Optimal International Asset Allocation with Time-varying Risk

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Abstract

This paper examines the optimal allocation each period of an internationally diversified portfolio from the different points of view of a UK and a US investor. A multivariate GARCH model is used to estimate the conditional covariance matrix of returns, and to rebalance their portfolios each period according to CAPM. Domestic equity is the dominant asset in the optimal portfolio for both investors, but the US investor bears less risk than the UK investor, and holds less foreign equity - 20% compared to 25%. Survey evidence indicates actual shares are 6% and 18%, respectively, making the home-bias puzzle more acute for US than UK investors. Put another way, there seems to be more potential gains from increased international diversification for the US than the UK investor.

Keywords*:* Asset allocation, international diversification, multivariate GARCH
JEL Classification: F3, G1.

1 Introduction

This paper is concerned with the potential benefits of holding an internationally diversified portfolio that is re-balanced each period to take account of time-variation in the covariance matrix of returns. In Flavin and Wickens (1998) it was shown that domestic asset allocation would be greatly improved by re-balancing in this way compared with using an allocation based on a constant covariance matrix. The analysis is conducted from the different perspectives of a UK and a US investor to see whether the location of the investor affects optimal asset allocation. Investors are allowed to hold three risky assets: domestic and foreign equity and a domestic long-bond. An optimal portfolio is then obtained based on the excess returns of these assets over the domestic risk-free rate.

Evidence that large benefits are available to investors who diversify their portfolio to hold foreign assets has been available from the early literature. Grubel (1968) and Levy and Sarnat (1970) were among the first to reach this conclusion. A number of more recent studies have reached the same conclusion. Grauer and Hakansson (1987) conclude that a US investor can reap "remarkably large" gains from including non-US assets in the portfolio of risky assets. Based on a paired t-test, they find that realised returns from an internationally diversified portfolio are significantly higher than those generated by a portfolio consisting entirely of domestic stocks. Furthermore, the gains increased as the investor becomes more risk averse. De Santis and Gerard (1997) provide evidence that even though equity market declines are contagious across countries, US investors may still earn expected gains of 2.1% on average and these have not fallen despite increased financial market integration. Eun and Resnick (1988) and Jorion (1985) both show that hedging foreign exchange risk can potentially increase the gains from international diversification. Many of these studies concentrate on equity markets, but Levy and Lerman (1988) find that a US investor who diversified across world bond markets could have realised more than twice the mean rate of return on a domestic US bond portfolio with the same risk level. By including both equity and bonds and taking account of the time-variation in the covariance matrix of asset excess returns, we expect to find that the gains to international diversification are even greater than found in these studies.

As in Flavin and Wickens (1998), our tactical allocation strategy is an adaptation of Markowitz's (1959) minimum variance portfolio selection theory in which excess returns are modelled by a version of the multivariate

GARCH (M-GARCH) process particularly suited to portfolio analysis. This enables us to determine how the covariance matrix of excess returns, and hence the minimum variance portfolio frontier and the optimal portfolio shares of risky assets, vary through time. The two-fund separation theorem together with either a target rate of return for the portfolio, or a mean-variance trade-off can then be used to choose the optimal mix of the risky and risk-free assets. One reason for focussing on minimising the portfolio variance is that, in practice, returns - especially equity returns - are not forecastable (i.e. they are virtually serially independent). We are able to reject the assumption that is usually made, namely, that the covariance matrix of returns is constant.[1] In the absence of transactions costs, this increases the need for the optimal portfolio to be re-balanced each period.[2] This suggests that, like portfolios that comprise only domestic assets, an international tactical asset allocation should aim to exploit the regularities in the covariance structure of excess returns in order to minimise risk.

For both sets of investors we find that although domestic equity dominates the optimal portfolio in each period, the foreign asset is also an important constituent of the portfolio, and on average has a greater share than the domestic bond. Comparing our results with survey results of actual asset holdings confirms that the home bias or international diversification puzzle is still alive. French and Poterba (1991) report that US investors hold 94% of their financial wealth in domestic securities, with Japanese and UK investors holding 98% and 82% of their respective portfolios in domestic assets. Cooper and Kaplanis (1994) estimate that the percentage of domestic equities in the total equity portfolio in US, UK and Japan is 98%, 79% and 87% respectively. We find that the home bias puzzle is much more acute in the US than the UK. This failure to exploit international diversification benefits has resulted in a large contemporary literature seeking to explain this puzzle.[3] Our analysis suggests that the US investor should, on average, apportion 20% of the funds held in risky assets to UK equity, while the optimal portfolio for the UK investor contains on average 25% of its asset holdings in US equity. We find that in the short run shares can differ considerably from these average values, and that most of the re-balancing is between domestic and foreign

[1] This has already been shown by Clare et al. (1998), Cumby, Figlewski & Hasbrouck(1994) etc.

[2] Of course, this is not the only reason to re-balance. In general, the portfolio will need to be re-balanced when expected returns are not equal to realised returns.

[3] This puzzle is reviewed in Uppal (1992), Lewis (1998).

equity. Since our investment opportunity set is restricted, our results may be interpreted as providing a lower bound on optimal foreign equity holdings.

The paper is set out as follows. Section 2 describes the econometric model. In Section 3 the data are described and the estimates are reported. The optimal portfolios are analysed in Section 4. Section 5 summarises our findings and has some conclusions. The appendix provides a brief review of the key results on portfolio optimisation used in the paper.

2 Econometric Model

The aim of this paper is to identify the optimal portfolio of risky assets in each period for both UK and US investors. The asset allocation strategy used is fully described in Flavin and Wickens (1998). It is assumed that investors use Markowitz's mean-variance portfolio theory to determine the optimal allocation of assets. Investors therefore select the weights of the portfolio that corresponds to a tangent from the origin to the portfolio frontier of excess returns on the risky assets. The risk-free asset in each case being the domestic risk-free bond.

Instead of assuming that the covariance matrix of excess returns is constant through time, we allow it to be time varying. This implies that the portfolio may need re-balancing every period in response to such time variation. We assume that the vector of excess returns is Normally distributed with a time-varying conditional mean and conditional variance generated by an M-GARCH(1,1) process. It is widely recognised that there is a major computational problem in the full information estimation of such models due to the large number of parameters they possess. We therefore adopt the M-GARCH representation set out by Flavin and Wickens (1998).[4] This model is a variant of the Berndt, Engle, Kraft & Kroner (BEKK) representation.[5] If n is the number of risky assets, then compared with the most general formulation of the model, this representation results in the number of parameters to be estimated increasing at the rate n^2 instead of n^4. When $n = 3$, the number of parameters to be estimated is reduced from 78 using

[4]This parameterisation is consistent with the covariance stationary model developed in Engle and Kroner (1995).

[5]For a full treatment of multivariate (G)ARCH models, the reader is referred to Bollerslev, Engle and Nelson(1994), Bera and Higgins(1993) or Bollerslev, Chou and Kroner(1992) for a survey of (G)ARCH models in finance

the most general model to 18, a substantial saving.

The model may be written

$$\begin{array}{rcl} \mathbf{r}_{t+1} & = & \boldsymbol{\upsilon} + \boldsymbol{\Gamma}\mathbf{r}_t + \boldsymbol{\Upsilon} dum87 + \boldsymbol{\xi}_{t+1} \\ \boldsymbol{\xi}_{t+1} & \mid & \Psi_t \sim N(0, \boldsymbol{\Omega}_{t+1}) \\ \boldsymbol{\Omega}_{t+1} & = & \mathbf{V}'\mathbf{V} + \boldsymbol{\Phi}'(\boldsymbol{\Omega}_t - \mathbf{V}'\mathbf{V})\boldsymbol{\Phi} + \boldsymbol{\Theta}'(\boldsymbol{\xi}_t\boldsymbol{\xi}_t' - \mathbf{V}'\mathbf{V})\boldsymbol{\Theta} \end{array} \tag{3.1}$$

where $\mathbf{r}_{t+1}$is the vector of excess returns in period t+1, $dum87$ is a dummy variable for the October 1987 stock market crash. $\boldsymbol{\nu}$ and $\boldsymbol{\Upsilon}$ are $nx1$ vectors, $\boldsymbol{\Gamma}$ is an nxn matrix and $\mathbf{V}$, $\boldsymbol{\Phi}$ and $\boldsymbol{\Theta}$ are nxn symmetric matrices of parameters.

By making the matrices symmetric rather than unrestricted we are able to economise on parameters since only $3n(n+1)/2$ parameters are required for the covariance matrix. It might seem that an equivalent specification would be to make the matrices triangular, but in fact this has the disadvantage of restricting the dynamic structure of the covariance matrix unnecessarily by introducing an additional lag involving cross-effects. By formulating the conditional variance-covariance structure in this way, we are able to obtain an estimate of both the unconditional (long-run) covariance matrix and the conditional covariance matrix (the short-run dynamics). This is important as it allows us to decide whether or not the short-run dynamics make a sufficiently useful contribution to justify their inclusion, and the time and effort to estimate them. It also allows us to identify which parameters are most significant in determining deviations from the long run. This formulation also guarantees a positive semi-definite unconditional and conditional covariance matrix.

Initially we choose $n = 3$ and $r = (ukeq, useq, gvbd)'$ where $ukeq$, $useq$ and $gvbd$ refer to the excess return over the domestic risk-free rate of UK equities, US equities and a domestic government bond respectively.[6] From an econometric point of view, there is a further benefit from working with excess returns, namely that all series are stationary and do not require differencing.[7]

[6] The main body of the paper is concerned with the case in which the foreign bond is omitted from the opportunity set of the investor. However, at the end of our analysis, the consequences of its inclusion are examined.

[7] A wide range of Unit root tests, such as (Augmented) Dickey Fuller tests, Stock Watson tests and Phillips Peron tests, were conducted on these series and all results confirm stationarity. Results are available from the authors upon request.

3 Data and Estimation Results

3.1 The Data

This paper uses time series data on broad classes of UK and US financial assets. The analysis is conducted, firstly, from the perspective of a UK investor, and secondly, with respect to a US investor. Each set of investors is assumed to choose an optimal portfolio consisting of three risky assets based on the excess returns of two domestic risky assets and one foreign risky asset over a domestic risk-free asset. The risky assets used in the analysis are UK equities, represented by the Financial Times All Share Index; US Equities represented by the S&P Composite Index, UK government bonds represented by the FT British government stock index; US bonds represented by a Datastream computed government bond index. In each case, the return on the foreign asset is converted into the domestic currency using the end of month exchange rate. The data used in this paper are annualised monthly total returns for each asset. The total return data is calculated so as to take account of dividend payments in the case of equities and coupon payments in the case of government bonds. For the UK investor, the rate of return on the UK government 30 day Treasury bill is taken as the risk-free rate of interest, while for the US investor the riskless interest rate is proxied by the Eurodollar rate, i.e. the rate available on one month US deposits in London.[8] These assets are riskless at least in the nominal sense. The data period is from January 1980 to March 1997. All data are sourced from DATASTREAM

By working with excess returns we are able to prevent volatility in the risk-free rate from incorrectly contributing to the risk of the optimal risky portfolio. Since the risk-free rate is perfectly predictable at the start of each period, and is therefore part of the investor's information set when the allocation decision is made, its inclusion would tend to lead to an overestimate of the total risk of the portfolio. As noted above, this also avoids unit root problems in the data since a unit root is rejected for all of the excess returns.

[8] We would prefer to have used the return on a 1-month Treasury bill but the series available would result in a shorter time frame. Despite not being perfectly correlated, we argue that the Eurodollar rate is a reasonable proxy.

3.2 Estimates

The estimates of the model for the representative UK and US investor are reported below; t-statistics are in parentheses.[9]

3.2.1 UK Investor

Conditional Mean

$$\boldsymbol{\nu} = \begin{bmatrix} 7.23 \\ (2.2) \\ 5.20 \\ (1.23) \\ -0.61 \\ (-0.33) \end{bmatrix}, \boldsymbol{\Gamma} = \begin{bmatrix} -0.06 & 0.07 & 0.20 \\ (-0.74) & (1.07) & (1.32) \\ -0.09 & 0.11 & 0.07 \\ (-1.05) & (1.39) & (0.43) \\ 0.01 & -0.06 & 0.10 \\ (0.27) & (-1.95) & (1.35) \end{bmatrix}, \boldsymbol{\Upsilon} = \begin{bmatrix} -403.51 \\ (-1.49) \\ -334.39 \\ (-2.01) \\ 0 \end{bmatrix}$$

Covariance Covariance (The matrices are symmetric.)

$$\mathbf{V} = \begin{bmatrix} 57.21 & & \\ (8.99) & & \\ 26.59 & 56.98 & \\ (4.51) & (11.94) & \\ 15.50 & -4.94 & 24.06 \\ (7.17) & (-2.17) & (11.4) \end{bmatrix}$$

$$\boldsymbol{\Theta} = \begin{bmatrix} 0.27 & & \\ (3.10) & & \\ -0.13 & 0.28 & \\ (-2.43) & (3.86) & \\ 0.15 & 0.02 & 0.43 \\ (4.28) & (0.51) & (4.08) \end{bmatrix}, \boldsymbol{\Phi} = \begin{bmatrix} -0.80 & & \\ (-7.45) & & \\ -0.19 & -0.73 & \\ (-2.01) & (-5.5) & \\ -0.15 & -0.01 & 0.39 \\ (-0.97) & (-0.07) & (3.05) \end{bmatrix}$$

3.2.2 Discussion of the Results

In the conditional mean the elements of $\boldsymbol{\Gamma}$ are generally not significant. This is consistent with the usual finding that total stock and bond returns are

[9] Maximisation of the likelihood function was achieved using the Berndt, Hall, Hall and Hausmann (BHHH) algorithm in RATS.

serially uncorrelated. The most significant element is Γ_{32} implying that the lagged excess return on the US equity has some explanatory power for the excess return on UK bonds, but it is difficult to think of a good reason why this should be. Consequently, we assume a constant vector of expected asset returns when we generate the portfolio shares. This has the added advantage that all of the variation in the estimated frontiers, and hence the portfolio shares, can be attributed to variation in the conditional covariance matrix of excess returns.

This is also the assumption made by Cumby, Figlewski and Hasbrouck(1994) who use the historical mean of each asset as its expected value. Jobson & Korkie(1981) advocate the use of global shrinkage based on Stein estimators whereby all assets of the same class have the same expected excess return. This is an extreme case of Stein estimation with the individual asset being assigned a weight of zero and the global mean having a weight of one. Jobson & Korkie show that this approach significantly improved the practical application of the mean-variance framework. Since we are working with financial asset indices as opposed to individual securities, these appoaches reduce to the same thing. Another reason for making this assumption is that the sensitivity of the portfolio shares to small variations in the mean is far greater than that to variations in the covariance matrix, Kallberg and Ziemba (1984). Best and Grauer (1991) show that even small changes in the mean vector can result in dramatic variation in the composition of the estimated optimal portfolio of risky assets.

Continuous re-balancing of the portfolio to changes in the predicted excess return would not only be expensive due to transaction costs, it would also be counter-productive because of the lack of persistence of the deviations of excess returns from their unconditional means. This is not true of re-balancing due to changes in the conditional variance because of their much higher degree of persistence and their lower volatility.

In contrast, the elements of the matrices determining the conditional covariance matrix are generally significant. All the elements of $\mathbf{V}$, which determines the unconditional or long-run matrix, and most of the elements of the matrices that determine the short run, $\boldsymbol{\Theta}$ and $\boldsymbol{\Phi}$ are highly significant. The significance of the diagonal elements of $\boldsymbol{\Theta}$ and $\boldsymbol{\Phi}$ indicates that the conditional variances differ considerably from the unconditional variances, showing considerable volatility clustering even at monthly intervals. The only off-diagonal elements not significant are Θ_{32}, Φ_{31} and Φ_{32}. The significance of the $\{3,1\}$ elements suggests that the allocation between UK equity and

UK bonds will need to be re-balanced in the short-run to achieve optimality.

The {2,1} elements of $\mathbf{\Theta}$ and $\mathbf{\Phi}$ show the volatility contagion between the UK and the US stockmarkets. This, together with the {2,1} element of the long-run covariance matrix, is the reason why investors may want to hold an internationally diversified portfolio in order to reduce risk. For example, the long-run covariance matrix is

$$H = \begin{bmatrix} 3273 & & \\ 1521 & 3953 & \\ 887 & 131 & 843 \end{bmatrix}$$

implying a correlation between the excess returns over the UK risk-free rate of UK and US equity returns of 0.42. This also implies that to achieve an optimal portfolio re-balancing between UK and US equity will be required.

3.2.3 US Investor

Conditional Mean

$$\boldsymbol{\nu} = \begin{bmatrix} 10.86 \\ (2.38) \\ 9.40 \\ (2.78) \\ 2.11 \\ (1.29) \end{bmatrix}, \mathbf{\Gamma} = \begin{bmatrix} -0.05 & 0.14 & -0.002 \\ (-0.51) & (1.17) & (-0.008) \\ 0.02 & -0.01 & 0.20 \\ (0.24) & (-0.05) & (1.35) \\ 0.02 & -0.07 & 0.20 \\ (0.65) & (-1.47) & (2.39) \end{bmatrix}, \mathbf{\Upsilon} = \begin{bmatrix} -350.93 \\ (-0.76) \\ -327.58 \\ (-1.50) \\ 0 \end{bmatrix}$$

Covariance Covariance

$$\mathbf{V} = \begin{bmatrix} 64.60 & & \\ (15.03) & & \\ 22.49 & 39.09 & \\ (5.96) & (16.17) & \\ 5.50 & 6.78 & 18.50 \\ (3.26) & (3.91) & (10.4) \end{bmatrix}$$

$$\mathbf{\Theta} = \begin{bmatrix} -0.32 & & \\ (-1.78) & & \\ 0.10 & -0.26 & \\ (0.09) & (-2.08) & \\ -0.02 & -0.21 & 0.22 \\ (-0.35) & (-3.46) & (2.65) \end{bmatrix}, \mathbf{\Phi} = \begin{bmatrix} -0.38 & & \\ (-0.95) & & \\ -0.05 & 0.24 & \\ (-0.15) & (0.60) & \\ 0.39 & -0.41 & 0.47 \\ (1.61) & (-1.99) & (2.10) \end{bmatrix}$$

3.2.4 Discussion of the Results

The results are similar to those for the UK. Again, $\mathbf{\Gamma}$ is almost insignificant, though here there does seem to be some significant persistence in the excess return on US bonds. The elements of $\mathbf{V}$ are all statistically significant. The estimates of the diagonal elements of $\mathbf{\Theta}$ are all significant showing considerable ARCH effects and hence differences between the long-run and short-run covariance matrices. The significance of the $\{3,2\}$ elements of $\mathbf{\Theta}$ and $\mathbf{\Phi}$ indicates that there will need to be a re-balancing between US equity and US bonds in the short run to achieve optimality.

The main difference is that there are no significant contagion effects between the US and UK stockmarkets. Taking together the UK and US results, this seems to indicate that causality runs from the US to the UK stockmarket. It would also suggest that the gains to the US investor from re-balancing the portfolio in the short run between US and UK assets are likely to be small. This is not to suggest that there aren't likely to be gains to the US investor to holding UK equity. The long-run covariance matrix is

$$H = \begin{bmatrix} 4173 & & \\ 1453 & 2034 & \\ 355 & 389 & 419 \end{bmatrix}$$

giving a correlation between the excess returns over the US risk-free rate on US and UK equity of 0.50.

4 Optimal Asset Allocation

4.1 Frontier Movements

Variations in the optimal portfolio weights when short sales are permitted are due entirely to movements in the portfolio frontier. These are brought about by new information on next period's conditional covariance matrix that causes it to vary over time. Estimates of the unconditional and conditional variances facing UK and US investors are plotted in Figures 1 and 2. For each country fluctuations in the exchange rate make foreign equity the asset with the most volatile excess returns. Nonetheless, since 1993, there has been a noticeable decline in volatility for all assets, and especially for equity returns expressed in sterling. This reflects the relative stability of the £/$ exchange rate over this period. The graphs also show that short-run deviations of the conditional variances from their long-run values can be quite large, and are therefore likely to have a significant impact on the portfolio frontiers and hence on asset allocation in the short run.

Some idea of the fluctuations in the portfolio frontiers facing UK and US investors can be obtained from Figures 3 and 4. Figure 3 shows for each country the frontier based on the long-run covariance matrix and the mean frontier in the short run. The position of the frontiers reflect the minimum portfolio standard deviation for a given portfolio return, hence this is just another way of comparing portfolio standard deviations. For each country the mean frontier lies to the left of the long-run frontier. It is therefore possible for investors to reduce their portfolio risk by re-balancing their portfolios each period. Another implication is that the actual portfolio risk borne by investors who use the long-run covariance matrix will be different from that shown by the long-run frontier. The frontier for the US investor lies to the left of that for the UK investor. This implies that the US investor bears less risk than the UK investor to achieve the same return. The optimal portfolio of risky assets available to the US investor should therefore deliver a higher Sharpe Performance Index than that for UK investor.

Figure 4 provides more information on the distribution of frontier movements. It displays the maximum, minimum and mean frontiers for each country. This shows that although the conditional distribution of frontiers for the US investor is shifted to the left of that for the UK investor, there is considerable overlap in the distributions. The graph also indicates that the conditional distributions are positively skewed, with a few periods when portfolio risk is much higher than the mean.

4.2 Optimal Portfolios

The optimal portfolio of risky assets when there are no restrictions on short sales is obtained from the point of tangency between the portfolio frontier and the Capital Market line which goes through the origin. The resulting portfolio weights will vary each period. The appendix provides a brief summary of the key results on portfolio optimisation used in the paper. Figures 5 and 6 show the time variation in the excess return and the standard deviation of the optimal portfolios of UK and US investors, and figure 7 shows the Sharpe Performance Index ($SPI = \frac{return}{risk}$). The US portfolio has a higher mean excess return (9.1% versus 8%) and achieves a higher (0.24 versus 0.18) and more stable SPI than the UK portfolio. For the UK, however, there has been a remarkable improvement in the SPI since 1992 due to the strong and persistent growth of equity prices, with the result that the SPI for the UK has exceeded that for the US since the end of 1994.

4.2.1 UK Investor

In examining the optimal asset allocation in more detail, we consider first the UK investor who is able to take unlimited short positions. We first look at the prescribed weights assuming a constant covariance matrix. This matrix is taken to be the long-run matrix, $\mathbf{V}'\mathbf{V}$, estimated above. The weight vector is as follows:

	UK Equity	US Equity	UK Bond
Asset Holding	64%	26%	10%

Table 1. Constant weight vector for UK investor.

In comparison with our time-varying allocation, this method of portfolio selection tends to overstate the importance of the UK bond, but even here we see US equity accounting for a large portion of the portfolio. The time series of shares in the optimal portfolio of risky assets of the UK investor is shown in Figure 8 and the summary statistics for the portfolios are reported in Table 2.

	Mean	Minimum	Maximum
UK Equity	77%	24%	210%
US Equity	27%	-2%	60%
UK Bond	-4%	-166%	55%

Table 2. Summary of Unrestricted Holdings for a UK investor

It is clear that UK equity dominates the portfolio, accounting on average for 77% of the investment. Although the optimal proportion of UK equity fluctuates a great deal, it is never held short. In many periods it is optimal to hold over 100% of total wealth in UK equity. The optimal proportion of US equity has a mean of 27% and is relatively stable. For only one period in the entire 200 period sample is it optimal to hold US equity short. In contrast to domestic and foreign equity, it is frequently optimal to go short in UK bonds. The mean optimal proportion of UK bonds is -4%, and varies between a maximum of 55% and a minimum of -166%. The volatility of this proportion is similar to that of UK equity. The usual reason for going short in UK bonds, therefore, is to buy UK equity. This is indicated very clearly by the optimal allocations after 1992, when the rise in the share of UK equity is matched by a corresponding fall in share of UK bonds. This is also the cause of the improvement in the SPI for UK investors after 1992.

It is unlikely that investors would follow this investment stragey for two reasons. First, the transactions costs of continuously rebalancing the portfolio in this way may be too high, although the use of indexed trackers or futures would help make it more feasible. Second, and in practice probably more important, UK mutual fund managers, the major holders of assets by far, are prohibited by law from going short. We therefore construct optimal portfolios subject to the constraint that asset shares must be non-negative. For the unconstrained portfolio it was possible to obtain a closed-form expression for the portfolio shares and hence find the return on the portfolio in each period. For the constrained portfolio we use quadratic programming to minimise the variance of the portfolio subject to a target rate of return which is chosen as the mean return on the unconstrained optimal portfolio. This implies that, in terms of the mean portfolio return, the investor is not penalised by the restriction and it aids comparison with the unrestricted case.

Figure 9 shows how the restricted shares vary over time, and Table 3 provides summary statistics.

	Mean	Minimum	Maximum
UK Equity	71%	62%	89%
US Equity	25%	0%	38%
UK Bond	4%	0%	11%

Table 3. Summary of Restricted Holdings for a UK investor

UK equity still dominates the portfolio with a mean of 71%, but its range of variation is reduced by a factor of about 9, having a maximum of 89% and a minimum of 62%. The mean share of US equity is similar and its range of variation is halved. The mean share of UK bonds is 4%, and its range of variation is reduced by a factor of about 20. Now that borrowing by selling domestic bonds is prohibited, portfolio re-balancing takes place mainly between domestic and foreign equity. This results in a considerable reduction in the degree of re-balancing.

Survey evidence shows that UK investors hold up to 18% of their wealth in foreign assets. Our estimate is that a UK investor faced with the opportunity to form an optimal portfolio from these three risky assets should hold about

25% of wealth in US equity. The difference between the two is a measure of the extent of home bias by UK investors.

4.2.2 US Investor

Once more, we begin our analysis by computing the vector of weights that would be suggested by adopting the constant covariance matrix. Table 4 presents these asset holdings.

	UK Equity	US Equity	US Bond
Asset Holding	20%	62%	18%

Table 1 Constant weight vector for US investor.

Once more, this approach places too much weight on the domestic bond to the detriment of the home equity. It also fails to recognise the benefits from adjusting the portfolio.

The time-varying asset shares for the US investor are shown in Figure 10 and summary statistics are reported in Table 5.

	Mean	Minimum	Maximum
UK Equity	20%	2%	109%
US Equity	64%	30%	332%
US Bond	16%	-342%	57%

Table 5. Summary of Unrestricted Holdings for a US investor

The results are similar to those for the UK investor. Domestic equity has the largest share and the greatest variation, and whenever in excess of 100% of wealth is invested in US equity, it is always funded by adopting a short position in the domestic bond. The mean shares are: domestic equity 64% (compared with 77% for the UK), foreign equity 20% (25%) and domestic bonds 16% (-4%). The last fluctuates wildly, moving between a range of 30% to 332%. Thus again this investment strategy looks excessively volatile.

Restricting the US investor to holding only non-negative positions provides Figure 11 and Table 6.

	Mean	Minimum	Maximum
UK Equity	20%	0%	43%
US Equity	63%	38%	86%
US Bond	17%	14%	20%

Table 6. Summary of Restricted Holdings for a US investor

Again the mean shares are hardly altered but the variation in the shares is greatly reduced compared with the unrestricted portfolio. The range of the share of US equity is from 38% to 86% and the share of the domestic bond is remarkably stable moving only between 14% and 20% of the portfolio. Even more than for the UK, re-balancing the portfolio is achieved mainly by substituting domestic for foreign equity, leaving the share of the US domestic bond relatively unchanged. Even so, UK equity still has a mean share of 20%.

Survey evidence shows that US investors hold as little as 6% of their wealth in foreign assets. This compares with our estimate that 20% should be allocated to UK equity. The home bias problem therefore seems to be much more a feature of US than UK investment.

4.2.3 The Foreign Bond

In the preceeding analysis, the opportunity set of the investor was limited by excluding the foreign bond. Here we examine the consequences of its inclusion. For both investors, we found that the unconstrained share held in the foreign bond is always negative and quite large. Table 7 reports the average unconstrained percentage holding.

	UK Equity	US Equity	UK Bond	US Bond
UK investor	73	54	11	-38
US investor	69	82	-40	-11

Table 7 Unconstrained asset holdings including the foreign bond.

Our main interest, however, is in the constrained portfolio that prohibits short sales. In this case the optimal share of the foreign bond is negligible. Consequently, we pay more attention to the portfolios excluding the foreign bond.[10] Table 8 summarises the mean holdings in the constrained allocation, while figures 12 and 13 plot the restricted time-varying weights.

	UK Equity	US Equity	UK Bond	US Bond
UK investor	69	29	2	0
US investor	30	67	0	2

Table 8 Constrained asset holdings including the foreign bond.

[10] Flavin & Wickens (2000) deal with extending the investment opportunity set to include many assets.

5 Conclusion

The aim of this paper has been to re-examine the issue of the optimal allocation each period of an internationally diversified portfolio. These results are then used to provide new ways of determining whether the location of an investor ought to affect portfolio selection, and of addressing the home-bias puzzle. The example used in the analysis is the optimal mix of domestic and foreign equity, and domestic bonds that should be held by UK and US investors, two countries that have not had significant barriers to investing abroad for some time. Two tactical investment strategies are compared. Both are versions of Markowitz's mean-variance portfolio theory in which investors use the joint conditional distribution of excess returns, which is time varying, to re-balance their portfolios each period. One allows investors to hold unlimited short positions; the other assumes that investors are constrained from going short, the situation faced by most fund managers. The conditional covariance matrix is estimated from a form of the multivariate GARCH model particularly suited to portfolio analysis.

We find that for both UK and US investors, although domestic equity is the dominant asset, it is optimal to hold between 20% and 27% of wealth in foreign equity. This compares with survey evidence which indicates that in practice UK investors hold around 18% in foreign assets, while US investors hold only about 6%. The home-bias puzzle seems therefore to be more acute for US than UK investors. Put another way, there seems to be more potential gains from increased international diversification for the US than the UK investor. We also stress that since our investment opportunity set is small, these figures represent a lower bound on the optimal holdings of foreign assets.

We also find that the location of the investor is important in determining the investment performance of the portfolio. The portfolio frontiers facing the US investor lie nearer the origin than for the UK investor, implying that US investors can achieve the same return as UK investors but with less risk. This 'risk-return' advantage is also shown in the higher average Sharpe Performance Index for the US - even though since 1993 the SPI for the UK has steadily improved and now lies above that for the US.

Finally, our results confirm the findings of Flavin and Wickens (1998) that using a constant covariance matrix to construct the assets shares produces a misallocation of resources and a false estimate of one-period risk, that it is optimal to re-balance the portfolio each period, and that prohibiting short positions greatly reduces the amount of portfolio re-balancing required.

6 Appendix: optimal asset allocation

It is assumed that investors are forming their portfolios using mean-variance analysis for one period only, at the beginning of period t, using the information then available. Let $\mathbf{R}_{t+1} = (R_{1t+1}....R_{nt+1})'$ denote an $nx1$ vector of risky asset returns realised during period t and paid at the beginning of period $t+1$. It is assumed that the conditional distribution of $\mathbf{R}_{t+1}$has mean $E_t\mathbf{R}_{t+1}$,which are not all equal, and non-singular covariance matrix $\mathbf{\Omega}_{t+1} = \{\sigma_{ij,t+1}\}$,for $i, j = 1, 2, ...n$. It is assumed that all funds are invested, thus the $nx1$ vector of weights $\mathbf{w}_t = (w_{1t}.....w_{nt})'$ satisfies $\mathbf{w}_t'\mathbf{i} = 1$,where $\mathbf{i}$ is an $nx1$ vector of ones. The conditional distribution of the return on the portfolio $R_{p,t+1}$therefore has expected return

$$E_t R_{p,t+1} = \mathbf{w}_t' E_t \mathbf{R}_{t+1} = \sum_i w_{it} E_t R_{it+1} \tag{A.1}$$

and variance

$$\sigma^2_{p,t+1} = \mathbf{w}_t' \mathbf{\Omega}_{t+1} \mathbf{w}_t = \sum_i \sum_j w_{it} w_{jt} \sigma_{ij,t+1} \tag{A.2}$$

The optimal portlio therefore has the standard Markowitz(1952) set-up

$$\begin{array}{cc} \text{Minimise} & \mathbf{w}_t' \mathbf{\Omega}_{t+1} \mathbf{w}_t \\ \text{subject to:} & \\ & \mathbf{w}_t' E_t \mathbf{R}_{t+1} = \mu_{t+1} \\ & \mathbf{w}_t'\mathbf{i} = 1 \end{array} \tag{A.3}$$

where μ_t is the target rate of return for the portfolio.

When there is no constraint on short sales the solution is

$$\mathbf{w}_t = \mathbf{\Omega}^{-1}_{t+1} \left[\begin{array}{cc} E_t\mathbf{R}_{t+1} & \mathbf{i} \end{array} \right] \mathbf{A}_t^{-1} \left[\begin{array}{c} \mu_{t+1} \\ 1 \end{array} \right] \tag{A.4}$$

$$\sigma^2_{p,t+1} = \mathbf{w}_t' \mathbf{\Omega}_{t+1} \mathbf{w}_t = \tfrac{1}{\Delta_t}(a_t - 2b_t\mu_{t+1} + c_t\mu^2_{t+1}) \tag{A.5}$$

where

$$\mathbf{A}_t = \begin{bmatrix} a_t & b_t \\ b_t & c_t \end{bmatrix} = \begin{bmatrix} E_t\mathbf{R}'_{t+1}\mathbf{\Omega}^{-1}_{t+1}E_t\mathbf{R}_{t+1} & E_t\mathbf{R}'_{t+1}\mathbf{\Omega}^{-1}_{t+1}\mathbf{i} \\ E_t\mathbf{R}'_{t+1}\mathbf{\Omega}^{-1}_{t+1}\mathbf{i} & \mathbf{i}'\mathbf{\Omega}^{-1}_{t+1}\mathbf{i} \end{bmatrix} \tag{A.6}$$

and $\Delta_t = (a_t c_t - b_t^2) > 0$. The optimal portfolio is of risky assets is obtained by choosing μ_{t+1} to correspond to the point of tangency of a line through the origin to the portfolio frontier when $E_t R_{p,t+1} = \mu_{t+1} = a_t/b_t$, i.e. it depends solely on the conditional covariance matrix. When short sales are not permitted no closed-form solution exists and it becomes necessary to solve the optimal problem using quadratic programming. For further discussion of the basic theory see, for example, Ingersoll (1987), and for the current implementation see Flavin and Wickens (1997).

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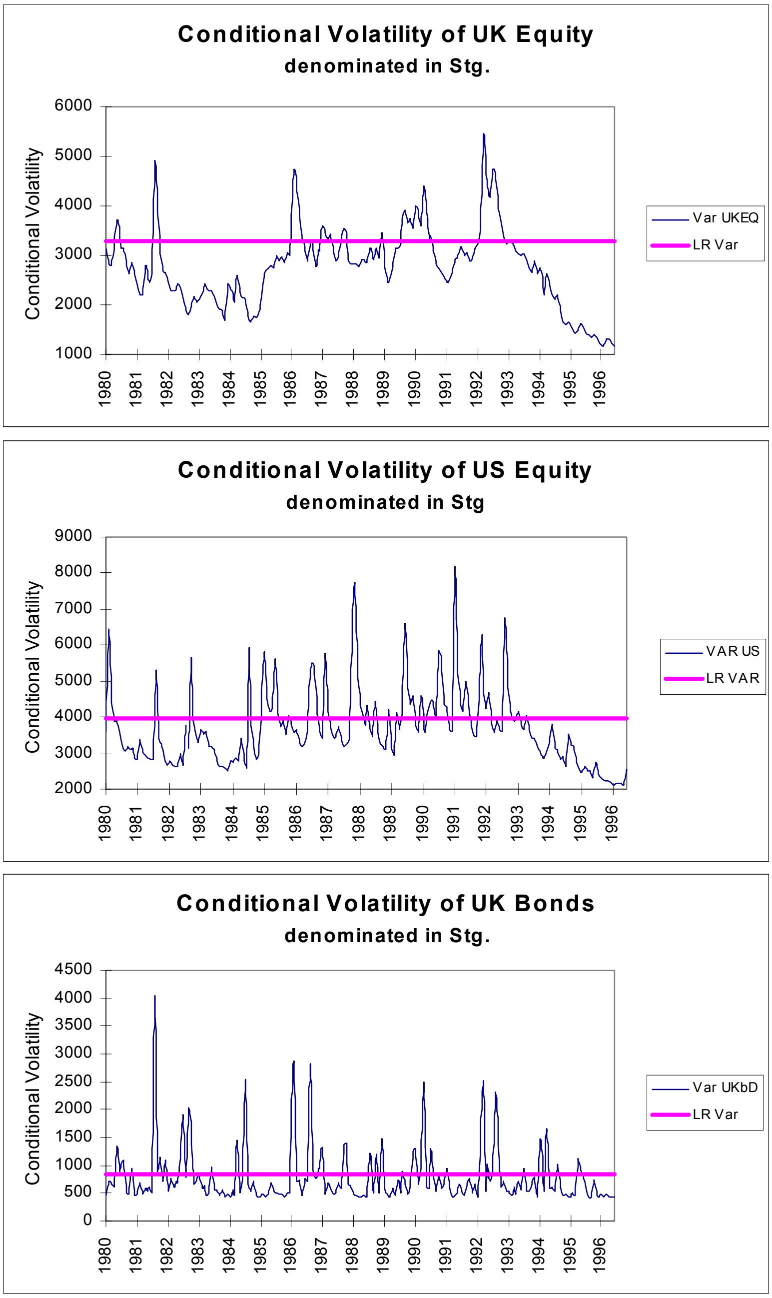

Conditional Volatility of UK Equity
denominated in Stg.
Conditional Volatility
6000
5000
4000
3000
2000
1000
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
Var UKEQ
LR Var
Conditional Volatility of US Equity
denominated in Stg
Conditional Volatility
9000
8000
7000
6000
5000
4000
3000
2000
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
VAR US
LR VAR
Conditional Volatility of UK Bonds
denominated in Stg.
Conditional Volatility
4500
4000
3500
3000
2500
2000
1500
1000
500
0
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
Var UKbD
LR Var

Figure 1:

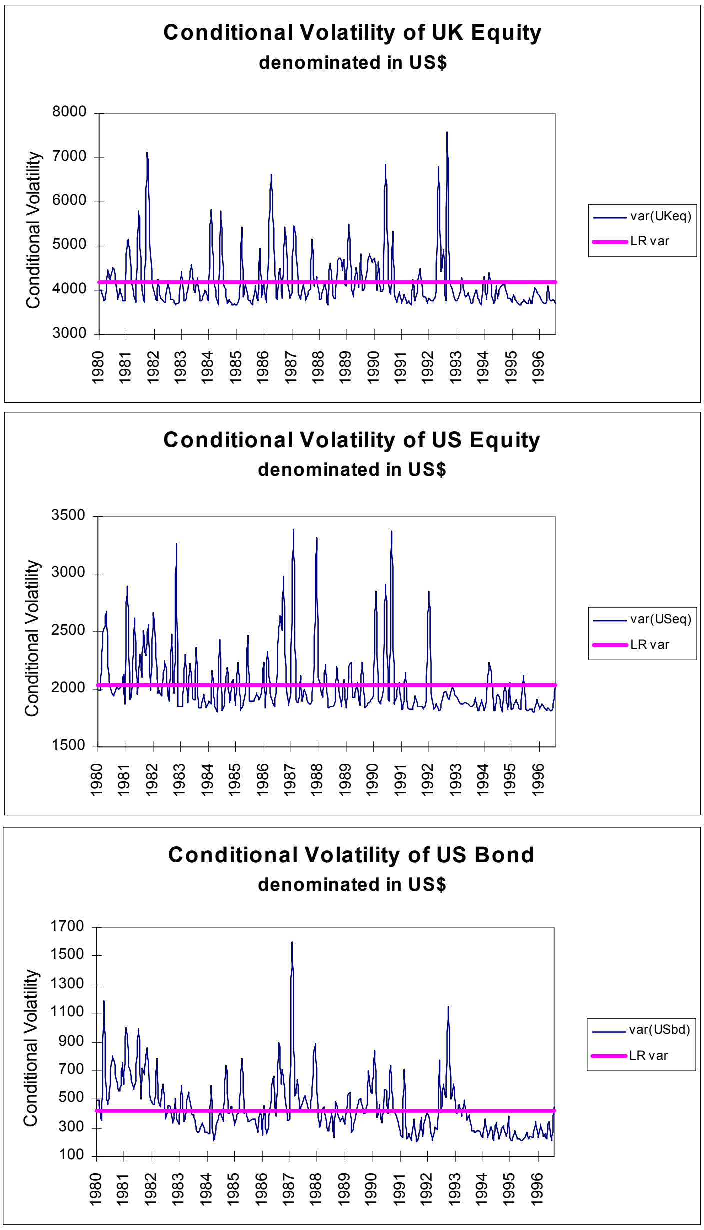
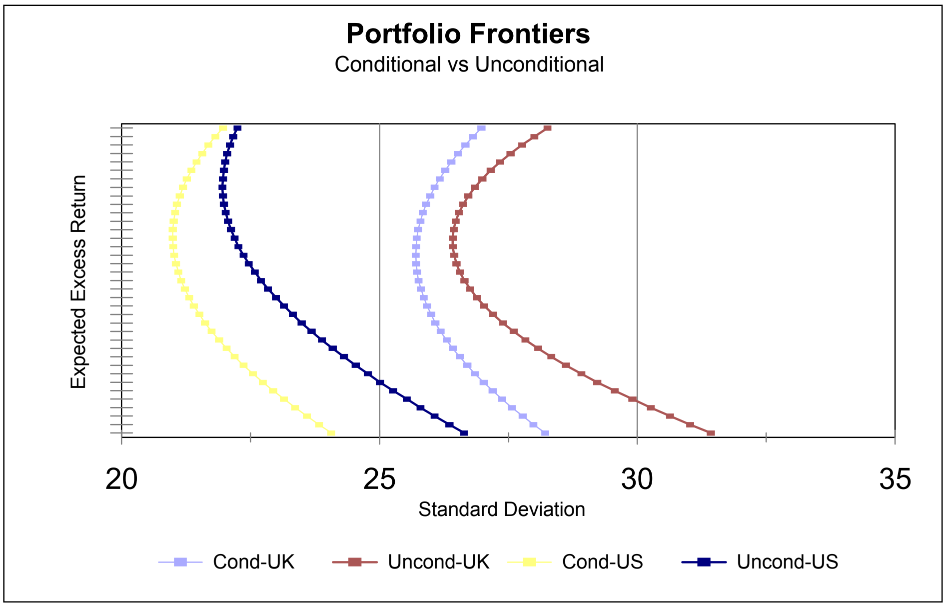

Conditional Volatility of UK Equity
denominated in US$
Conditional Volatility
8000
7000
6000
5000
4000
3000
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
var(UKeq)
LR var
Conditional Volatility of US Equity
denominated in US$
Conditional Volatility
3500
3000
2500
2000
1500
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
var(USeq)
LR var
Conditional Volatility of US Bond
denominated in US$
Conditional Volatility
1700
1500
1300
1100
900
700
500
300
100
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
var(USbd)
LR var

Figure 2:



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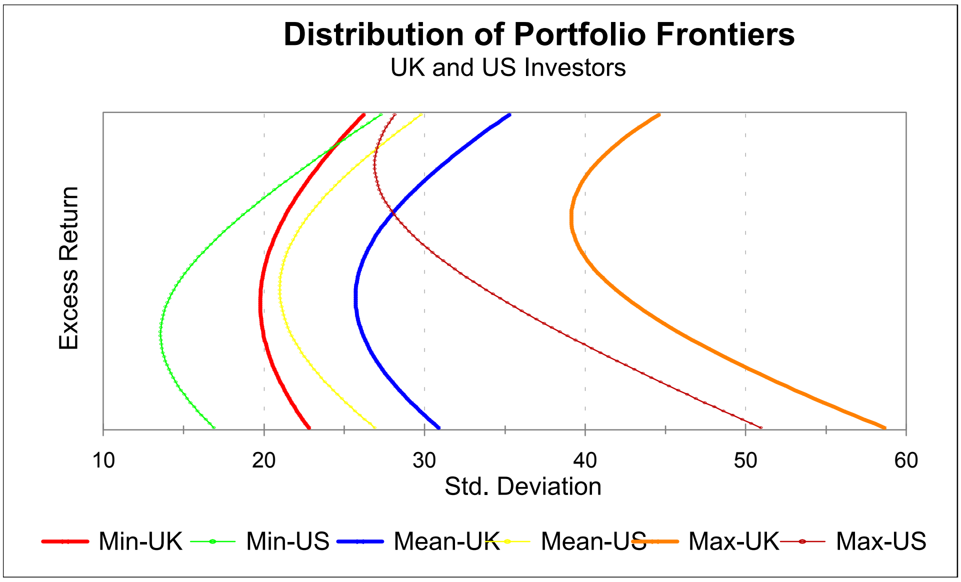

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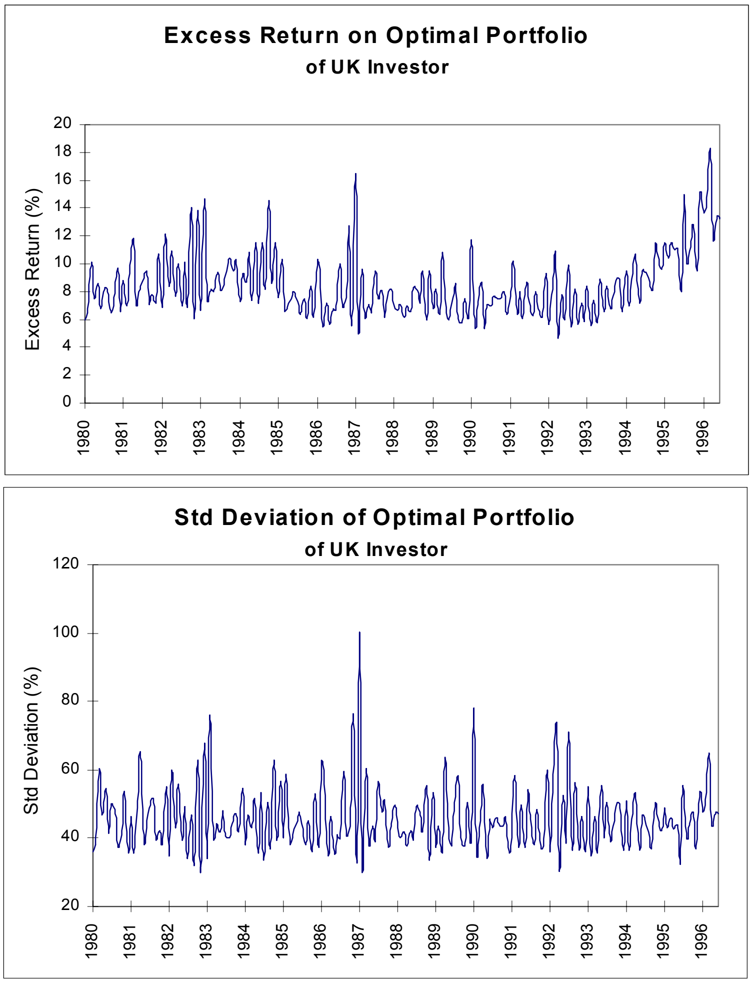

Excess Return on Optimal Portfolio
of UK Investor
Excess Return (%)
20
18
16
14
12
10
8
6
4
2
0
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
Std Deviation of Optimal Portfolio
of UK Investor
Std Deviation (%)
120
100
80
60
40
20
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996

Figure 5:

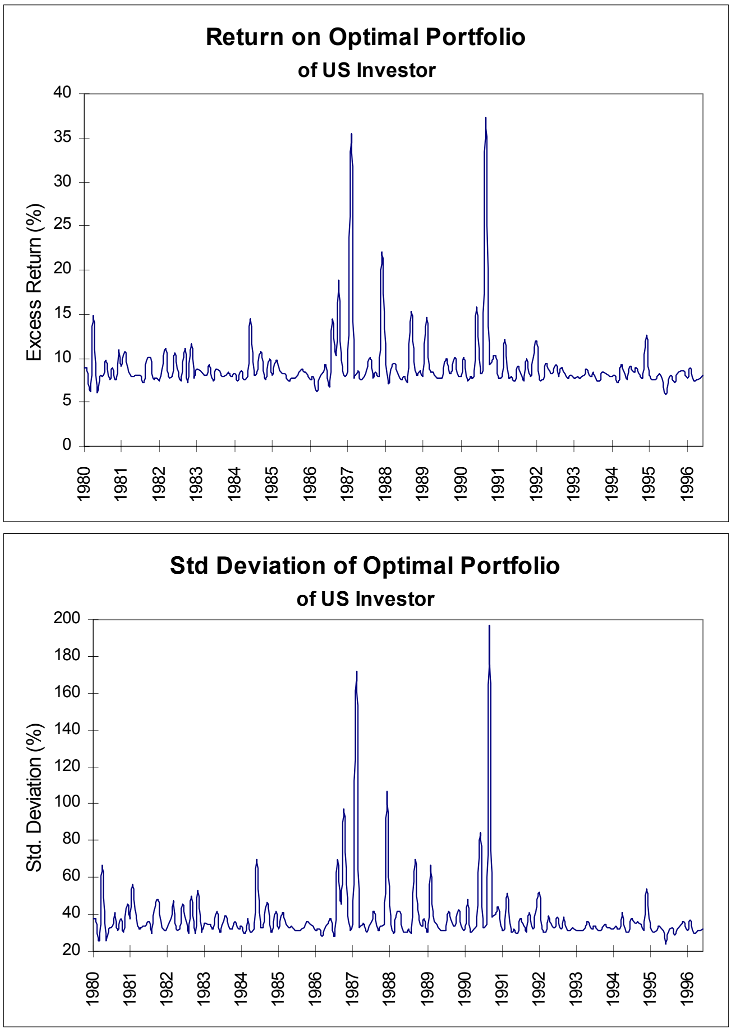
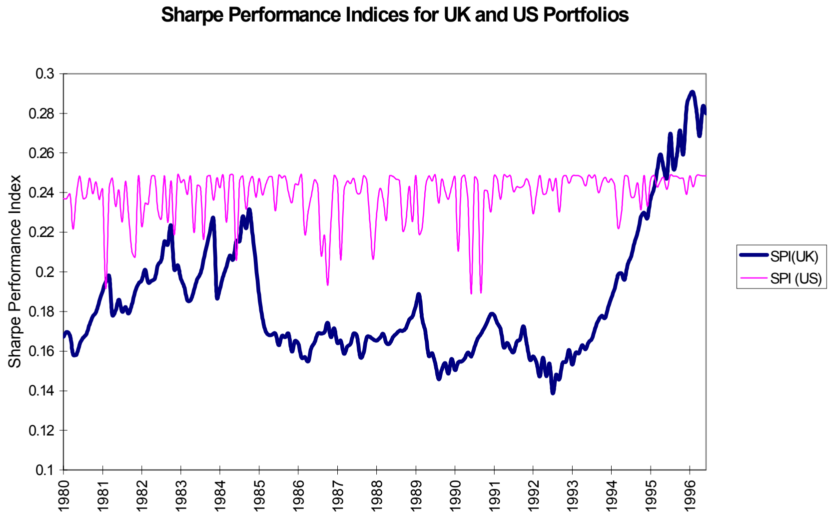

Return on Optimal Portfolio
of US Investor
Excess Return (%)
40
35
30
25
20
15
10
5
0
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
Std Deviation of Optimal Portfolio
of US Investor
Std. Deviation (%)
200
180
160
140
120
100
80
60
40
20
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996

Figure 6:



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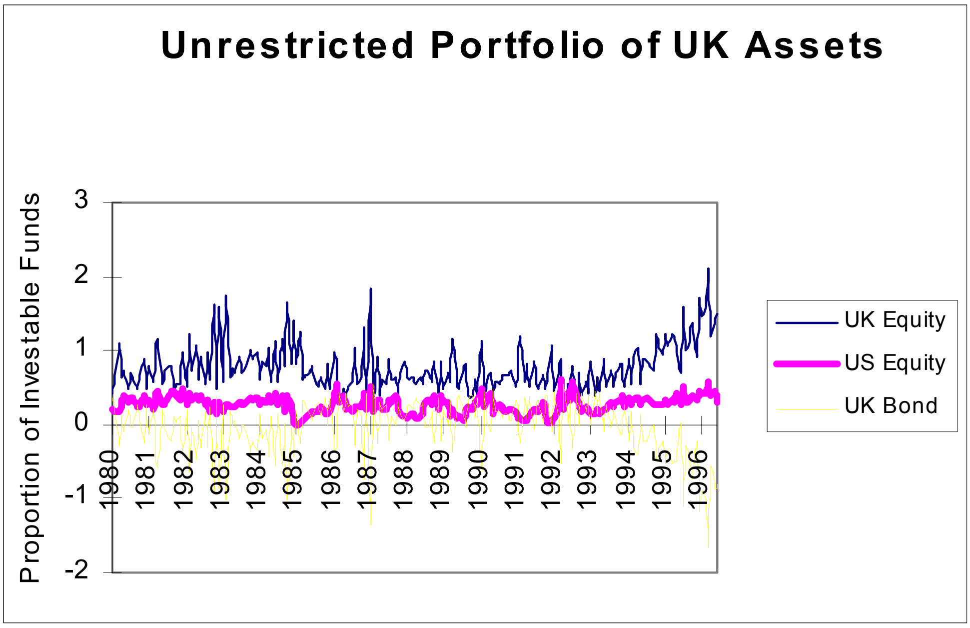

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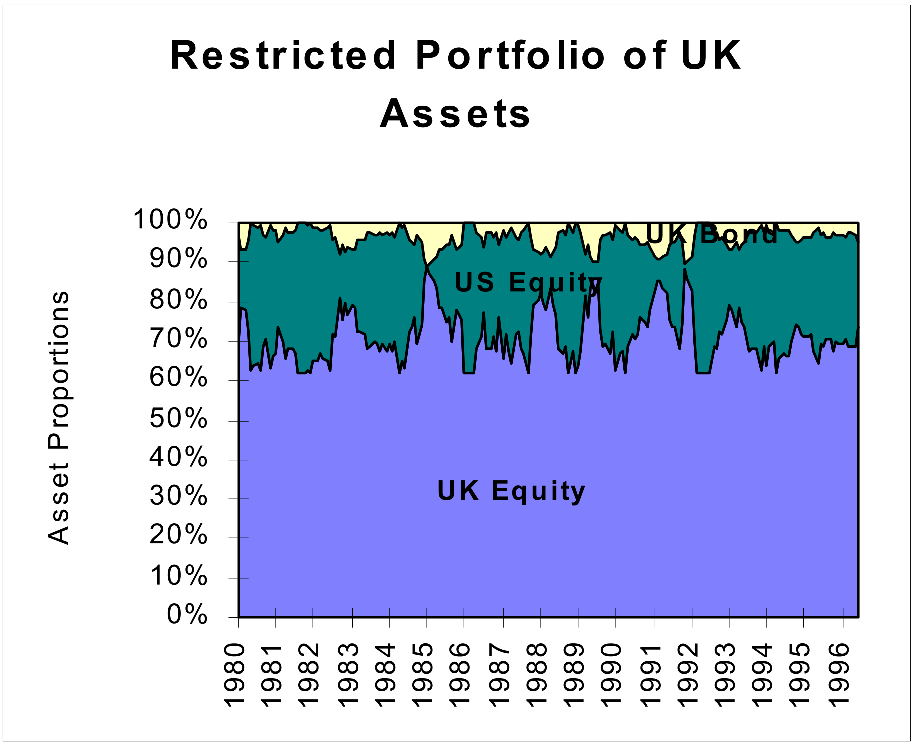
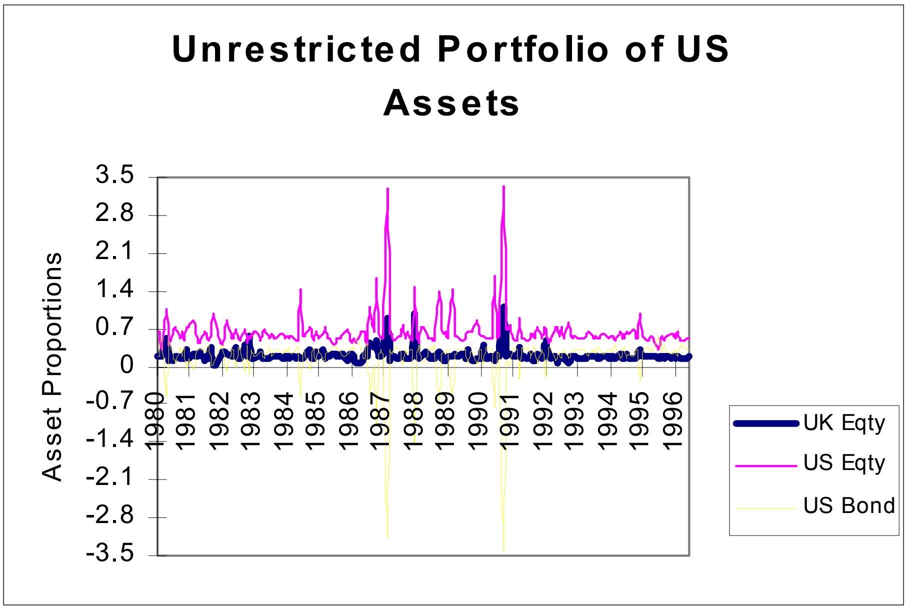

Restricted Portfolio of UK Assets
Asset Proportions
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
UK Bond
US Equity
UK Equity
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996

Figure 9:



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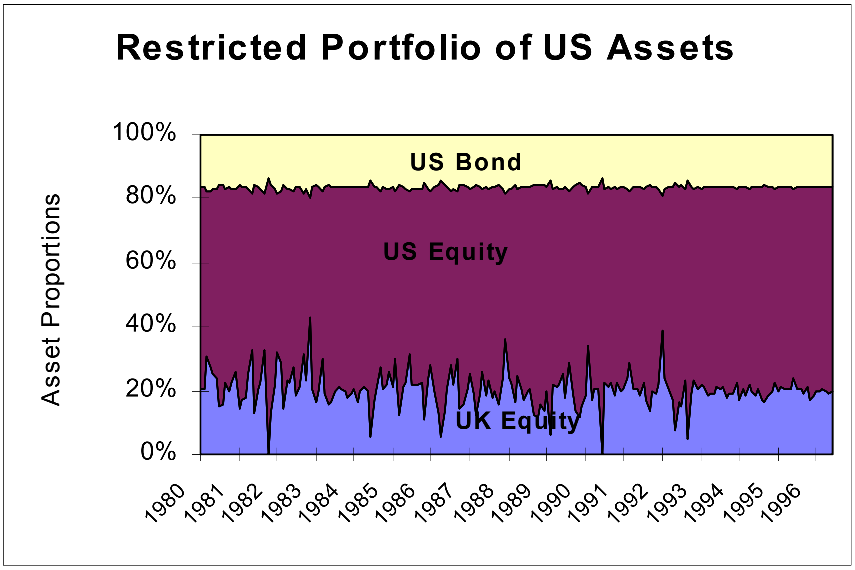

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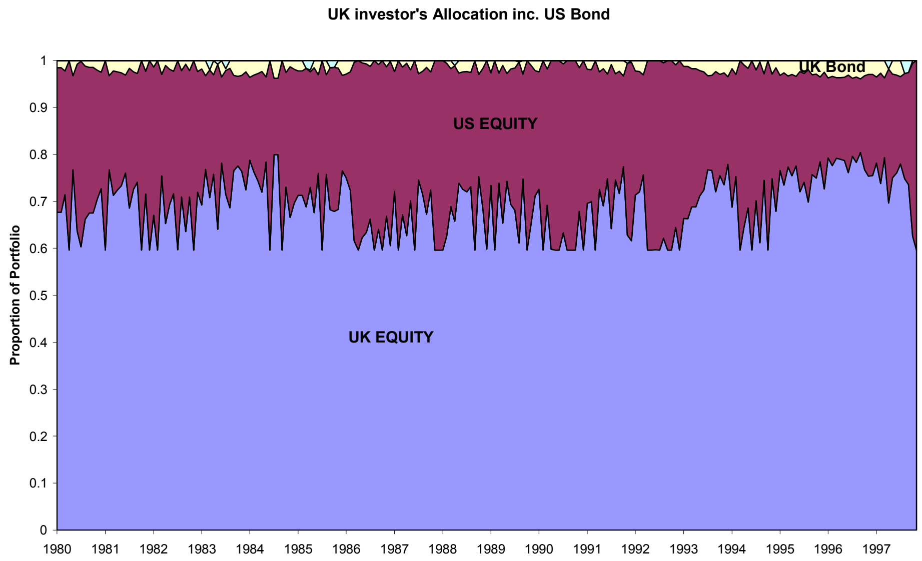

Figure 12:

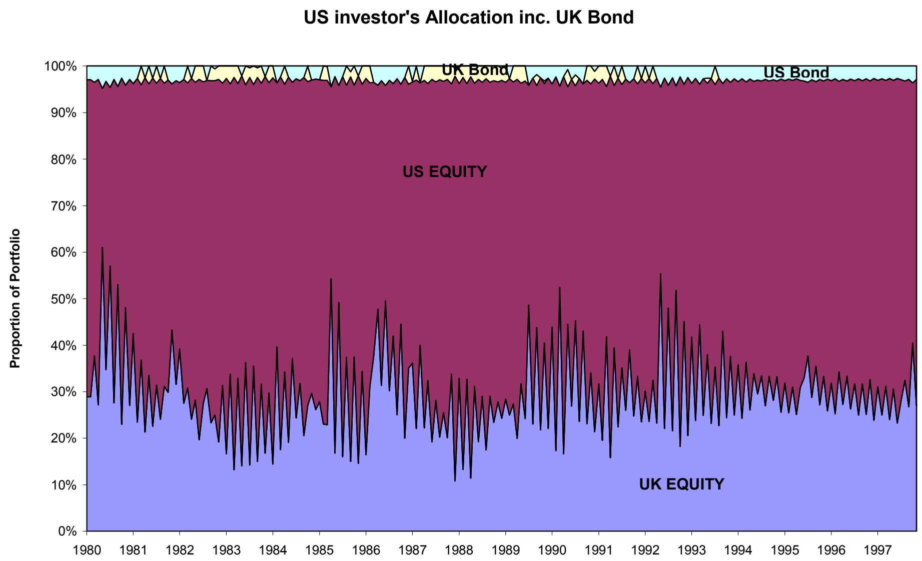

Figure 13: