion

When dealing with duration data researchers must confront how to deal with competing risks. If the primary outcome of interest is the exit rate to a given state then one could estimate a cause-specific hazard function. For those interested in the overall incidence of exits to different states

then cause-specific hazards are insufficient and by themselves may potentially give misleading results. In the absence of censoring one could simply estimate population duration averages by their sample counterparts. However, in the presence of censoring this approach will lead to inconsistent results. In such cases, researchers rely on estimating Cumulative Incidence Functions. In this paper, I consider how to deal with competing risks when the outcome of interest is the difference in average duration between two groups. I propose a competing risks decomposition, which identifies the contribution of each exit state to the overall difference in duration between the two groups, distinguishing between differences in duration, conditional on an exit state, and differences in the incidence or absolute risk of exits to each state.

I use the decomposition to examine the impact of a 50% cut to unemployment benefit for 18 year-old claimants in Ireland. I consider the effect separately for men and women. For both genders, the aggregate effect of the benefit cut was substantial, with unemployment duration falling by 33 weeks for men and 31 weeks for women. However, the competing risks decomposition reveals substantial gender differences in the channels through which this effect operates. For men, the treatment has little impact on the relative proportions exiting to each state. Instead, those subject to the benefit cut exit to all states quicker. This is particularly true of the exits to training, to work and to “other” states. I find similar duration effects for women, particularly for exits to work and to “other” states. However, for women, the benefit cut also has a significant impact on the relative incidences across states. Those subject to the benefit cut are much more likely to exit to work and less likely to exit to an inactive state. The additional incidence effects apparent for women indicate another channel through which the benefit cut effects behaviour for young women.

Although the focus of this paper is on the initial duration of unemployment, the estimated differences in the relative importance of incidence effects by gender suggest that the long-run effects of the benefit cuts may also differ by gender. In so far as shorter unemployment durations reduce the potential scarring effects associated with unemployment, one might expect the short-run effect to persist into the future. However, the change in the distribution of claimants across exits states for women may result in additional long run effects, over and above those normally associated with reduced scarring. This additional channel may have important policy implications, implications that are only evident following the competing risks decomposition.

Table 1: Summary Statistics by Month of Entry to Unemployment

	Males		Females	
	May	April	May	April
Average unemployment Duration (weeks)	66.53	99.98	61.20	92.00
Proportion Exiting to Alternative Labour Market States				
Proportion exit to Training	0.35	0.31	0.31	0.30
Proportion exit to Work	0.31	0.36	0.36	0.21
Proportion exit to Education	0.05	0.06	0.06	0.11
Proportion exit to Other	0.26	0.25	0.20	0.19
Proportion exit to Inactive	0.03	0.03	0.06	0.18
N	428	377	297	210

Table 2: Cause-Specific Hazard Functions

(standard errors in parentheses)

Cause-Specific Hazard	Males	Females
	Treatment Effect	Treatment Effect
Training	0.47*** (0.13)	0.37*** (0.16)
Work	0.26** (0.13)	0.89*** (0.18)
Education	0.40 (0.30)	-0.13 (0.31)
Other	0.41*** (0.14)	0.40* (0.21)
Inactive	0.69 (0.44)	-0.43 (0.29)

Table 3: Competing Risks Decomposition (Duration in Weeks)

non-parametric 95% bootstrap confidence interval in parentheses (number of bootstrap replications 1000).

	Males			Females		
Treatment Effect	-33.50** [-44.78:-22.00]			-30.79** [-45.54:-16.67]		
Decomposition	Overall Decomposition (1)	Duration Effect (2)	Incidence Effect (3)	Overall Decomposition (4)	Duration Effect (5)	Incidence Effect (6)
Training	-6.35 [-14.36:2.89]	-10.20** [-18.14:-2.86]	3.85 [-1.98:9.97]	-3.04 [-11.91:5.55]	-3.63 [-10.40:4.03]	0.59 [-5.56:6.17]
Work	-14.80** [-23.72:-5.67]	-10.15** [-17.03:-3.35]	-4.65 [-11.42:2.44]	-1.42 [-11.82:8.08]	-16.60** [-28.79:-5.32]	15.18** [6.46:25.31]
Education	-2.83 [-7.19:1.87]	-1.94 [-4.64:0.42]	-0.90 [-5.08:3.18]	-5.24 [-11.22:0.04]	-1.00 [-3.74:1.53]	-4.25 [-9.37:0.36]
Other	-6.94 [-14.48:0.34]	-8.06** [-14.05:2.48]	1.12 [-4.12:6.26]	-7.10 [-15.16:0.27]	-7.82* [-14.82:-1.63]	0.72 [-5.30:7.00]
Inactive	-2.57 [-7.53:2.08]	-1.84 [-4.38:0.48]	-0.73 [-5.79:3.77]	-13.99** [-22.93:-5.16]	0.04 [-2.89:3.04]	-14.03** [-21.76:-6.43]

Table 4: Competing Risks Decomposition of Restricted Means at 2 years (Duration in Weeks)

non-parametric 95% bootstrap confidence interval in parentheses (number of bootstrap replications 1000).

	Males			Females		
Restricted Mean Difference (2 years)	-13.02** [-18.00:-8.12]			-13.09** [-19.41:-6.99]		
Decomposition	Overall Decomposition (1)	Duration Effect (2)	Incidence Effect (3)	Overall Decomposition (4)	Duration Effect (5)	Incidence Effect (6)
Training	3.07* [0.47:5.76]	-0.15 [-2.20:1.78]	3.22** [1.08:5.34]	0.61 [-2.63:3.60]	-0.28 [-2.28:1.76]	0.87 [-1.73:3.24]
Work	1.04 [-1.55:3.74]	0.36 [-1.42:2.28]	0.67 [-1.09:2.60]	5.18** [1.82:8.44]	-1.07 [-4.73:2.68]	6.20** [3.41:9.53]
Education	0.25 [-1.17:1.62]	-0.20 [-0.99:0.53]	0.45 [-0.84:1.98]	-1.25 [-3.51:0.72]	-0.06 [-0.94:0.84]	-1.19 [-3.05:0.41]
Other	1.16 [-1.26:3.53]	0.75 [-2.38:0.80]	1.91 [-0.30:4.00]	0.69 [-2.13:3.40]	-0.77 [-3.31:1.40]	1.43 [-0.74:4.24]
Inactive	.02 [-0.57:0.53]	-0.22 [-4.99:6.17]	0.23 [-5.66:5.34]	-2.27 [-5.09:0.35]	0.47 [-0.14:1.24]	-2.74* [-5.30:-0.53]
Residual	-18.55 [-24.86:-12.09]			-16.05 [-23.99:-7.59]		

Table 5: Competing Risks Decomposition of Restricted Means at 4 years (Duration in Weeks)

non-parametric 95% bootstrap confidence interval in parentheses (number of bootstrap replications 1000).

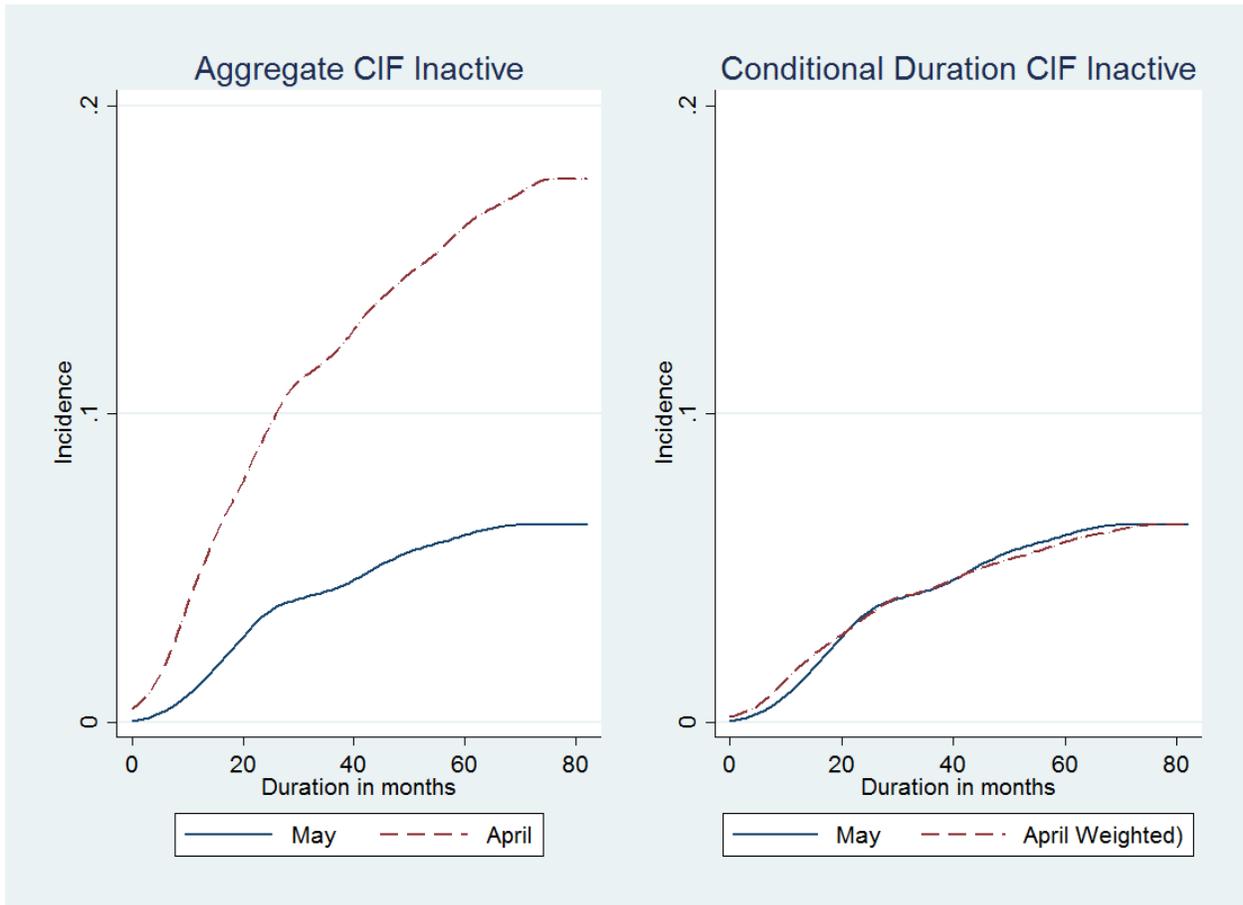
	Males			Females		
Restricted Mean Difference (4 years)	-26.54** [-35.68:-17.43]			-25.97** [-37.35:-14.42]		
Decomposition	Overall Decomposition (1)	Duration Effect (2)	Incidence Effect (3)	Overall Decomposition (4)	Duration Effect (5)	Incidence Effect (6)
Training	-2.05 [-6.97:2.42]	-5.57** [-9.72:-1.69]	3.53 [-0.22:7.68]	-2.67 [-9.04:3.22]	-3.34 [-8.31:1.58]	0.67 [-4.03:5.06]
Work	-2.92 [-8.99:2.83]	-2.65 [-6.94:1.19]	-0.26 [-4.65:3.92]	4.99 [-0.22:10.63]	-4.56 [-10.80:1.71]	9.55** [5.16:15.20]
Education	-1.16 [-4.01:2.00]	-1.04 [-2.74:0.56]	-0.12 [-2.77:2.91]	-2.56 [-7.03:1.38]	0.15 [-1.84:2.17]	-2.71 [-6.58:0.28]
Other	-0.04 [-4.36:4.38]	-2.23 [-5.53:1.06]	2.19 [-1.19:5.67]	-2.29 [-7.74:2.89]	-3.83 [-8.10:0.24]	1.54 [-2.95:6.07]
Inactive	.01 [-2.19:0.51]	-0.64 [-2.12:0.51]	.64 [-1.41:2.80]	-8.31** [-13.97:-1.98]	0.16 [-1.34:1.81]	-8.47** [-13.66:-3.17]
Residual	-20.38 [-30.38:-11.14]			-15.13 [-26.49:-3.82]		

Table 6: Competing Risks Decomposition of Restricted Means at 6 years (Duration in Weeks)

non-parametric 95% bootstrap confidence interval in parentheses (number of bootstrap replications 1000).

	Males			Females		
Restricted Mean Difference (6 years)	-33.06** [-44.11:-21.63]			-30.67** [-45.08:-16.53]		
Decomposition	Overall Decomposition (1)	Duration Effect (2)	Incidence Effect (3)	Overall Decomposition (4)	Duration Effect (5)	Incidence Effect (6)
Training	-6.04 [-13.18:0.99]	-9.70** [-16.55:-3.37]	3.66 [-1.58:9.32]	-3.05 [-11.91:5.55]	-3.63 [-10.40:4.03]	0.58 [-5.56:6.17]
Work	-10.26* [-19.17:-2.21]	-7.36* [-14.00:-1.09]	-2.95 [-9.29:3.36]	-0.71 [-10.07:7.71]	-14.669** [-25.67:-4.98]	13.95** [6.24:22.83]
Education	-1.95 [-5.86:2.32]	-1.42 [-3.81:0.91]	-0.53 [-4.19:3.26]	-3.65 [-8.55:0.81]	-0.30 [-2.38:1.92]	-3.35 [-7.74:0.05]
Other	-6.82* [-13.67:-2.64]	-7.92** [-13.27:-2.65]	-0.81 [-6.54:4.92]	-3.95 [-10.57:2.71]	-5.29 [-11.19:0.18]	1.33 [-3.76:6.72]
Inactive	-1.61 [-5.76:2.08]	-1.51 [-3.79:0.49]	-0.09 [-4.21:3.91]	-13.99** [-22.93:-5.16]	.04 [-2.89:3.04]	-14.03** [-21.76:-6.43]
Residual	-6.38 [-13.22:0.46]			-5.33 [-12.32:1.23]		

Figure 1: Graphical Representation of Total Effect (left panel) and Duration Effect (right panel) accounted for by exits to inactivity for Females



Appendix 1:

Theorem: Let Y be any random variable

$$E(Y|K = k, Y \leq \tau) = \tau - \int_0^\tau \left[\frac{CIF_k(y)}{CIF_k(\tau)} \right] dy$$

Proof:

For any random variable X

$$E[X] = \int_0^\infty S(x) dx \text{ where } S(x) = \Pr(X > x).$$

Specifically,

$$E(Y|K = k, Y \leq \tau) = \int_0^\infty \Pr(Y \geq u|K = k, Y \leq \tau) du$$

Using the conditional probability law we can rewrite this as

$$= \int_0^\infty \frac{\Pr(Y \geq u, Y \leq \tau, K=k)}{\Pr(Y \leq \tau, K=k)} du$$

By definition of $CIF_k(\tau)$ this can be written as

$$\begin{aligned} &= \frac{1}{CIF_k(\tau)} \int_0^\infty \Pr(Y \geq u, Y \leq \tau, K = k) du \\ &= \frac{1}{CIF_k(\tau)} \left[\int_0^\tau \Pr(Y \geq u, Y \leq \tau, K = k) du + \int_\tau^\infty \Pr(Y \geq u, Y \leq \tau, K = k) du \right] \end{aligned}$$

However, since the last term above is zero we can rewrite this

$$= \frac{1}{CIF_k(\tau)} \left[\int_0^\tau \Pr(Y \geq u, Y \leq \tau, K = k) du \right]$$

Using the fact that $\Pr(Y > y, K = k) = 1 - \Pr(Y < y, K = k) - \Pr(K \neq k) = \Pr(K = k) - \Pr(Y < y, K = k)$ this equals

$$\begin{aligned}
&= \frac{1}{CIF_k(\tau)} \left[\int_0^\tau [S_k(u) - S_k(\tau)] du \right] \text{ (where } S_k(y) \equiv \Pr(Y > y, K = k) \\
&= \frac{1}{CIF_k(\tau)} \left[\int_0^\tau [\Pr(K = k) du] - \left[\int_0^\tau CIF_k(u) du \right] - \left[\int_0^\tau S_k(\tau) du \right] \right] \\
&= \frac{1}{CIF_k(\tau)} \left[\tau \cdot \Pr(K = k) - \int_0^\tau [CIF_k(u) du] - [\tau \cdot S_k(\tau)] \right]
\end{aligned}$$

Again using the fact that $\Pr(Y > \tau, K = k) = \Pr(K = k) - \Pr(Y < \tau, K = k)$ this equals

$$\begin{aligned}
&= \frac{1}{CIF_k(\tau)} \left[\tau \cdot \Pr(K = k) - \int_0^\tau [CIF_k(u) du] - [\tau \cdot [\Pr(K = k) - CIF_k(\tau)]] \right] \\
&= \frac{1}{CIF_k(\tau)} \left[\tau \cdot CIF_k(\tau) - \int_0^\tau CIF_k(u) du \right] \\
&= \left[\tau - \int_0^\tau \left[\frac{CIF_k(u)}{CIF_k(\tau)} \right] du \right] \text{ as required.}
\end{aligned}$$

Note; If $K=1$ so that there is only one exit state this simplifies to the standard expression for the conditional mean.

$$E(Y | Y \leq \tau) = \left[\tau - \int_0^\tau \left[\frac{CIF(y)}{CIF(\tau)} \right] dy \right] = \int_0^\tau \left[\frac{(S(y) - S(\tau))}{1 - S(\tau)} \right] dy$$

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