SIGNALLING WITH DEBT MATURITY CHOICE

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Abstract

This paper, presents a game theoretic approach to the choice of the debt maturity by ¯rms. The maturity of the debt can be viewed as a signal about the ¯rm's quality sent to the ¯nancial sector. Two situations are investigated when the ¯rm declares bankruptcy: the ¯rm's assets may have zero or positive value.

In the ¯rst scenario, it is shown that under positive reputational loss concerns from the part of the ¯rms, we can achieve a separating equilibrium where the good quality ¯rm issues short maturity for its debt whereas the bad quality ¯rm issues long maturity.

In the second scenario, again the same type of separating equilibria occur. However, some equilibria do not require a costly signal to get separation of the two types.

Keywords: Signalling; Debt Maturity Choice; Short Term Debt; Long Term Debt

JEL Classi¯cation: G32; G33

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1 Introduction

A bank, providing funds to di®erent ¯rms requiring capital, faces an adverse selection problem given that it cannot observe the ¯rms' quality. The ¯rms' quality is represented by their default probability, where bad ¯rms have a higher probability to default. Consequently, the lender applies a uniform rule to price the ¯rms' debt, proposing a unique funding rate (demanded interest rate) that is computed considering the average level of quality in the economy. This funding rate, compared to the one paid in the full information case, turns out to be too high for good ¯rms and too low for bad ¯rms. On one hand, this leads good ¯rms to consider some projects with positive net present value too costly so that they do not undertake them. On the other hand, bad ¯rms have incentives to undertake projects that result expensive for the bank because of the high probability of default these ¯rms induce. This result is similar to the market for lemons in Akerlof [1]. Hence, good ¯rms are willing to signal themselves to obtain the possibility of paying a lower default premium (that is funding rate times capital) and to undertake projects with positive net present value. Many papers focused on the Signalling aspect of the corporate debt ¯nancing decision in order to explain the ¯rms' capital structure. Basically two areas emerge: One considering endogenous debt level and the other exogenous debt level. In each of these areas the use of di®erent tools for Signalling purposes is considered.

The ¯rst area is composed of very di®erent models. In many of them, the debt-equity ratio is used as a signal.[1] Their main result states that the ¯rm value and the debt-equity ratio are positively correlated. In other words, the issuance of debt conveys the information of a good quality ¯rm to the ¯nancial market. Indeed, a manager ¯nancing most of the ¯rm's capital with debt, increases the probability of ¯ling for bankruptcy and therefore, having to su®er the costs of bankruptcy such as loss of reputation or quasirents. Debt ¯nancing being costly, ¯rms with higher distribution earnings will adopt higher debt level. Hence, a high level of debt signals the ¯rm's high quality.

The second area, to which our work belongs, also admits di®erent models. Bester [2], for instance, constructs a model where the lender can o®er di®erent loan contracts with variable collateral requirements and a decreasing interest rate with the level of the collateral. He obtains that the low risks put down some collateral and pay a lower interest rate than the high risks who do not put down any collateral. Other papers, as this one, explore the Signalling aspect of the debt maturity choice.

Flannery [3] considers a situation where a ¯rm wants to undertake a project lasting for two periods. In each period, the project can either increase or decrease in value. It increases with some positive probability that depends upon the ¯rm's quality. The ¯rm's quality is private information to the ¯rm. Besides, he considers

[1] See for instance Ross [10] and Poitevin [9].

positive transaction costs assuming that when a ¯rm issues short term debt it has to pay twice the cost of issuing long term debt. The existence of a Signalling equilibrium where good ¯rms issue short term debt and bad ¯rms issue long term debt is shown to depend upon the distribution of ¯rms' quality and the magnitude of underwriting costs for corporate debt. As a matter of fact, higher transaction costs make the short term debt contract less attractive for the bad ¯rm and lead the good ¯rm only to choose this contract.
Kale and Noe [5] having the same basic model, without any transaction costs, impose a positive correlation in the good ¯rm value changes which takes the following form. If the project increases in value in the ¯rst period, the probability of getting a high result in the subsequent period is higher than the initial one. Whereas if it decreases in value in the ¯rst period, the probability of getting a high result in the second period is lower than the bad ¯rm's probability of getting a high result. The good ¯rm is then de¯ned as the ¯rm having the project with the highest initial net present value. They show the existence of separating equilibria in which, again, good ¯rms issue short term debt and bad ¯rms issue long term. These equilibria are shown to depend upon the long term default probability of both types of ¯rms. Given the positive correlation in the good ¯rm value changes, the separating equilibrium exists if the bad ¯rm's long term default probability is larger than the good ¯rm's long term default probability. The intuition is the following. Given the positive correlation in the good type's cash °ows, the bad type does not want to mimic the good type's decision since in case of a low ¯rst period realization, the short term debt is priced on the good type's higher default risk leading the bad type to su®er mispricing losses from mimicking. On the other hand, the good type does not mimic the bad type because the default premium on the long term debt is based on the bad type's default probability which induces the good type to su®er mispricing losses.

The model we consider is as follows. A ¯rm wants to undertake a sequence of two projects where each project requires an initial investment composed by a ¯xed and a variable cost. Each project lasts for a period. To ¯nance them, the ¯rm has the choice between two di®erent contracts: A short term debt (STD) contract lasting for a period and a long term debt (LTD) contract lasting for the two periods. The two debt contracts di®er in their maturity and in two other features as well. As in Flannery [3] and Kale and Noe [5], the STD contract allows the release of some information at the intermediate date. Besides, it allows possible ¯nancial exchanges between the bank and the ¯rm at the intermediate date. If intermediate bankruptcy occurs, the ¯rm incurs some reputational losses linked to the search of a new source of ¯nance. The possibility of intermediate ¯nancial exchanges is not present in both Flannery [3] and Kale and Noe [5] as the projects returns take place at the end of period 2 only. A precise debt composition for a project makes possible the consideration of a liquidation value for the ¯rm. This liquidation value gives the value of the ¯rm when it ¯les for

bankruptcy if ¯nanced with a STD contract. According to intuition this liquidation value is linked to the value of the ¯rm's physical assets. In our model this value is represented by the ¯xed cost. The liquidation value may either be zero, i.e. the physical assets depreciated completely or be positive but lower than the ¯xed cost, i.e. the physical assets depreciated partially only. The consideration of a precise debt composition is not present in Flannery [3] and Kale and Noe [5]. Because of those two main di®erences, we ¯nd the existence of separating equilibria with the good ¯rm issuing STD and the bad ¯rm issuing LTD where the previous authors do not have any. It is the case when the two types of ¯rms are not very di®erent in terms of quality. The existence of separating equilibria do not require the two types of ¯rms to be very di®erent. In the two papers cited it is a necessary condition. When the two types of ¯rms are not very di®erent the long term rate is much smaller than the short term one. The long term funding rate is computed using the bad ¯rm's long term non default probability whereas the short term one is computed using the good ¯rm's short term non default probability. As a consequence, the STD contract is seen as more expensive. The introduction of a positive liquidation value gives a secure revenue for the bank when the ¯rm ¯les for bankruptcy. It plays the role of a collateral. This reduces the short term funding rate. As this reduction is greater for the good ¯rm than for the bad ¯rm both issuing STD, a separating equilibrium may emerge. We show that some separating equilibria do not necessarily imply the use of a costly signal by the good ¯rm. Some separating equilibria exist even if the reputational loss is zero. This di®ers from Flannery [3] who assumes a costly signal. Kale and Noe [5] do not assume a costly signal. However the \learning process", i.e. the correlation in the ¯rm value changes, concerns the good ¯rm only.
Besides the type of separating equilibria described above, we do get separating equilibria where a positive reputational loss is necessary. This is the case when, for instance, the liquidation value is equal to zero. Since the bad ¯rm has a higher probability (by de¯nition) than its good counterpart to incur this loss, it helps to separate the two types of ¯rms.

Our work is organized in four sections. We present the model and the basic assumptions in section 2. In section 3, we derive the bank's optimal strategies for the full information setting. In section 4, considering no liquidation value and non-negative reputational loss, we give the necessary and su±cient conditions for the existence of separating equilibria. A necessary condition is that ¯rms must be su±ciently di®erent in quality. We then introduce, in section 5, a non-negative liquidation value leading to the existence of separating equilibria even when the ¯rms are not too di®erent in quality terms. A conclusion summarizes our results and presents more comments. Finally, unless provided in the text, proofs are gathered in the Appendix.

2 The Model

Consider a two-period Signalling model in which a bank and a ¯rm interact. Assume they are both risk neutral. Let the bank be a representative bank from the ¯nancial sector. We assume that the market for corporate debt is competitive. With no loss of generality we set the interest rate equal to zero. Consider that the ¯rm possesses a real investment opportunity that has a positive present value. This investment opportunity is represented by a sequence of one-period projects. We assume that the outcomes of the two projects are iid random variables. The project's cash in°ows follow a binomial distribution. They can be high ($X > 0$) or low (0). The probability that the project is successful (i.e. of getting X) depends upon the ¯rm's quality or type (q). To simplify, assume that the ¯rm's type can either be good ($q = G$) or bad ($q = B$). One may think of the type as re°ecting the ¯rm manager's ability to deal with a project. Let p_q denote the probability of getting a high result for a type-q ¯rm, we have

$$0 < p_B < p_G < 1; \tag{1}$$

which simply means that a good ¯rm is more likely to get X.

The project's cash in°ows for a type-q ¯rm are given by

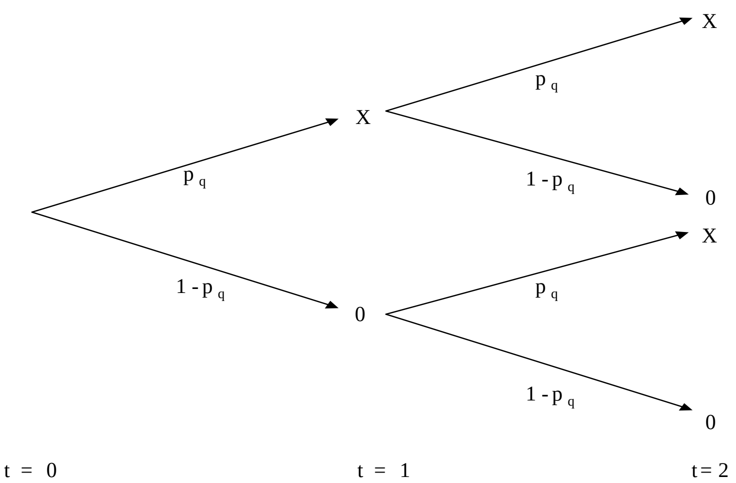

Figure 1: Project returns for a type-q ¯rm.

The ¯rst period project requires an initial investment of $(F + v)$ where F $(F > 0)$ and v $(v > 0)$ stand for the ¯xed and variable cost respectively. We define the fixed cost as the machines whereas wages, rents are variable cost.[2] If bankruptcy does not occur at $t = 1$, the settled down infrastructure can be used in the second period. In that case, the firm has to incur the variable cost only. If bankruptcy occurs, the cost of the second project is identical to the first period one.

The timing and strategies for both the bank and the firm are the following. At $t = 0$, once Nature determined its type the firm chooses a maturity (C^0) and a level for its debt (D^0). The maturity can either be long (LTD) or short (STD).[3] The level of debt specified in the contract is at least equal to $(F + v)$. For each maturity choice the bank proposes a funding rate ($R(C^0)$). The firm may accept or refuse the contract. Once the firm accepts the contract, it completes the first project. The second project starts then. To finance the second project, the firm asks for an additional loan (D^1) (necessarily STD) when one of the two following cases occurs. Either the firm issued a LTD contract at $t = 0$ with a level of debt insufficient to cover the second period project. Or it issued a STD contract and the first project failed (result of zero). In the latter case, the firm files for bankruptcy. Limited liability limits its financial losses to its assets.[4] However bankruptcy induces a reputational loss (Y). This loss can be assimilated to the cost of finding a new source of finance for the entrepreneur. When the first period project is successful, we assume for simplicity that the firm has enough cash to cover the second period project's cost. Formally we assume

$$\frac{F + v(1 + p_B)}{p_B} < X. \qquad (2)$$

If bankruptcy occurs at $t = 2$, it does not lead to a reputational loss as the game ends.

The information structure is as follows. The distribution of the project's cash inflows is common knowledge and verifiable. The firm's quality is private information to the entrepreneur. Before setting the funding rates, the bank observes both the firm's maturity and the level of the firm's debt.

We now define the expected payoff function of the firm. It depends upon the firm's type (q^*), upon the maturity (C) that leads to a certain funding rate and upon the level of debt (D) demanded. Let $V_{q^*}(C^0; D^0; D^1)$ denote a quality q^*-firm's gross expected payoff when financed with a debt contract C^0 at $t = 0$ with a debt level D^0 for date 0 and D^1 if it resorts on the financial sector at $t = 1$. Let

[2]Our definition of fixed costs is different from the one given by Glazer [4]. According to him long term debt is described as fixed costs. In our model fixed costs are not linked to the type of debt contract itself but to the production process.

[3]The LTD contract lasts for the two periods whereas the STD lasts for a period only.

[4]Because of limited liability all debt contracts are Standard Debt Contracts. Standard Debt Contracts are defined as contracts generally promising a fixed repayment but corrected by a limited liability rule for the borrower: the reimbursement cannot exceed the result of the investment (return of the project).

$U_{q^*}(S;D^0;D^1) = V_{q^*}(S;D^0;D^1) - (1-p_{q^*})Y$ be the short term expected payoff net of the reputational loss if the firm's quality is q^*.

The main idea is to show that firms can use the maturity choice of their corporate debt (C^0) as a signal of their quality in this model. We also want to analyze how the ability to signal the quality is affected by introducing a precise debt composition. The bank observes this signal and uses it to compute the funding rates. The first period funding rate is denoted by $R^0_q(C^0;D^0)$ with $R^0_q(C^0;D^0) \geq 1$ where q stands for the bank's beliefs concerning the firm's quality. Let $R^1_q(C^0;D^0;D^1) \geq 1$ be the second period funding rate. We point out that in the following analysis only the short term funding rates are indexed by the date at which they are provided. We solve this model for a Perfect Bayesian Nash equilibrium. We now turn to the resolution of this model.

3 Full Information Setting

We start by computing the default premium set by the bank when it knows the firm's quality. Indeed a separating equilibrium corresponds to a situation where the bank correctly anticipates the firm's quality from its signal. The bank's expected payoff has to be equal to zero given that there is perfect competition in the financial market.

3.1 Long Term Debt Contract

When a type-q firm chooses to finance a level of debt $D \leq X(2-p_q)p_q$ with a long term contract, the long term funding rate, $R_q(L;D)$, is such that[2]

$$(2-p_q)p_q D R_q(L;D) = D. \quad (3)$$

Thus the long term funding rate is given by

$$R_q(L;D) = \frac{1}{(2-p_q)p_q}. \quad (4)$$

The right hand side of expression (3) represents the amount of the loan whereas the left hand side represents the expected gains of the bank. The bank gets the reimbursement of the loan with probability $(2-p_q)p_q$. This probability represents the long term non default probability.

Now let us consider the short term debt contract.

[2] When the firm chooses $(2-p_q)p_q X < D \leq 2Xp_q$, the funding rate $R_q(L;D)$ paid is such that

$$p_q[p_q D R_q(L;D) + 2(1-p_q)X] = D.$$

If the firm chooses an amount higher than $2Xp_q$, the bank will never lend as it will never recover the losses. Moreover it can be shown by plugging these interest rates values into the expressions of the payoff functions that a firm of quality q is better off choosing a level of debt smaller than $X(2-p_q)p_q$.

3.2 Short Term Debt Contract

Proceeding in the same way as for the long term debt contract and for a level of debt $D^0 \leq Xp_q$, the first period funding rate for a type-q firm is given by

$$R_q^0(S, D^0) = \frac{1}{p_q}. \tag{5}$$

In setting this funding rate, the bank uses the fact that the firm has zero value in the default states.
We now study the second period. Given condition (2), when the firm has previously obtained a high result, it internally finances the second project. Given the verifiability of the project's first period result, if the firm were demanding funds to the bank it would pay a funding rate of 1. The firm is then indifferent between borrowing the variable cost or internally financing it. We, then, assume that it chooses to internally finance the second period variable cost. After a low first period result and for a level of debt D^1 such that $D^1 \leq Xp_q$, the bank sets the second period funding rate such that

$$R_q^1(D^1) = \frac{1}{p_q}. \tag{6}$$

Because the first and the second period projects are identical, funding rates for the two periods have the same form.
Obviously all funding rates are decreasing in p_q, meaning that the good firm in the symmetric information case pays lower funding rates.

The firm, when it chooses a short term structure, does not need to commit itself to roll over its debt. Indeed, as some financial exchanges take place between the firm and the bank at $t = 1$, the first short term debt contract is concluded at the end of the first period. It is ended either by the repayment of the loan or by bankruptcy, in which case the bank has no way to recover its loss.

4 Asymmetric Information Setting

In this section, we assume that the bank does not know the type of the firm it faces. We are going to investigate the conditions under which there exists a Signalling equilibrium where, at $t = 0$, the good firm issues STD while the bad firm issues LTD.

When there is asymmetry of information, the bank demands a unique funding rate. The good firm, finding it too high wishes to signal itself by selecting the short term debt contract. This debt contract is more expensive (in expected terms) because of the reputational loss. As the bad firm has a higher probability to incur the reputational loss, the bank believes that the good quality firm only can select it. We then look for Signalling equilibria of the following form.

In order to finance its sequence of projects, the good firm, at $t = 0$, issues STD and may borrow again D^1 in the second period if bankruptcy occurs at $t = 1$. For the same purpose the bad firm, at $t = 0$, chooses to issue LTD and depending upon the amount financed with the LTD contract, it may borrow an extra amount with short term maturity. The bank, when observing the selected contract, believes that the firm's type is good whenever the term of the contract is short at $t = 0$ and its subsequent debt is priced as such, and that it is bad whenever the chosen term at $t = 0$ is long and its subsequent debt is priced as such. As a consequence, the bank's strategies are defined as in the full information setting with the incorporation of the beliefs above. When $C^0 = L$, the demanded funding rate is $R_B(L; D^0) = \frac{1}{(2-p_B)p_B}$ with $D^0 \leq X(2-p_B)p_B$ and, if the same firm needs to borrow again, the demanded funding rate is $R^1_B(L; D^0; D^1) = \frac{1}{p_B}$. If the chosen contract is $C^0 = S$, the first period funding rate is $R^0_G(S; D^0) = \frac{1}{p_G}$ with $D^0 \leq Xp_B$. If the first period result is X, the second period funding rate is 1 and the firm internally finances the project, whereas if the previous result is 0, the second period funding rate is equal to the first period funding rate $R^1_G(S; D^0; D^1) = R^0_G(S; D^0) = R_G(S) = \frac{1}{p_G}$ with $D^1 \leq Xp_B$.

We now establish the following intermediate results.

Lemma 1 *For the two types of firms issuing STD at $t = 0$ and given the bank's beliefs, financing $F + v$ dominates financing any amount of debt D such that $F + v < D \leq F + 2v$.*

Proof. Before starting the proof of this lemma, we give the general expression of the q^*-type firm's expected payoff when issuing STD. If the firm chooses, at $t = 0$, a STD contract with a debt level D^0 such that $F + v \leq D^0 \leq Xp_B$ knowing that if it goes bankrupt at $t = 1$ it needs to borrow $F + v \leq D^1$, we get

$$U_{q^*}\left(S; D^0; D^1\right) = V_{q^*}\left(S; D^0; D^1\right) - (1 - p_{q^*})Y; \tag{7}$$

with $V_{q^*}(S; D^0; D^1) = p_{q^*}\left[2X - D^0R^0_q(S) - v - (1-p_{q^*})D^1R^1_q(S)\right]$.
We now go to the proof of the lemma.
In the second period, the firm never chooses a debt level higher than $F + v$ as issuing debt is costly and the firm only requires $F + v$ in the case of intermediate bankruptcy. It is now straightforward to prove the lemma. Calculate the payoff functions incorporating the bank's beliefs (good firm issuing short term debt) $U_{q^*}(S; F + v; F + v)$ and $U_{q^*}(S; D; F + v)$ and see that $F + v < D$ is equivalent to $V_{q^*}(S; D; F + v) < V_{q^*}(S; F + v; F + v)$ for $q^* = B$ and G. $\square$

Given the bank's beliefs, for both types of firms the debt costs the same in period 1 and period 2. Therefore, as issuing debt is costly, a firm never wants to borrow more than necessary. By choosing $F + v$ they avoid paying a funding rate on the second period variable cost which could be internally financed when the first project is successful.

Lemma 2 When issuing LTD at = 0 and given the bank's beliefs:
The bad firm is indifferent between financing $F + v$ and financing any amount D such that $F + v < D \leq F + 2v$,
Whereas the good firm is better off financing $F + v$ rather than financing any amount of debt D such that $F + v < D \leq F + 2v$.

Proof. We first define the general form of the expected payoff of a q^*-type firm issuing LTD. The firm can choose a LTD with a loan D^0, such that $D^0 < F + 2v$. In that case if the firm obtains a low intermediate result it has to borrow again in the second period an amount D^1 in such a way that it can undertake the second project. Its expected payoff is given by

$$V_{q^*}(L; D^0; D^1) = p_{q^*}\Big[2X - (2 - p_{q^*})D^0 R_q(L; D^0) - D^1\left(1 + (1 - p_{q^*})R^1_q(L; D^0; D^1)\right)\Big]: \tag{8}$$

If the firm chooses to finance $D^0 = F + 2v$ with a LTD, replace D^1 by zero in expression (8). We now prove the lemma.
Given that issuing debt is costly, firms will never choose to borrow more than $F + 2v$ for the first period and more than v for the second period. It is direct to prove the lemma by writing the expressions, given the bank's beliefs, of the payoff functions $V_{q^*}(L; F + v; v)$ and $V_{q^*}(L; D^0; D^1)$ for a q^*-quality firm such that $D^0 + D^1 = F + 2v$. For the bad firm ($q^* = B$), the two payoffs are equal (given the bank's beliefs). While, for the good firm, as $p_B < p_G$, we have $V_G(L; D^0; D^1) < V_G(L; F + v; v)$ (again given the bank's beliefs). □

Given the bank's beliefs, it is the case that the issuance of debt is more expensive in the second period than in the first period ($\frac{1}{b} < \frac{1}{p_B}$).[3] The bad firm when issuing, at $t = 0$, a level of debt $F + 2v$ on one hand, benefits from a better funding rate on the variable cost for the second period, but on the other hand may not need it if it obtains a high result in the first period. However, because the two strategies are fairly priced for the bad firm (given the bank's beliefs), the size of those two opposite effects makes that the bad firm is indifferent between the two strategies. In the subsequent analysis, we then omit the level of debt when writing the interest rate value. We use $R_B(L)$ as the long term interest rate. If the firm ever needs to resort to the financial sector after having chosen a LTD, the interest rate demanded is written as $R^1_B(L)$. We now consider the good firm. The good firm is not indifferent between those two strategies. Given the bank's beliefs when the good firm issues LTD at $t = 0$, it is priced as a bad firm. Consequently, the good firm's debt is over valued when issuing LTD. However

[3] b denotes the bad firm's long term non-default probability. In other words we have that $b = (2 - p_B)p_B$. In the remaining analysis b is used instead of $(2 - p_B)p_B$.

the disadvantage of choosing $F + 2v$ is higher than when choosing $F + v$. Indeed as before the ¯rm can avoid paying any funding rate when the ¯rst project is successful. This compensates the higher funding rate that has to be paid if it borrows v in the second period.

Those two lemmas simplify the analysis of the separating equilibria we are looking for. They restrict the number of pro¯table deviations available to each type of ¯rms. We need to look at the following deviations only: ¯nancing $F + v$ with a LTD for the good ¯rm and ¯nancing $F + v$ with a STD for the bad ¯rm.

Considering this, we now establish the existence of a separating equilibrium. Let $A_q = \frac{V_q(S; F+v; F+v) - V_q(L; F+v; v)}{1 - p_q}$, we get the following result.

Proposition 1 There exist Signalling equilibria in the market for corporate debt where, at $t = 0$, the good ¯rm issues STD for a level of debt $F + v$ while the bad ¯rm issues LTD for a level of debt $F + v$ if and only if the reputational loss is such that

$$0 < A_B \leq Y \leq A_G. \tag{9}$$

Proof. See Appendix. In the Appendix a more general proof is provided that is with non-negative liquidation value. The proof of this proposition can go through by setting $l = 0$.

Corollary 1 Condition (9) de¯nes a non-empty set for the reputational loss if the long term and short term default premia are such that

$$-\frac{1}{1+(1-p_G)(1-p_B)} v(1-p_G)(1-p_B) R_B^1(L) \leq (F+v)(R_B(L) - R_G(S)). \tag{10}$$

Proof. Condition (9) de¯nes a non-empty set for the reputational loss when the upper bound is greater than the lower bound. Doing so, it is straightforward to get condition (10).¥

The interpretation of condition (9) on the reputation is the following. The loss of reputation in case of bankruptcy has to be su±ciently high to lead the bad ¯rm to select the long term contract (the bad ¯rm is not able to bear this loss). In addition, it has also to be su±ciently small to lead the good ¯rm to select the ¯rst period short term contract (the good ¯rm is able to bear the loss without switching to the long term contract). The expected loss due to the reputational loss is compensated by a better funding rate.

This reputational loss can be compared to the transaction costs in Flannery [3]. In our case the reputational loss is not incurred with certainty. Since the expected cost linked to the reputational loss is lower for a good ¯rm than for a bad ¯rm ($p_G > p_B$), the two types of ¯rms separate.

It is stated in proposition 1 that the measure of the reputational loss has to be strictly positive. This means that to have the speci¯ed separating equilibria

$0 < Y$ is a necessary condition. When $Y = 0$, the good firm would like to signal its type but the "tool" it has is not sufficient. Indeed, in that case, when a firm files for bankruptcy at $t = 0$ its payoff is zero. Because of limited liability, the bad firm does not suffer any loss from going bankrupt. The bad firm has then a profitable deviation if issuing STD. As a consequence a separating equilibrium fails to exist.

Now let us consider condition (10). This condition for intermediate values of p_G, i.e. $\overline{p}_G < p_G < b$, determines a relationship between the long and the short term default premia.[4] Whenever condition (10) is verified, the expected cost of the two projects financed with a LTD is higher than the one financed with a STD. This gross expected cost does not incorporate the reputational loss. However, as $p_G < b$, the short term funding rate is higher than the long term one. In order that condition (10) is verified, a low level of fixed cost is required. The long and short term expected costs are respectively given by

$$(2 - p_q)\, p_q\, (F + v)\, R_B(L) + p_q v\, (1 + (1 - p_q)\, R_B^1(L))\,;$$
$$(2 - p_q)\, p_q\, (F + v)\, R_G(S) + p_q v.$$

Holding the bank's beliefs constant, both firms find a LTD too expensive. Then a STD contract is more attractive to finance their projects. This behavior can be corrected by introducing a positive reputational loss when bankruptcy occurs. Because this reputational loss affects more the bad firm than the good firm ($1 - p_G < 1 - p_B$), the two types of firms are selecting a different maturity term. If condition (10) is not verified the two expected costs are either very close or the expected cost of financing the two projects with a STD is larger than financing them with a LTD. In both cases, the introduction of a reputational loss leads the two types of firms to select the same contract. The existence of separating equilibria fails. This can be due to very close up probabilities for both types of firms ($p_G < \overline{p}_G$). The long term funding rate is then much smaller than the short term one. Consequently, the reputational loss must be set sufficiently small such that the good firm selects the short term contract. However, this in turn leads the bad firm to mimic the good firm by selecting the same maturity term. The other way round if the reputational loss is set in such a way that the bad firm selects the long term contract (Y is set at a high value). In that case the good firm mimics the bad firm.
It can easily be seen that when $b < p_G$, condition (10) is trivially satisfied as $R_G(S) < R_B(L)$. This leads to the fact that it is cheaper for both types of firms to finance their debt with a STD.

From condition (9), we can state a result concerning the behavior, in equilibrium, of F and Y.

[4] The existence of $\overline{p}_G$ is relegated to the Appendix.

Proposition 2 Y and F can be seen as being strategic complements if and only if $b < p_G$, whereas they can be seen as being strategic substitutes if and only if $\bar{p}_G < p_G < b$.

Proof. The proof is direct when rewriting A_G and A_B in terms of F and v. Doing so we get

$$A_q = \frac{p_q}{1 - p_q}\left[(2 - p_q)\left(\frac{F + v}{b} - \frac{F + v}{p_G}\right) + (1 - p_q)\, vR_B^1(L)\right]: \qquad (11)$$

It can be seen that if $\bar{p}_G < p_G < b$, an increase of F implies a decrease of both A_B and A_G that may decrease Y to get the separating equilibrium.
The other way round if $b < p_G$, an increase of F implies an increase of both A_B and A_G that may increase Y in order to have the separating equilibrium. This ends the necessary part. The sufficient part is also straightforward. This ends the proof of the claim given in proposition 2.■

This proposition analyses the behavior of F and Y for a separating equilibrium. From proposition 1, we know that the reputational loss must be positive. Proposition 2 tells us that even though $0 < Y$, some room is left (for $\bar{p}_G < p_G < b$) in order to set the equilibrium level of Y and F.

The reputational loss and the fixed cost can have a similar or an opposite effect on the firms' incentives to behave as specified in proposition 1. The influence of the reputational loss is independent of the range of p_G. It always decreases the expected payoff of financing the two projects with a STD contract from a non-financial point of view. As Y increases, the payoff of issuing STD decreases. This makes issuing LTD relatively more attractive for both types of firms. An increase of the fixed cost always decreases the firm's profit margin. Nevertheless, the magnitude of this decrease depends upon the difference in quality between the two types of firms. As a matter of fact, their choice is determined by the relative position of the short term non default probability with respect to the long term one. This is now discussed.
For all the subsequent analysis, we consider a parameter configuration where proposition 1 holds initially. We then make F move.
When $b < p_G$, the short term funding rate is lower than the long term one. As a consequence, the bad firm's debt is under valued with a STD contract. Besides, an increase of the fixed cost, F, has a higher impact for a LTD contract than for a STD contract. Therefore, if Y is held constant and we start from a situation where a separating equilibrium exists, an increase of F increases the incentives to deviate for a bad firm and eventually leads it to issue STD if F increases too much. An increase of the loss of reputation is then required to avoid this deviation.
When $\bar{p}_G < p_G < b$, the opposite effect takes place. The long term funding rate

is lower than the short term one. The good ¯rm's debt is then over valued with a STD. In that case, an increase of the ¯xed cost has a larger e®ect for a STD contract than for a LTD contract. For a ¯xed Y, an increase of F increases the incentives to deviate for the good ¯rm. If F increases too much the good ¯rm ends up issuing LTD. A decrease of the reputational loss is needed to overcome the e®ect of increasing F.

Obviously when the short term and the long term non-default probabilities coincide, the two funding rates are identical. Then an increase of the ¯xed cost has no in°uence on the ¯rm's incentives to deviate as the reduction of the pro¯t margin is identical when issuing STD or LTD.

We now turn to the case where the liquidation is non-negative: When bankruptcy occurs the ¯rm has a positive value.

5 Non-Negative Liquidation Value

The assumption relative to the ¯xed cost being sunk cost assets is now relaxed. From now on, when a ¯rm ¯les for bankruptcy at $t = 1$, its remaining assets have a non-negative value so that it has a non-negative ¯rst period liquidation value. This ¯rst period liquidation value, denoted by l, is assumed to be common knowledge and exogenously given such that its maximum value is F. Then the liquidation value can enter the speci¯cations of the contract. The liquidation value concerns the physical assets of the ¯rm only. As a consequence, it does not embody neither human assets (skill,...) nor the brand value of the ¯rm. Because we assume this de¯nition for the liquidation value, the restriction on the maximum value of l seems realistic. Indeed the physical assets of the ¯rm cannot have a value greater than F which was their initial value. Incorporating human assets in the de¯nition of the liquidation value would complicate the analysis without adding any insights to the problem analyzed here.[5] For simplicity, it is assumed that the ¯xed cost depreciates totally at the end of period 2 so that the liquidation value is zero et the end of period 2. Assuming a positive liquidation value at the end of period 2 would complicate the analysis without changing the results and their intuition. All the rest of the model remains the same.

A non-negative ¯rst period liquidation value in°uences the bank's revenue in case of bankruptcy only. It plays the role of a collateral as it gives a secure revenue for the bank when the ¯rm ¯les for bankruptcy. However the ¯rm does not choose its level nor does it a®ect it directly through the production process. This liquidation value enters the computations of the short term ¯rst period

[5] Incorporating human assets into the liquidation value could lead to a liquidation higher than the ¯xed cost. As the project starts, human asset could increase in value through a learning by doing process for instance. However, when bankruptcy is declared the bank liquidates the ¯rm and therefore derives pro¯t from the sale of the ¯rm's physical assets.

default premium , $D\overline{R}^0_q(S)$, in the following way (see the full information case)

$$p_q D\overline{R}^0_q(S) + (1-p_q)l - D = 0. \tag{12}$$

Due to competition in the market for corporate debt, the bank's expected payoff is zero. In the default states occurring with probability $1-p_q$ the bank receives l. The default premium is then given by

$$D\overline{R}^0_q(S) = \frac{D-(1-p_q)l}{p_q} = DR^0_q(S) - \frac{1-p_q}{p_q}l. \tag{13}$$

As it gives an additional revenue for the bank in case of bankruptcy, a positive liquidation value leads to a lower first period default premium.
The payoff from choosing a short term debt contract is changed as follows:

$$\overline{U}_q\left(S;D^0;D^1\right) = \overline{V}_q\left(S;D^0;D^1\right) - (1-p_q)Y, \tag{14}$$

where $\overline{V}_q(S;D^0;D^1) = p_q\left[2X - D^0\overline{R}^0_q(S) - v - (1-p_q)D^1R_q(S)\right]$.

The long term funding rate is not affected by the liquidation value. Thus the firm's payoff from issuing LTD is not affected.

We are now focusing on the separating equilibrium defined in the previous section. We point out that lemma 1 and lemma 2 are still true in this section. Then, the analysis of the separating equilibria is as simple as before. The introduction of a non-negative liquidation value modifies the previous proposition 1 in the following way:

Proposition 3 There exist Signalling equilibria in the market for corporate debt where, at $t = 0$, the good firm issues STD for a level of debt $F+v$ while the bad firm issues LTD for a level of debt $F+v$ if and only if the reputational loss is such that

$$A_B + \frac{p_B(1-p_G)}{p_G(1-p_B)}l \le Y \le A_G + l. \tag{15}$$

Proof. See Appendix.

Corollary 2 Condition (15) defines a non-empty set for the reputational loss if the long and short term default premia are such that

$$-\frac{1-p_G}{1+(1-p_G)(1-p_B)}\left((1-p_B)vR^1_B(L) + \frac{l}{p_G}\right) \le (F+v)\left(R_B(L) - R_G(S)\right). \tag{16}$$

Proof. Condition (15) defines a non-empty set for the reputational loss when the upper bound is greater than the lower bound. Doing so, it is straightforward to get condition (16). ∎

The intuition concerning condition (15) on the reputational loss is identical to the one provided in proposition 1. The liquidation value is now entering this condition. The impact of introducing the liquidation value depends upon the firm's quality and upon the bank's beliefs. The improvement of the funding rate by introducing a positive liquidation value is the same for both types of firms. However the good firm has a higher probability of profiting from this improvement as it has a higher probability of getting a high result. The following discussion illustrates this point. As the bank believes that only the good firm issues STD, a reputational loss of $Y < F$ can exactly be offset by setting a liquidation value $l = Y$, for the good firm issuing STD. However this is not true for the bad firm issuing STD. In that case, in order to compensate the same loss of reputation, the liquidation value has to be set at $Y < \frac{p_G(1-p_B)}{p_B(1-p_G)}Y = l$ (with $\frac{p_G(1-p_B)}{p_B(1-p_G)}Y < F$). The introduction of the liquidation value is more beneficial to the good firm than to the bad one. As a consequence, this positive liquidation value provides a useful additional tool to separate the good firm from the bad one. By setting a suitable liquidation value, this enables us to get the existence of separating equilibria where before it was not possible (firms very close in quality or a too high fixed cost compared to the variable cost). From the comparison of conditions (10) and (16), one can see that condition (10) is now relaxed. The introduction of a non-negative liquidation value decreases the expected cost of the two projects when financed with a STD. Indeed, the first period funding rate decreases. This induces that even if condition (10) is not satisfied, separating equilibria may exist.

The separation of the two types of firms is now possible even if they are very close in quality (p_G is close to p_B). The closer is p_G to p_B, the larger the difference between the short and the long term funding rates (when $l = 0$). Given the bank's beliefs both types of firms find the LTD contract more attractive. The introduction of a positive liquidation value decreases the difference between those two funding rates. Since the impact of an increase of l is greater for the good firm issuing STD than for the bad firm issuing the same term, there exists a way to separate both types of firms even if they are very close in quality.

The following proposition gives the different types of separating equilibria occurring.

Proposition 4 There exist non-empty sets of parameters such that the following types of separating equilibria occur:

type 1: A positive reputational loss is required whereas the liquidation value may be zero,

type 2: Both a positive reputational loss and a positive liquidation value are required,

type 3: A positive liquidation value is required whereas the reputational loss may be zero.

Proof. See Appendix.

All the following discussion is made holding the bank's beliefs identical to the ones de¯ned previously.

The ¯rst type of equilibria exists for even zero liquidation value:

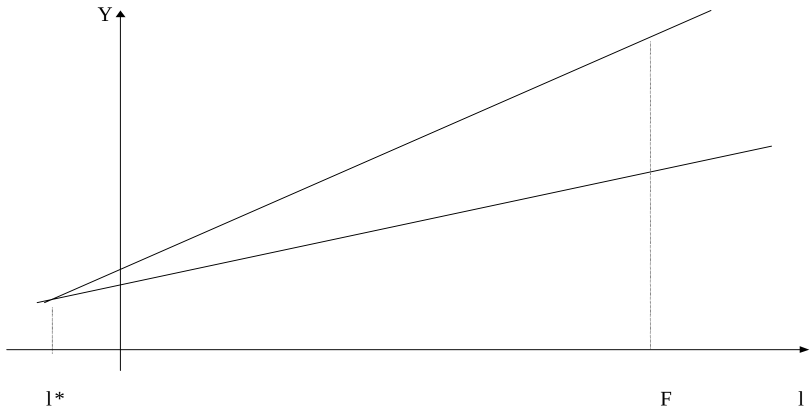

Figure 2: Type 1 equilibria.

In this ¯gure, the line with the higher slope corresponds to the upper bound given in condition (15) whereas the line with the lower slope corresponds to the lower bound given in condition (15). Then the level of reputational loss must stand in between the two straight lines to get the separating equilibria described in proposition 3. This ¯gure corresponds to the case where $0 < A_B < A_G$. Equilibria for which $l = 0$ are the equilibria found in proposition 1. Since $0 < A_B < A_G$, if $Y = 0$ and $l = 0$ both types of ¯rms want to issue STD. The introduction of a positive liquidation value with $Y = 0$ exacerbates this problem as it decreases the cost of issuing STD. A positive reputational loss in case of bankruptcy is then required to separate them. The interpretation of this case has already been discussed in the previous section when analyzing proposition 3.

The second and the third type of equilibria are now analyzed. It should be pointed out that they correspond to situations where in Flannery [3] separating equilibria do not exist. Indeed, in those situations, a positive reputational loss alone leads the good ¯rm to deviate and to issue LTD. This is the case when the decrease in the funding rate is smaller than the cost of Signalling the good ¯rm's type. In that situations, the good ¯rm never signals its type as the tool is costly. However it should not be forgotten that the good ¯rm pays a funding rate that is higher than the one paid in the symmetric information case. As a consequence, it is willing to signal its type. The only way it could do it, is by using a non-costly signal. This is achieved by the introduction of a non-negative liquidation value. A non-negative liquidation value reduces the cost of the STD contract in such a way that this reduction is greater for the good firm than for the bad one. We now look more specifically at each type of equilibria.

The second type of equilibria occurs for intermediate values of the ratio $\frac{F}{v}$ and $p_G < b$.[6]

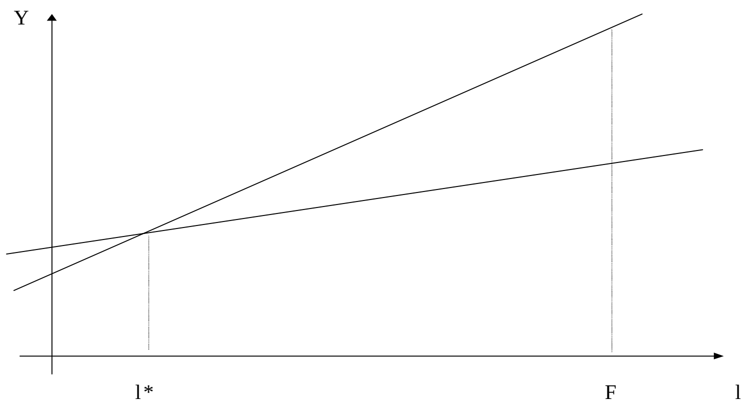

Figure 3: Type 2 equilibria.

As in the preceding figure the two straight lines represent the upper and lower bounds given in condition (15). In the present case as $p_G < b$, the long term funding rate is smaller than the short term one. In order to have the specified separating equilibria, the reputational loss must stand in between the two straight lines and be greater than l^*. The conditions on the default premia are more complicated. The upper and lower bounds for $\frac{F}{v}$ are respectively given by the following conditions

$$(F + v) R_G (S) < (F + v) R_B (L) + vR_B^1 (L); \tag{17}$$

$$(F + v) R_B (L) + \frac{1}{1+(1-p_G)(1-p_B)} v (1 - p_G)(1 - p_B) R_B^1 (L) < (F + v) R_G (S): \tag{18}$$

We point out that the lower bound corresponds to condition (10) being violated. This type of equilibrium is new and was not occurring in the first part of this work. In that situation, if $Y = 0$ and $l = 0$, the two types of firms may select the STD or the LTD depending upon the value of the expected cost of the two projects financed with a STD. It can be the case that this expected cost is high (low) compared to the one when issuing LTD that the two types of firms issue LTD (STD). The parameter configuration is such that we need both a positive

[6] See the proof of proposition 4 in the Appendix for the derivation of those conditions.

reputational loss and a positive liquidation value to separate the two types of firms. Indeed, the use of one of these tools only implies that the two types of firms issue either STD if $l > 0$ and $Y = 0$, or LTD if $Y > 0$ and $l = 0$. The use of both tools enables us on one hand to increase the loss from issuing STD ($Y > 0$) and on the other hand to decrease it with positive liquidation value. This can be done in such a way that the two types of firms separate themselves.

The last type of equilibria are situations that can be depicted as follows:

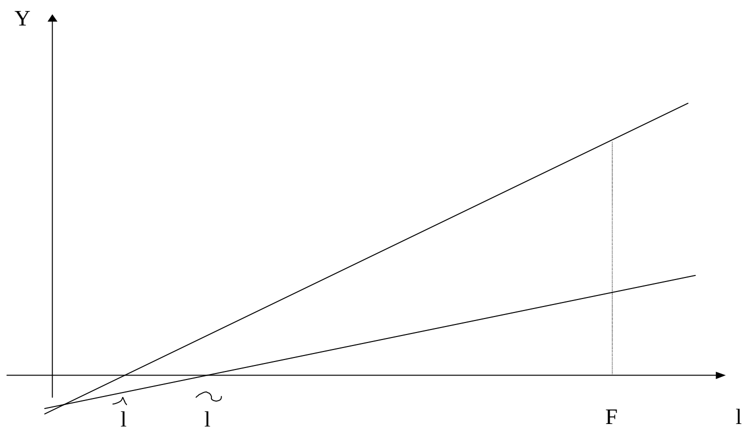

Figure 4: Type 3 equilibria.

This case corresponds to the case where $A_G < A_B < 0$. Again, in order to have the separating equilibria, Y must stand in between the two straight lines (given by condition (15)) and be non-negative. This type of equilibria occurs whenever $p_G < b$ and condition (17) is violated.[7] In that case when $Y = 0$ and $l = 0$ the expected cost of the two projects when financed with a LTD is smaller than the one financed with a STD. Thus the two types of firms prefer the LTD to the STD whenever both Y and l are zero. The introduction of a positive reputational loss with $l = 0$ exacerbates the problem as it increases the cost of issuing STD. Consequently, a positive liquidation value is necessary to separate the two types of firms. If the level of the liquidation value is too high both firms issue STD. By setting a suitable reputational loss we correct this behavior and the good firm separates from the bad firm according to the behavior described in the proposition.

This particular case deserves more comments. It can be seen from figure (4) that separating equilibria without reputational loss exist. This is true when the

[7] See the proof of proposition 4 for the derivation of those conditions.

liquidation value belongs to the interval $I = \left[\underline{l}, \overline{l}\right]$. This separating equilibrium crucially depends upon the ability for the good ¯rm to provide a positive revenue, belonging to I, to the bank when intermediate bankruptcy is declared. The reputational loss is not necessary for situations where the two types of ¯rms are similar enough in quality and when the cost of ¯nancing the two projects with a STD is higher than with a LTD. Meaning that the two types of ¯rms would issue LTD when both l and Y are zero.

6 Conclusion

In this paper, we investigate the e®ects of separating ¯xed and variable costs, as well as concern for reputation and bankruptcy with limited liability, on the existence of Signalling-separating equilibria, in a game of debt maturity choice by ¯rms of di®erent quality. Firms are also assumed to choose their debt level. We ¯nd that, if the concern for reputation is su±ciently high, the existence of such equilibria is guaranteed for ¯rms di®erent enough in terms of quality. It means that the ¯rms must su®er losses when bankruptcy occurs. The ¯rms that are not protected by limited liability for those losses, do care about them. Therefore the bad ¯rm does not take the risk to bear them since they occur with a high probability. From this proposition, we see that the concern for reputation and the ¯xed costs are not acting in the same way. This result is established in our second proposition.

Allowing for non negative intermediate liquidation value gives the possibility to the bank to lower the ¯rst period funding rate in such a way that the good ¯rm chooses the short term debt and the bad ¯rm the long term debt. We can achieve a separating equilibrium even when both types of ¯rms are very similar in terms of quality. A particular equilibrium is shown to exist. Whenever the cost of ¯nancing the two projects with a STD is higher than with a LTD, an equilibrium without reputational loss exists. In that case both types of ¯rms would issue LTD with $Y = 0$ and $l = 0$. The introduction of a non-negative liquidation value, by decreasing the ¯rst period funding rate decreases the expected cost of the two projects ¯nanced with a STD. Thus there exists an interval for values of liquidation value such that the two types of ¯rms are separated. As most of the existing literature admits a costly signal, this result is in contrast with their result. A non costly signal is provided to the good ¯rms to signal themselves. As the good ¯rm has a higher probability to bene¯t from the decrease of this funding rate, we can separate them.

In our general setting (non-negative liquidation value), it is always possible to separate both types of ¯rms when the undertaken project requires high ¯xed costs. Whereas, when the project presents small ¯xed costs relatively to the variable costs, it is not always possible to separate them (it is not possible when the ¯rms are very similar). This means that, it is easier for the good ¯rm to signal itself by its debt maturity choice in industries requiring relatively high ¯xed costs as compared to the variable ones. It can be interpreted in the following way. The good ¯rm, by de¯nition, has a higher probability to support the repayment of the loan which is high. Hence, by taking a sequence of short term contracts, the ¯rm sends a costly signal (reduction in the pro¯t margin and loss of reputation). This signal leads the bank to believe that only the good ¯rm has sent it.

A natural step further, for this work, would be to consider the following modi¯ed model. We could introduce the possibility for each ¯rm to choose a technology in a previous stage to the debt maturity choice. Each technology would be characterized by the level of ¯xed cost. If the choice of this technology can be observed by the bank and as the level of this ¯xed cost determines the liquidation value, the technology could also be used as a signal. Then, it is interesting to determine whether this would help in separating the two ¯rms and whether it increases the set of parameters leading to Signalling equilibria. This will be studied in future research.

One could test the robustness of our results to the reduction of the ¯rm's margin when increasing the value of the ¯xed cost. This could be done by keeping constant $F + v$, and therefore when we would move up the ¯xed cost we would move down the variable cost by the same amount.

Appendix

Proof of proposition 3

In step 1, we prove that conditions (15) and (16) are necessary conditions for the existence of the separating equilibrium. In step 2, we prove that those two conditions are sufficient for the existence of the separating equilibrium.

Step 1: Necessary part.

Given the bank's beliefs, both types of firms behave as specified in the separating equilibrium if the good (bad) firm maximizes its expected profit issuing STD (LTD). This is given by the two incentive constraints

$$p_G\left[2X - (2-p_G)(F+v)R_B(L) - v\left(1+(1-p_G)R_B^1(L)\right)\right] \leq p_G\left[2X-(2-p_G)(F+v)R_G(S)-v\right]-(1-p_G)Y; \tag{19}$$

$$p_B\left[2X-(2-p_B)(F+v)R_G(S)-v\right]-(1-p_B)Y \leq p_B\left[2X-(2-p_B)(F+v)R_B(L)-v\left(1+(1-p_B)R_B^1(L)\right)\right]. \tag{20}$$

Condition (19) gives the good firm's incentive constraint whereas condition (20) gives the bad firm's incentive constraint. Those two constraints can be rewritten as follows

$$(1-p_G)Y \leq (2-p_G)p_G(F+v)(R_B(L)-R_G(S))+(1-p_G)\left(p_G v R_B^1(L)+I\right); \tag{21}$$

$$b(F+v)(R_B(L)-R_G(S))+p_B(1-p_B)\left(vR_B^1(L)+\frac{I}{p_G}\right) \leq (1-p_B)Y. \tag{22}$$

From condition (21) we obtain an upper bound for Y while from condition (22) we obtain a lower bound. Putting those two bounds in one condition, we find condition (15).
We must verify that condition (15) does not define an empty set. It can be proved that condition (16) is equivalent to the fact that the upper bound is greater than the lower bound. This ends the proof of the necessary part.

Step 2: Sufficient part.

Conditions (15) and (16) are now satisfied. The sufficient part is straightforward when fixing the banks' beliefs as before. Indeed when conditions (15) and (16) are satisfied, firms have no incentive to deviate from the behavior depicted by the banks' beliefs. This ends the sufficient part of proposition 3. The proof of proposition 3 is now finished.

We now look more precisely at the proof of proposition 1.
Setting $I = 0$ in the above proof leads to the necessary and sufficient conditions

(9) and (10). We still have to prove that $A_B > 0$ for the separating equilibrium. By definition we have $0 \le Y$. From condition (10) we have that $Y \le A_G$. This leads to $0 \le A_G$. This is equivalent to

$$-\frac{v}{(2 - p_G)}(1 - p_G) R_B^1(L) \le (F + v)(R_B(L) - R_G(S)): \tag{23}$$

Now let us consider A_B. It can be rewritten as

$$A_B = \frac{p_B}{1 - p_B}((2 - p_B)(F + v)(R_B(L) - R_G(S))) + v p_B R_B^1(L): \tag{24}$$

Applying (23) and using that $R_B^1(L) = \frac{1}{p_B}$ on condition (24), we get that

$$\frac{v}{(1 - p_B)(2 - p_G)}[p_G - p_B] \le A_B: \tag{25}$$

As $p_B < p_G$, the term in bracket in the R.H.S. of (25) is strictly positive. This proves that $0 < A_B$. This ends the proof of proposition 1.□

Existence of $\bar{p}_G$

The existence of the lower bound $\bar{p}_G$ is got by rewriting condition (10) in the following way

$$v\frac{P}{(1 + (1 - p_G)(1 - p_B))} \le F(p_G - b); \tag{26}$$

with

$$P = p_G^2(3 - p_B)(1 - p_B) - p_G(2 - p_B)\left(2 - p_B^2\right) + b(2 - p_B): \tag{27}$$

It can be checked that this polynomial admits two real roots by computing the discriminant of this polynomial. The discriminant is strictly positive. This proves the existence of two different real roots. It can be checked that the smallest one, $\bar{p}_G$, is greater than p_B and smaller than b. One can also check that the largest root is greater than 1.

Given the above, condition (26) defines an empty set for p_G strictly smaller than $\bar{p}_G$. Indeed in this case the polynomial is positive whereas the expression multiplied by $F + v$ is negative. For $p_G \in [\bar{p}_G; b)$, expression (26) gives us a condition on F and v. Finally, for $p_G \ge b$, condition (26) is always verified.

Proof of proposition 4

We need to prove the existence of type 2 and type 3 equilibria only. Indeed we already proved the existence of type 1 when proving proposition 1.

Before starting the proof, we define l^* as the liquidation value such that the upper and lower bounds in (15) are equal. We also define $\hat{l}$ and $\tilde{l}$ as the liquidation values such that the upper and lower bounds are respectively equal to zero.

We start with a result used to prove the existence of type 2 and type 3 equilibria.

For convenience, we define the following condition

$$(F+v)R_B(L)+vR_B(L)<(F+v)R_G(S). \tag{28}$$

Lemma 3 Whenever condition (28) is verified, we have $0<\hat{l}<\tilde{l}<F$ and if this condition is not satisfied we have $\tilde{l}<\hat{l}<l^*$.

Proof. We prove it by taking their difference and show that it has the appropriate sign under the conditions of the lemma.
Let us begin when condition (28) is satisfied.
The positive sign of $\hat{l}$ is equivalent to prove that $A_G<0$. This is straightforward when using condition (28).
We now compute the difference of $\tilde{l}$ and $\hat{l}$. Their difference is given by

$$\tilde{l}-\hat{l}=\frac{p_G(1-p_G)}{1-p_G}\left[(F+v)R_G(S)-(F+v)R_B(L)-vR_B(L)\right]. \tag{29}$$

This difference is positive given condition (28).
We now compute the following difference

$$F-\tilde{l}=\frac{p_G-p_B}{p_B(1-p_G)}\left[F(1-p_B)+v(2-p_B)\right]>0.$$

This ends the proof of the first point of the lemma.
When condition (28) is not satisfied it is direct to prove that expression (29) is negative proving that $\tilde{l}<\hat{l}$. Now, calculating the difference $l^*-\hat{l}$, we get

$$l^*-\hat{l}=\frac{p_G}{1-p_G}\left((F+v)R_B(L)+vR_B(L)-(F+v)R_G(S)\right).$$

This proves the last claim of this lemma. ∎

Type 3 equilibria:

Using the first point of lemma 3, the fact that both the upper and lower bounds are increasing in l and the slope of the upper bound is higher than the lower bound's one, we get the existence of type 3 equilibria.

Type 2 equilibria:

First, we point out that $l^*<0$ is equivalent to condition (10) being satisfied. Therefore when condition (10) is violated l^* is positive. We already defined $\overline{p}_G$ as being the smallest root of the polynomial P, where the form of P is given by expression (27).

Lemma 4 If $\overline{p}_G < p_G < b$, we have $l^* < F$.

Proof. Let us calculate the difference $F - l^*$, we get

$$vP < F(1 - p_B)\left(-p_G^2 + p_G(2 - p_B)(1 + p_B) - b\right). \qquad (30)$$

It can be checked that over the relevant interval for p_G the polynomial multiplied by F is positive. Moreover for $\overline{p}_G < p_G$ P is negative. This proves lemma 4. ∎

It can be checked that condition (28) and condition (10) being both violated do not define an empty set. Taking the second point of lemma 3, lemma 4, and the fact that the slope of the upper bound is greater than the lower bound one, we get that if conditions (28) and (10) are violated and $\overline{p}_G < p_G < b$ we have the existence of type 2 equilibria. This ends the proof of proposition 4. ∎

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