BLUFFING: AN EQUILIBRIUM STRATEGY

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Abstract

The present work studies the behavior of a monopolistic informed trader in a two-period competitive dealer market.

We show that the informed trader may engage in stock price manipulation as a result of the exploitation of his informational advantage (su±cient conditions are provided). The informed trader achieves this manipulation by not trading in the ¯rst period according to the information received. This trader attempts to jam his signal or to blu®. In equilibrium this behavior is anticipated by the market maker, but still the informed continues to blu® with a positive probability. Equilibria with blu±ng behavior are mixed strategies equilibria where the informed both follows and jams his information with positive probabilities. We also show that under those su±cient conditions, a pure strategy equilibrium where the informed does not blu® does not exist.

Keywords: Blu±ng; Trade-Based Manipulation; Dealer Market

JEL Classi¯cation: G14

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1 Introduction

It has long been argued that stock price manipulation through trades of the stock only could not be pro¯table.[1] Indeed, in that case, in order to manipulate the stock price, the trader establishes a trend (or a bubble) with some trades and then trades against this trend. This strategy leads the trader to buy at a high price and sell at a low one. Whereas, to make a pro¯t he should be able to \buy cheap" and \sell dear".

The present work aims at showing that the previous reasoning may fail in some situations as such type of strategy may result pro¯table. More precisely, this work determines the market conditions leading to the use of this strategy in equilibrium. Throughout the paper, this strategy is called blu±ng strategy. The reason will become clear as we de¯ne the model. The analysis is performed in a simple two-period model often used in the market microstructure literature.
The market is organized as a dealership market where the dealer or market maker sets the prices before the traders' order submission. This dealership market operates as a continuous mechanism, i.e., the traders' orders are executed sequentially. The traders can be one of two types: non-myopic with private information of the future asset value or myopic liquidity trader. We assume that there exists a single non-myopic informed trader (henceforth informed trader). They all submit discrete orders to the market maker.[2] The quantity submitted by the liquidity traders is exogenously ¯xed. The market maker when facing a trader does not know the trader's identity. Obviously, the informed trader is the one that might attempt to manipulate the stock price. Indeed, this trader has monopoly in information. Because of that, he might use in equilibrium the blu±ng strategy as a result of a strategic exploitation of his information. In order to establish the trend in the ¯rst period when blu±ng, this trader uses the converse side of the market given by his private information. In the second period, he follows his private information. If the trader received, for instance, low information he purchases in the ¯rst period and then sells in the second period. On one hand, the trader bears some cost in the ¯rst period as he purchases the stock at a price above its conditional expected value. On the other hand, he raises the second period bid price.
Given such a model, it is always the case that the per unit cost of inducing the trend overcomes the second period gain in the price. Indeed, the informed trader discloses part of his private information in the second period. This has a direct

[1]This type of manipulation is called trade-based manipulation.

[2]Two models are used in the analysis. The ¯rst one is based on Glosten and Milgrom [11] where traders submit discrete orders of a unit size. The second model is a slight modi¯cation of Easley and O'Hara [8]. In our model the non-myopic trader has the choice between two sizes for his discrete order (large and small) in the ¯rst period. In the second period this trader is constrained to trade the large size only.

implication: The informed trader never blu®s if he is constrained to trade orders of the same size in both periods.[3] Thus the informed trader should be allowed to choose among di®erent order sizes to overcome the ¯rst period cost. The second model allows him to choose among two order sizes in the ¯rst period. In the second period he trades the large quantity only. Given that model, we show in proposition 5 that the informed trader might ¯nd pro¯table to blu® even though the market maker anticipates it. Proposition 5 illustrates a situation where the informed blu®s if he receives low information whereas he follows his information if it is high. The trader mixes between selling a large quantity and purchasing a small one if his information is low, whereas he mixes between purchasing a large quantity and a small one if his information is high. Three points are important for the use, in equilibrium, of the blu±ng strategy. The ¯rst point deals with how the large quantity compares with the small one. When the two sides of the market (bid and ask sides) are su±ciently wide , i.e. the large quantity is su±ciently large compared with the small one, the informed does not blu® in equilibrium. Indeed the advantage of trading large quantities compensates any better price induced by blu±ng or following the private information with a small quantity. The second point concerns the extent of the informed's private information disclosure when he trades against the trend. If the second period price reacts too much when he follows his private information, blu±ng cannot be pro¯table. Indeed in that case blu±ng does not imply a \so much better" second period price. In other words, the price should exhibit some price momentum when the trader follows his private information in the second period. Finally, the cost of inducing the trend also a®ects the pro¯tability of the blu±ng strategy. An example gives some parameter values for such a situation.

In related literature, Allen and Gale [1] and Allen and Gorton [2] speci¯cally look at trade-based manipulation, whereas Jarrow [13] uses a stochastic dynamic model to study stock price manipulation.[4]

Allen and Gale [1] study trade-based manipulation in a rational expectations framework with ¯nite horizon (3 dates). Three types of agents are present in their model, risk averse investors (continuum) and two risk neutral large traders (informed and the manipulator). By de¯nition, large traders' orders move the price. The traders are able to trade two assets: cash and a risky asset. The informed knows whether an announcement is forthcoming but he is not informed of the future value of the asset. If the announcement concerns bad (good) news, it is made at $t = 2$ ($t = 3$). Prior to $t = 1$, only the investors who hold all the stock are present in the market. At $t = 1$, the informed enters only if he anticipates an announcement. If there is no announcement, the manipulator

[3]This is an illustration of the reasoning previously mentioned concerning the pro¯tability of trade-based manipulation strategies.

[4]Cherian and Jarrow [7] provide a review of the literature on stock price manipulation. Their review is based on the work of Jarrow [13].

may enter. Once a large trader enters the market he stays until the second date. The investors are not able to observe the date 1 trader's identity. As a consequence, they do not know whether the large trader purchase results from share undervaluation or manipulative motives. Manipulation occurs provided investors attach a positive probability to the manipulator being an informed trader. If at t = 2, no announcement is made and a purchase took place, there is still the possibility of receiving good news at t = 3. This pushes up the price at t = 2 and enables the manipulator to unwind his position with positive gains.

Allen and Gorton [2] also study the case of trade-based manipulation by uninformed traders in a four-period model, where each period is based on Glosten and Milgrom [11]. They exhibit an example of pro¯table manipulation when liquidity selling is more likely than liquidity buying or, when the probability of a buyer being informed is di®erent from the one of a seller being informed. The former case can be motivated by the fact that liquidity buyers are more free to time their purchases. The latter case deals with the fact that because of short sale constraint, good news are easier to exploit than bad news. Those two cases imply a di®erent price response for a sale order and for a buy order. This creates pro¯table manipulations, meaning that the uninformed can use strategies leading to positive pro¯t. Once the market maker anticipates this possibility, the uninformed continues to use this strategy with some positive probability and gets zero pro¯t.

Jarrow [13] investigates stock price manipulation by large traders in a stochastic dynamic model of asset markets. Large traders are de¯ned as in Allen and Gale [1]. Jarrow [13] adopts a partial equilibrium analysis where the equilibrium price process properties are exogenously given. He shows that market manipulation strategies including bubbles, corners and short squeezes exist under reasonable hypothesis on the equilibrium price process. For trade-based manipulation, the large trader induces a trend in asset prices with his ¯rst periods trades and then trade against it. Jarrow [13] shows that this strategy may result pro¯table if the exogenously given price process exhibits price momentum. In other words, the prices should not react too much when the large trader trades against the trend and revert back to quickly to their initial level. Example 3, in Jarrow [13], describes such a situation in a three-period model. The price process is built up such that its last period sensitivity to trades is lower than the ¯rst two periods sensitivities. Given this price process, trade-based manipulation may be pro¯table.

Those three papers share the view that trade-based manipulation results from uninformed traders, i.e. from traders who do not have private information concerning the asset future value. Whereas, we do believe that such a behavior may be the result of a strategic exploitation of information. To our knowledge no paper investigates precisely that possibility. Because of that, this paper displays an e®ect not present in the three previous papers. The informed is concerned

with his aggregate position on the asset when blu±ng. Indeed, because of the presence of noise trading the informed pro¯tably exploits his information when following his information. As a consequence, when blu±ng he must end up with an aggregate position (net seller or net buyer after the two period trades) in accordance with his private information. In the three papers analyzing trade-based manipulation, the manipulator derives pro¯t when manipulating even if he ends up with a zero aggregate position on the asset.
Besides, in contrast to Jarrow [13], the price process is derived as part of the equilibrium. Indeed, the price process depends upon the traders' behavior through the dealer's beliefs concerning the information received by the informed trader.

Another paper related to our work is the one by Kyle [14]. In this seminal paper, Kyle [14] studies the dynamic behavior of an informed trader in a batch-clearing market. All (market) orders are submitted to a market maker who sets a price which makes the market clear. In that case the market maker does not observe the size of the individual trades but rather the aggregate order °ow. In this framework, a monopolistic informed trader selects his order size and trading intensity taking explicitly into account the e®ect his trades have on the current and future prices. As a result, the informed continuously trades without attempting to manipulate prices with the blu±ng strategy. Such strategies would lead to unbounded pro¯t and therefore are ruled out in equilibrium. In our framework, as the informed trader trades discrete orders he cannot realize unbounded pro¯t when blu±ng.

Besides trade-based manipulation, the literature on stock price manipulation also looks at two other types of manipulation. The ¯rst type can be described as action-based manipulation. Manipulation is based on actions that change the actual or perceived value of the assets. The second category falls into information-based manipulation, in that case, manipulation is based on the release of false information or the spread of false rumors. Many real examples of such manipulation can be found in Allen and Gale [1]. The U.S. Securities Exchange Act of 1934 outlawed information-based and action-based manipulation.

Vila considers the two types of manipulation previously mentioned whereas Bagnoli and Lipman [3] consider the ¯rst type only. In Vila's [20] model analyzing action-based manipulation, the manipulator pools with a purchaser of the stock before the occurrence of a takeover bid that will increase the value of the ¯rm. In Bagnoli and Lipman [3], a bidder manipulates the ¯rm's stock price through a takeover bid. As his motives (manipulation or real takeover) are private information to the agent, the market price of the ¯rm's stock price increases. This increase generates pro¯t for the manipulator. Moreover, they show that banning manipulation has ambiguous e®ects on welfare.

Three works can be classi¯ed in the second category: Vila [20], Kyle and Vila [15] and Benabou and Laroque [4]. In both Kyle and Vila [15] and Vila [20], the manipulator shorts the stock, releases false information and then buys back at a lower price. Extra payo®s are derived from false information release. Benabou and Laroque [4] consider the manipulator as someone who has privileged information about the future return of the stock. If his statements are credible for the investors, he can bene¯t from manipulation by misleading announcements and tradings.

The second model, analyzed in this chapter, uses the dealership model developed by Easley and O'Hara [8] as a basis. We, now, describe and comment in some details this model. Easley and O'Hara [8] do not assume the existence of new information. In other words, there exists uncertainty as to the presence of new information in the market. They model that in the following way. With some probability a subset of traders receives a signal concerning the future value of a risky asset, while with the converse probability nobody receives a signal. They assume, given a market side (ask side or bid side), that the liquidity traders can be one of two types: large trader or small trader. The informed traders, if there are any, trade any quantity they want. In addition, they behave myopically. All traders trade with a risk neutral dealer who, competitively, sets ask and bid prices before knowing the size and the side of the incoming individual order. As usual, the dealer does not observe the trader's identity he is facing.

The ¯rst feature of the equilibria is that the informed traders choose to trade the quantities exchanged by liquidity traders only, i.e., the informed either submit the small or the large quantities. By doing so, they bene¯t from the \camou°age" provided by liquidity trading. Indeed, if they were trading a di®erent quantity, they would be \recognized" and as a consequence, would pay a price incorporating all their private information.

We now look at the second feature of the equilibria. The equilibria can either have a semi-separating form or a pooling form. The form of the equilibria depends upon both the width of the market de¯ned as the relative largeness of the large quantity with respect to the small quantity and the level of information-based trading. When the market is wide (the large quantity is large enough) or there is a little level of information-based trading (the probability to trade with an informed is small and/or the probability of new information is small) the informed trade, in equilibrium, the large quantity only. The advantage of trading large quantities compensates the better price available for small quantities. As a consequence, the equilibrium is a semi-separating equilibrium. The informed traders are separated from small liquidity traders but not from large liquidity traders. For small orders, the bid-ask spread is equal to zero (no information-based trading), i.e., the ask price is equal to the bid price which are in turn equal to the unconditional expected stock value. For large orders, as there is some possibility of information-based trading, a bid-ask spread emerges. When the market is not wide enough or there is a high level of information-based trading, the equilibrium has a pooling form. In that case, the advantage of trading large quantities does not compensate the better price available for small quantities. The informed

traders mix between trading large and trading small and they are pooled with the liquidity traders for the two quantities. A bid-ask spread emerges for both the large and the small quantities.
In our model, we assume that there is no uncertainty as to the existence of new information. The probability that a subset of traders receives private information is one. We also assume that the informed trader has the choice among the large and the small quantity, traded by the liquidity traders, only. As a consequence of our framework, we also get equilibria with a semi-separating form and pooling form. They refer to the informed trader's behavior when following his private information.

An outline of the paper is as follows. The benchmark and its notation are presented in Section 2 as well as its resolution giving the impossibility to blu® in equilibrium in this framework. Section 3 is divided in two subsections. The ¯rst subsection gives the notation of the two-quantity model considered now. The second subsection provides the results of this model with among them the su±cient conditions for a particular equilibrium with blu±ng. A conclusion ends the analysis. Finally, unless provided in the text proofs are gathered in the Appendix.

2 One-quantity model

This section is used as a benchmark for the analysis of dynamic trading. In particular we assume that traders can trade orders of a unit size of the stock in each period. This framework enables us to pin down some of the basic elements for the use, by the informed, of the blu±ng strategy in equilibrium.

We show in this section that blu±ng is a strictly dominated strategy as it leads to negative expected payo®.

2.1 The model

Consider a two-period model for a ¯nancial market. Each period of this model is based on Glosten and Milgrom [11].

The participants of this market are divided in two groups: the traders and the market maker (or dealer). The market maker is a risk neutral agent and faces perfect competition. Let him be, without any loss of generality, a representative market maker (or dealer). Two types of traders are present: A single risk neutral non-myopic trader and myopic liquidity traders.

In each period, equivalent to a trading round, the traders exchange a single risky asset with the market maker. A trader is assumed to submit a discrete order normalized to a unit of the asset. The non-myopic trader is the only trader able to choose the side of his orders (i.e. whether to purchase or sell). The liquidity traders trade for some exogenous liquidity reasons. Let $X_S > 0$ and $X_B = (1 - X_S) > 0$, be the proportion of liquidity traders who want to sell and purchase one unit of the asset respectively. The market maker sets a price at which he takes the opposite side of the order. He expects to trade with the non-myopic trader with probability μ and with the myopic liquidity traders with the converse probability.

The information structure is as follows. When submitting an order all traders know the execution price. In addition to that, the single non-myopic trader receives some private information concerning the future value of the risky asset. Henceforth this trader will be called informed trader.[5] The future value of the asset is represented by a random variable $\tilde{V}$. The private information takes the form of a signal. This signal can take one of two values, high or low. It is low with probability δ. We define $\underline{V} = E\left[\tilde{V} \mid s = L\right]$ and $\overline{V} = E\left[\tilde{V} \mid s = H\right]$, with $\overline{V} > \underline{V}$ and $V^* = \delta \underline{V} + (1 - \delta)\overline{V}$ the unconditional expected stock value. The informed receives his private information at the beginning of the first period. As the value of the asset is realized at the end of the second period, the informed's private information lasts for the two periods. Finally, both the market maker and the informed observe some public information incorporating past price quotes and past trades. The dealer does not know the trader's identity when facing one, i.e., he does not know whether a trade is information based.

The informed trader, due to the risk neutrality assumption, is an expected wealth maximizer. He maximizes the sum of his first and second period expected wealth conditional on his information.

The trading game unfolds as follows. First, the market maker determines his price quotes. He uses the probabilistic arrival process of the traders to compute them. Second, a trader arrives at the dealer and asks for the competing bid and ask quotes. Considering them, he either does not trade, or takes the best quote if he is uninformed, or takes the profit-maximizing quote if he is informed. Once the first period trade is executed, the market maker revises his beliefs and proposes new price quotes incorporating this new information. A new trader arrives according to the stochastic arrival process that is independent of the period. This trader asks for the market maker's price quotes and then decides whether to trade. Once this trade has taken place, the true value of the stock is realized.

Before going to the characterization of the equilibrium, let us make some general comments on the model. First, the market maker sets his prices on each trade such that the losses made on the informed trader are exactly balanced by the gains made on the liquidity traders. This is obviously due to the competitive assumption. Second, it is assumed that liquidity traders have inelastic demand

[5]Several papers assume a monopolistic informed trader as well, among them, Kyle [14] and Bhattacharya and Spiegel [5].

or supply. This ensures that, even if the asymmetry of information between the market maker and the informed is very high or if the probability to trade with the informed is very high, the market always remains open. No price precludes liquidity trades so that the market maker can always expect to break even on a given trade. This is made by setting worse prices at the expense of the liquidity traders. Hence, the case of the no trade theorem of Milgrom and Stokey [17] never occurs here.[6]

2.2 Characterization of the equilibrium

In this part we prove that blu±ng is a strictly dominated strategy.

In order to solve this model, we use the concept of Perfect Bayesian Nash Equilibrium. An equilibrium is a situation where the market maker correctly anticipates the informed's behavior. He correctly incorporates into his beliefs and therefore into his prices the information received by the informed. Moreover given these prices, the informed trader behaves as the market maker thinks he does.

The following proposition states the impossibility for the informed to blu® in equilibrium.

Proposition 1 *Blu±ng is a strictly dominated strategy.*

Proof. See Appendix.

The aim of a blu±ng strategy is to establish a trend and then to trade against it. If the informed received a low (high) signal, he attempts to increase (decrease) the second period bid (ask) price. This can be realized by deviating from his private information in the ¯rst period.[7] Consider the case where the informed receives a low signal. When this trader blu®s, he ¯rst purchases the asset and then sells it back in the second period. In the ¯rst period, this strategy is costly since he purchases the asset at a price higher than its expected value. However a ¯rst period buy order pushes down the ¯rst period updated beliefs of ± conditional on the buy order. This leads to a higher second period bid price. Even though the informed bene¯ts from better second period prices when blu±ng he will never do it. The ¯rst period per unit cost of inducing the trend overcomes the second period per unit bene¯t. Indeed, as the informed follows his information in the second period, the market maker re¯nes his beliefs toward the \correct" private information. Thus the second period execution price is always lower than the ¯rst period one. If the market maker anticipates this blu±ng behavior, the payo® from

[6] See for instance Madhavan [16] and Glosten [12] for the analysis of market breakdown in ¯nancial markets with strategic traders.

[7] In the Industrial Organization literature this strategy is referred to as \signal jamming " strategy. See Riordan [18] and Fudenberg and Tirole [9] for models developing this idea.

blu±ng decreases even more. In that case two e®ects are at work. The ¯rst e®ect increases the expected payo® from blu±ng and the second one tends to decrease it. On one hand as the ¯rst period prices react less to a speci¯c order the ¯rst period cost is reduced. On the other hand, the informed also bene¯ts from \less" better second period prices. Indeed the market maker puts more weight on the fact that there is a tentative of blu±ng for a sequence of two orders with opposite side. The second e®ect always dominates the ¯rst one. Because he is informed and has the possibility to hide behind the liquidity traders, the informed pro¯tably exploits his information by following it. He then derives a positive expected pro¯t by achieving a position on the asset in accordance with his private information since the prices never completely reveal his private information. At the light of this discussion, larger second period quantities may make possible for the informed to recover his ¯rst period losses. Blu±ng could then be pro¯table and appear in equilibrium. This is considered in the next section.

This proposition may seem to contradict Allen and Gale [1]. However it does not. In their paper, because the manipulator is uninformed, he does not want to achieve a particular position on the risky asset. He wants to sell at $t = 2$ the quantity purchased at $t = 1$ ($B > 0$) with positive pro¯t. This is only possible if he is able to sell it at a price higher than the one he paid. Due to their information structure, an order to sell coupled with no information announcement at $t = 2$ increase the price. When no announcement is made at $t = 2$, there is a nonzero probability to have good information at $t = 3$. Thus the price is increased.

3 Two-quantity model

The informed has now the choice between di®erent quantities in the ¯rst period. We analyze whether this leads him to use the blu±ng strategies in equilibrium. In the ¯rst subsection, we describe the modi¯ed model and in the second subsection we solve this model.

3.1 The model

The modi¯cations made to the benchmark concern the orders submitted by the traders only. All the rest of the framework remains identical to the benchmark (information structure, timing of the game,...).

In the ¯rst period, the informed has the choice between two discrete orders on each side of the market (ask and bid sides).[8] He can sell or purchase either a small quantity ($S^1 > 0$ and $B^1 > 0$ for a small sell or buy order respectively) or a large quantity ($S^2 > 0$ and $B^2 > 0$ for a large sell or buy order respectively)

[8] This framework is related to the one used by Easley and O'Hara [8]. RÄell [19] provides the continuous quantity case of this model.

with $0 < S^1 < S^2$ and $0 < B^1 < B^2$. For convenience, we set $|S^1| = |B^1| = 1$ and $|S^2| = |B^2|$. The structure of the first period noise is modified as follows. Let $X_S^i > 0$ and $X_B^i > 0$, $i = 1; 2$ be the proportion of liquidity traders who want to trade S^i and B^i, $i = 1; 2$. The liquidity traders do not have the choice of their order.

We now define the second period. In order to simplify the analysis, we assume that the informed trader can trade one quantity only. This assumption reduces the number of potential equilibria to investigate as all strategic considerations of the second period are removed. This assumption is made for tractability only. It does not affect how the informed bluffs nor does it affect its intuition. We strongly believe that an equilibrium with bluffing should occur in a two-period game where the informed is allowed to choose among two discrete quantities in each period. The model we are dealing with could be understood as a situation where the original model (two discrete quantities in each period) admits a semi-separating equilibrium in the second period. Therefore, the informed either sells S^2 or purchases B^2. Due to that, we assume that, in the second period, the liquidity traders are also trading large quantities. As in the benchmark X_S (X_B) refers to the proportion of liquidity sellers (buyers), with $X_S + X_B = 1$.

In this framework the informed chooses both the size and the side of his order. This order maximizes the sum of his first and second period expected profit conditional on his private information.

Table (1) gives a summary of the results obtained in this chapter.

3.2 Characterization of the equilibria

In this subsection we characterize part of the different equilibria occurring in the two-quantity model.

As in Easley and O'Hara [8], different forms of equilibrium arise in our framework. In their analysis, the informed traders have no monopoly power in information and as a consequence they never bluff in equilibrium. They always follow their private information in equilibrium. Given a market side two types of equilibrium can arise: a semi-separating or a pooling equilibrium. In a semi-separating equilibrium, the informed traders trade one quantity with probability 1. This quantity is the large one. As a result they are separated from the small liquidity traders only. In a pooling equilibrium, the informed traders trade the two quantities with positive probability and are not separated from the small liquidity traders.

In our case, an equilibrium can be a more complex situation. It is worth pointing out that the informed never bluffs with probability one in equilibrium. As a consequence he always follows his signal with a positive probability (≤ 1).[9] An

[9] If the market maker anticipated that the trader bluffs with probability 1, when bluffing the informed would have some of his private information incorporated into the prices and would

equilibrium must de¯ne the informed's behavior when following his signal (semi-separating or pooling equilibrium as in Easley and O'Hara [8]) and whether he is blu±ng. As we show next, if he is blu±ng he must do it trading a small quantity.

We showed in proposition 1 that the informed does not blu® when he is restricted to the same quantities in both the ¯rst and the second periods. A similar result holds in the two-quantity framework. This result does not depend on the informed's behavior when following his signal. This is stated in the next proposition.

Proposition 2 Blu±ng with a large quantity in the ¯rst period is a strictly dominated strategy.

Proof. Updated beliefs are computed in the same way as in proposition 1, using now the proportion of large liquidity traders. Then using the same argument as in the proof of proposition 1, we get that ¦ $(S^2; S^2 j s = L) > 0$ whereas ¦ $(B^2; S^2 j s = L) < 0$. The same thing can be done for a high signal.¥

The intuition is similar to proposition 1. As a consequence of this proposition, blu±ng with a large quantity will never form part of an equilibrium. Henceforth, when we speak about blu±ng we always refer to the situation in which the informed blu®s with a small quantity. Moreover, unless confusing, when the informed follows his signal we do not specify it. In other words we say that the informed trades the quantity q implicitly meaning that he trades the quantity q when following his signal.

In the remaining of this chapter, we characterize some of the equilibria arising in this model. We do a case by case analysis, starting with the situation where the informed trades the large quantity on the two sides of the market when following his private information. In the following analysis, S stands for semi-separating and P for pooling. They refer to the informed's behavior when following his private information.

3.2.1 The S-S case

In this part we analyze the case where the informed trades the large quantity only, when following his private information, on both the ask and the bid sides (S-S case). We derive the necessary and su±cient conditions for the existence

have to incur the ¯rst period costs. By following his information, none of his private information would be incorporated into the prices and the trader would not have to incur the ¯rst period costs. In summary, second period prices when blu±ng would be worse than the ones when not blu±ng and the informed incurs some cost in the ¯rst period when blu±ng. Clearly, following the private information leads to higher conditional expected payo®. As a consequence blu±ng with probability 1 cannot be an equilibrium. The informed follows his information with a probability less or equal than one. When blu±ng does not take place in equilibrium , the informed follows his information with probability one (proposition 3 shows such a situation).

of a semi-separating equilibrium without bluffing on both the ask and bid sides. We then show that bluffing cannot occur whenever there exists a semi-separating equilibrium, when following the private information, on the two sides of the market.

Let $\delta_1(\delta; x)$ and $\delta_2(\delta; x; y)$ be defined as in the proof of proposition 1. Define $\Delta_{S^1;S^2}$ and $\Delta_{B^1;B^2}$ as follows

$$\Delta_{S^1;S^2} = \delta_2\left(\delta; S^1; S^2\right) - \delta_2\left(\delta; S^2; S^2\right); \tag{1}$$

$$\Delta_{B^1;B^2} = \delta_2\left(\delta; B^1; B^2\right) - \delta_2\left(\delta; B^2; B^2\right). \tag{2}$$

The difference between the second period prices when trading small and large is given by $\Delta_{B^1;B^2}$ and $\Delta_{S^1;S^2}$. Hence, $\Delta_{B^1;B^2}$ and $\Delta_{S^1;S^2}$ give the relative advantage on the second period prices of trading small in the first period.

In the following proposition, we give the necessary and sufficient conditions for the existence of a semi-separating equilibrium without bluffing.

Proposition 3 There exists a unique equilibrium (where the informed takes into account the effect his current trade has on current prices and on future trading opportunities) which is a S-S equilibrium without bluffing if and only if

$$S^2\left[1 - \delta_1\left(\delta; S^2\right) + \Delta_{S^1;S^2}\right] \geq 1 - \delta; \tag{3}$$

$$B^2\left[\delta_1\left(\delta; B^2\right) - \Delta_{B^1;B^2}\right] \geq \delta. \tag{4}$$

Proof. See Appendix.

By definition of a semi-separating equilibrium, small quantities are traded by the liquidity traders only. As a consequence, first period prices for small quantities do not incorporate any information about the future value of the asset. Both the first period ask and bid prices are equal to the unconditional expected value of the asset V^*. The informed when trading a small quantity delays the release of part of his private information to the second period. Whereas when he trades a large quantity, he discloses part of it which is incorporated by the market maker in both the first and the second period prices. Thus the first and the second period quotes for large quantities are worse than the quotes for and after a small quantity. For a wide enough market (B^2 and S^2 sufficiently large) the advantage of the large quantities outweighs the better prices (for both the first and the second periods) available with a first period small trade. This intuition is similar to Easley-O'Hara [8] for their semi-separating equilibrium. In their model, the informed behave competitively (i.e. myopically) and do not take into account the influence of today's trades on tomorrow's trading opportunities. Given a trading round, informed traders trade large quantities only when the size of the large quantity compensates the better prices when trading small. In our model, the informed takes into account the e®ect of current trades on future trading opportunities and therefore behaves strategically.

In the above proposition, the necessary and su±cient conditions (3) and (4) do not incorporate conditions concerning the blu±ng strategies. This can be explained as follows. By de¯nition of the semi-separating equilibrium, small traded quantities do not lead the market maker to revise his beliefs. This implies that the second period prices are independent of the ¯rst period small trade side. If the informed deviates and trades a small quantity blu±ng or not, he obtains the same second period price. However when he blu®s, he must bear a nonzero cost due to his ¯rst period price destabilizing trade. This cost is not present when he trades small.

Then a natural question arises. Can we have an equilibrium with blu±ng when there exists a semi-separating equilibrium for the two sides of the market? We answer this question in the following proposition.

Proposition 4 There cannot exist a S-S equilibrium with blu±ng at least on one side of the market.

Proof. See Appendix.

It has been seen that when the dealer does anticipate blu±ng and when there exists a S-S equilibrium the cost of delaying the information disclosure with a small quantity is higher when blu±ng than when following the private information. Then the informed gets higher payo® when trading a small quantity than when blu±ng. When the market maker anticipates blu±ng this di®erence is larger. The anticipation by the dealer of blu±ng has two implications. First, it decreases the payo® from blu±ng. Some of the private information is incorporated into the prices. Second, it increases the payo® of following the signal with the small quantity on the side used to blu®. The market maker takes into account informed trading but with the converse information. This shows that when the market maker anticipates blu±ng and when there exists a semi-separating equilibrium on both sides of the market, the informed gets a higher payo® when following his signal with a small quantity than when blu±ng. This contradicts the existence of blu±ng in equilibrium. In a market where both the ask and bid sides are wide enough, blu±ng will never occur. In that case, large quantities are su±ciently high to compensate any better ¯rst and second period prices as a result of the informed strategic behavior.

From the previous proposition, a direct result can be stressed. This is done in the next corollary.

Corollary 1 A necessary condition for blu±ng in equilibrium is that the informed must trade, in equilibrium, small quantities with positive probability when following his signal. This has to be true for one side of the market at least.

This corollary defines the situations in which bluffing may occur in equilibrium. They are situations where there exists a pooling equilibrium on one side of the market at least. We then check whether in that case bluffing is consistent with an equilibrium behavior.

3.2.2 The P-S or S-P case

We now concentrate our attention on cases where the informed trades the two quantities with positive probability on one side of the market whereas he only trades the large on the other side. Such cases are referred to as S-P or P-S case.

Whenever (3) or (4) is violated, a pooling equilibrium exists on the side of the market for which the condition is violated. This can arise for two reasons. The first reason deals with the fact that if one side of the market is not wide enough, the better prices when trading a small quantity in the first period outweigh the advantage of large quantity. The second reason is as follows. As the informed is strategic he takes into account the effect of his current trade on the second period prices ($\Psi_{S^1;S^2}$ and $\Psi_{B^1;B^2}$). It can be the case that $\Psi_{S^1;S^2}$ and $-\Psi_{B^1;B^2}$ are so negative that a pooling equilibrium always exists. This occurs when the second period price after a small trade is very low (ask price) or very high (bid price).

In this subsection, we look at the S-P case only: The informed when following his signal sells large and purchases both the large and the small quantity with positive probability. The P-S case is symmetric to the previous case so that we do not develop it.

The previous corollary provides a necessary condition to have bluffing in equilibrium. However this condition is not sufficient. Even though the small quantity is traded by the informed it can be the case that bluffing does not arise in equilibrium. As we concentrate our attention on the S-P case, a S-P equilibrium without bluffing may exist. The sufficient conditions for the existence of a S-P equilibrium without bluffing are given in the Appendix.

Let us define the following incentive constraints:

$$S^2\left[1-\delta_1\left(\delta;S^2\right)+\delta_2^*\left(\delta;B^1;S^2\right)-\delta_2\left(\delta;S^2;S^2\right)\right]^* \geq -\left(1-\delta_1^*\left(\delta;B^1\right)\right); \quad (5)$$

$$B^2\left[\delta_1^*\left(\delta;B^2\right)+\delta_2^*\left(\delta;B^2;B^2\right)-\delta_2\left(\delta;S^1;B^2\right)\right]^* \geq -\delta. \quad (6)$$

When those incentives constraints are satisfied, the informed does not bluff on any side of the market. Moreover the star stands for the fact that the informed mixes between the small and the large quantity on the ask side. On the bid side he is trading the large quantity with probability one. We are in the S-P case.

Until now we know that the informed never bluffs if he is constrained to trade discrete orders of the same size in both periods. This has been proved in proposition 1 of the benchmark. We then concluded that the informed should be able to trade a larger quantity when following his signal in the second period.

Allowing that in the second model, we demonstrated that the informed does not blu® if the large quantity is \too large" (proposition 4 and corollary 1).

The next proposition provides su±cient conditions to get a S-P equilibrium with blu±ng when $s = L$.

Proposition 5 If conditions (3) and (6) are satis¯ed whereas conditions (5) and (4) are violated then a S-P equilibrium with blu±ng when the signal is low exists. Moreover under those conditions there does not exist any equilibrium where the informed does not blu®.

Proof. See Appendix.

Although we have more equilibria where blu±ng occurs, we focus on the one depicted in proposition 5 as it has a simpler form and a similar intuition. The other equilibria are relegated to the Appendix.

Under the su±cient conditions described in this proposition the informed is indi®erent, in equilibrium, between following his signal with a large quantity (S^2) and blu±ng with a small quantity (B^1) when the signal is low. He then mixes between trading S^2 and blu±ng when his signal is low while he mixes between trading B^2 and B^1 when his signal is high.

Two points are important for the pro¯tability of the blu±ng strategy. First, the price movement induced and second the quantity traded by the informed when blu±ng. Let us have a closer look at the price movements. We already know that the blu±ng strategy takes the following form. The informed induces a trend in the ¯rst period against which he trades in the second period. In the second period, he follows his private information. A crucial point is how much of this private information is incorporated into the second period price. Indeed if a substantial amount is revealed the second period prices when blu±ng and not blu±ng tend to be close. Because of the ¯rst period cost borne by the informed, blu±ng is not as pro¯table as following his signal. This is the case when the L.H.S. term in bracket in (5) is positive. However if prices exhibit a \price momentum", i.e. they do not react \a lot" to the second period trade when blu±ng it may be pro¯table to blu®. In that case $\pm_2^{¤}(\pm; B^1; S^2)$ is small implying that the L.H.S. term in bracket in (5) is negative. Obviously this price reaction depends upon the scope for the informed to hide from the market maker. In turn, the extent of this scope depends upon the proportion of liquidity traders trading the same quantity as the informed in the second period. Moreover it is related, through the dealer's updated beliefs to the form of the equilibrium that exists on the side used to blu®.

We now look at the second point that concerns the quantity traded. From condition (5), it can be seen that even if few information is revealed in the second period blu±ng may not take place in equilibrium. Indeed, if the large quantity is not su±ciently large the informed cannot overcome the ¯rst period cost.

We provide an example where the sufficient conditions of proposition 5 are satisfied.

Example 1 Let us consider the following example: $X_S^1 = X_S^2 = 0.43$, $X_B^1 = 0.12$, $X_B^2 = 0.02$, $\delta = 0.5$, $\mu = 0.33$ and $S^2 = B^2 = 6$. For this parameter configuration, we are in the situation described in the preceding proposition. The informed mixes between following his information with S^2 and bluffing with B^1 when his private information is low. In that case, he follows his private information with probability $.99$ and therefore bluffs with the converse probability. When mixing between those two strategies he obtains a conditional expected payoff of $3.29\left(\overline{V} - \underline{V}\right)$. Whereas if he sells the small quantity when he has low information he gets $2.83\left(\overline{V} - \underline{V}\right)$. On the other side of the market, the informed mixes between purchasing large and small. When he has high information, he trades the large quantity with probability $.411$ and the small one with the converse probability. He obtains conditional expected payoff of $.612\left(\overline{V} - \underline{V}\right)$. If he bluffs when he has good information, he gets $.587\left(\overline{V} - \underline{V}\right)$.

In that case, the fact that when the informed has good information he purchases both the large and the small quantity with positive probability leads him to bluff with a small quantity when he has low information.

The bluffer's behavior deserves one more comment. It is possible from the previous proposition to have more insights concerning his behavior when bluffing. This is done in the following corollary.

Corollary 2 Comparative Statics
In equilibrium, the probability of bluffing when $s = L$ is increasing with respect to the probability of trading B^1 when $s = H$, everything else being equal.

The intuition of this corollary is straightforward. In a pooling equilibrium, the informed with a high signal mixes between B^1 and B^2. A small purchase can possibly comes from the informed and this is reflected in the ask price. The higher the intensity (probability) by which the informed trades the small quantity (B^1), the more "high" information is conveyed by a small purchase. Hence the higher is the corresponding ask price. Then if we focus on the informed's incentive to bluff when $S = L$, the higher is the first period cost of bluffing. However the higher is the second period bid price. The second effect (increase of the bid price) is higher than the first one (increase of the cost). Thus the payoff from bluffing increases. The probability by which the informed with a high signal purchases B^1, provides to the informed with the converse signal a scope to hide behind it. Then, the higher, ceteris paribus, the probability by which the informed with $s = H$ trades B^1 in equilibrium, the higher is the scope for the informed with $s = L$ to hide, when bluffing, behind this increased probability. Hence he increases the intensity by which he bluffs i.e. by which he trades B^1.

We now look at the P-P case.

3.2.3 The P-P case:

In that case when the informed follows his signal and for the two values of the signal he trades the large or the small quantity with positive probability. For this situation we provide two examples only. They show the occurrence in equilibrium of this manipulative behavior.

Example 2 The first example is given by: $X_S^1 = X_B^1 = 0.4$, $X_S^2 = X_B^2 = 0.1$, $\delta = 0.5$, $\mu = 0.31$ and $S^2 = B^2 = 12$. The other example is given by: $X_S^1 = X_B^1 = 0.35$, $X_S^2 = X_B^2 = 0.15$, $\delta = 0.5$, $\mu = 0.31$ and $S^2 = B^2 = 20$. In these two examples the informed bluffs whatever the value of his signal. As the two sides of the market are symmetric, for each side of the market, he bluffs and trades the small or large quantity with the same intensity (i.e. probability). In the first example, he bluffs with probability .1 and trades the large quantity when following his signal with probability .58. He then trades, when following his signal, the small quantity with probability .32. He obtains conditional expected payoff of $4.1\left(\overline{V} - \underline{V}\right)$. For the second example, the informed bluffs with probability .02 and when following his information, he trades the large (small) quantity with probability .88 (.1). He gets conditional expected payoff of $6.8\left(\overline{V} - \underline{V}\right)$.

In both examples the informed bluffs whatever is the value of his signal. The intuition follows the same line as for proposition 5.

4 Conclusion

In this paper, we focused on dynamic strategic trading by a monopolistic informed trader in a two-period dealer market. Traders trade discrete orders. In a first part traders can trade a unit of the stock only. This is considered as our benchmark. In this framework, the informed never fools the market maker. He always follows his private information and does not attempt to manipulate the market makers' beliefs through bluffing strategies. This is due to the fact that the first period per unit cost is always higher than the second period per unit benefits. Indeed as the informed always follows his signal in the second period, this enables the market maker to refine his beliefs towards the "correct" information. This effect is present even if the market maker anticipates bluffing. The only way for the informed to recoup the first period losses would be to allow him to trade larger quantities in the second period. This would allow him to get a better position on the share when bluffing. This position would be according to his private information. The benefit derived from this position when the asset value is realized may compensate the loss borne from the first period in such a way that he bluffs in equilibrium.

In order to verify this intuition, the informed trader is allowed to choose between two di®erent quantities in the ¯rst period. For tractability, all traders trade the large quantity only in the second period. In this model, it is shown that an equilibrium where the informed when following his signal and for its two values trades large and blu®s with small quantity at least on one side cannot exist. Large quantities compensate any better ¯rst and second period prices. Then they compensate the e®ect the informed has on the second period prices when blu±ng. As a consequence a pooling equilibrium is necessary at least on one side of the market. We give su±cient conditions for the existence of an equilibrium with blu±ng. This is done for the situation where the informed, when following his signal, sells large and purchases the two quantities with positive probability and moreover the informed with a low signal blu®s with a small quantity. We provide an example for the occurrence of this equilibrium. In order to show that blu±ng can also arise when there exists a pooling equilibrium on both sides of the market, we give two di®erent examples. These results are proved in a very restricted setting as we use a discrete orders model and as the trader is not allowed to choose the quantity he wants to trade in the second period. However, as said in the introduction, we do believe that the results carry over to a situation where the informed can choose the quantity he wants to trade in the second period. The discrete order property of the model is obviously an important aspect of the model as blu±ng does not lead to in¯nite pro¯t as it is the case in Kyle [14].

Appendix

Proof of proposition 1

In a first step, we compute the dealer's bid and ask quotes for both periods. In a second step, we calculate the informed trader's conditional expected payoff when bluffing and not bluffing (following the signal). We show that bluffing leads to negative conditional expected payoff, whereas not bluffing implies a positive conditional expected payoff.

Step 1: Dealer's quotes.

First period: As the dealer is competitive he realizes zero conditional expected profit on each trade. This leads to a ask price, a_1, and a bid price, b_1, such that

$$\begin{aligned} a_1 &= E\left[\tilde{V} \mid \text{buy order}\right] = \overline{V}\Pr\left[\tilde{V} = \overline{V} \mid \text{buy order}\right] + \underline{V}\Pr\left[\tilde{V} = \underline{V} \mid \text{buy order}\right]; \\ b_1 &= E\left[\tilde{V} \mid \text{sell order}\right] = \overline{V}\Pr\left[\tilde{V} = \overline{V} \mid \text{sell order}\right] + \underline{V}\Pr\left[\tilde{V} = \underline{V} \mid \text{sell order}\right]. \end{aligned}$$

All conditional probabilities are determined using Bayes rule. Let us point out that $\Pr\left[\tilde{V} = \underline{V} \mid x\right] = \Pr[s = L \mid x]$ with x being the incoming order. Denote $\delta_1(\delta; x) = \Pr[s = L \mid x]$. Let us define $\left(\varepsilon_i^H, \varepsilon_j^L\right)$, the probability that the market maker uses to updated his beliefs. He anticipates that the informed with a high (low) signal purchases when $i = B$ ($j = B$) or sells when $i = S$ ($j = S$) such that $\varepsilon_S^H + \varepsilon_B^H = 1$ $\left(\varepsilon_S^L + \varepsilon_B^L = 1\right)$. The dealer's updated beliefs are given by

$$\delta_1(\delta; x) = \delta \frac{\mu \varepsilon_x^L + (1-\mu) X_x}{\delta \mu \varepsilon_x^L + (1-\delta)\mu \varepsilon_x^H + (1-\mu) X_x} \text{ for } x = S, B.$$

Second period: In the second period, the dealer uses the information arising from the first period trade side. The dealer still behaves competitively. Given a trade x ($x = S, B$) in the first period, the second period ask, $a_2(x)$, and the bid price, $b_2(x)$, are given by

$$\begin{aligned} a_2(x) &= E\left[\tilde{V} \mid x; \text{buy order}\right] = \overline{V}\Pr\left[\overline{V} \mid x; \text{buy order}\right] + \underline{V}\Pr[\underline{V} \mid x; \text{buy order}]; \\ b_2(x) &= E\left[\tilde{V} \mid x; \text{sell order}\right] = \overline{V}\Pr\left[\overline{V} \mid x; \text{sell order}\right] + \underline{V}\Pr[\underline{V} \mid x; \text{sell order}]. \end{aligned}$$

Again we use Bayes rule to determine the conditional probabilities. Let us denote $\delta_2(\delta; x; y) = \Pr[\underline{V} \mid x; y]$ where y stands for the incoming order. It is worth pointing out that

$$\delta_2(\delta; x; y) = \delta_1(\delta_1(\delta; x); y). \tag{7}$$

Using (7) and the fact that the informed is always better off following his signal in the second period, we have

$$\delta_2(\delta;x;S) = \delta_1(\delta;x)\frac{\mu + (1-\mu)X_S}{\delta_1(\delta;x)\mu + (1-\mu)X_S};$$
$$\delta_2(\delta;x;B) = \delta_1(\delta;x)\frac{(1-\mu)X_B}{(1-\delta_1(\delta;x))\mu + (1-\mu)X_B}.$$

Step 2: Informed trader's payoff

Let $\Pi(x;y|s=l)$ be the informed's conditional expected payoff with signal l ($l = H;L$) for the sequence of trade $(x;y)$.As the cases for which $s = H$ or $s = L$ are symmetric, we only prove the proposition for $s = L$. The expected payoff from following the signal and bluffing are given respectively by

$$\Pi(x;S|s=L) = \begin{cases} (b_1 - \underline{V}) + (b_2(S) - \underline{V}) & \text{if } x = S; \\ (\underline{V} - a_1) + (b_2(B) - \underline{V}) & \text{if } x = B. \end{cases}$$

Using the bid and ask quotes, it is straightforward to get that $\Pi(S;S|s=L) > 0$ $\forall\, 0 < \varepsilon^L_S < 1$, whereas $\Pi(B;S|s=L) < 0$ $\forall\, 0 < \varepsilon^L_B < 1$. This proves the claim of proposition 1.□

Proof of proposition 3

In the first step of the proof, we compute the market maker's first period updated beliefs in the most general case. In the second step we compute the bid and ask quotes in the particular case considered in proposition 3 (S-S equilibrium without bluffing). The necessary part of the proof is shown in step 3. The sufficient part is provided in step 4. Finally, in step 5 we show the uniqueness of this equilibrium. In order to simplify the analysis we use the result of proposition 2. Hence in the following equilibrium analysis we do not consider the possibility of bluffing with large quantities as it gives negative payoff.

Step 1: Updated beliefs.

As far as the updated beliefs are concerned, we keep the same notation as in the benchmark. Let σ^s_x with $x = S^1;S^2;B^1;B^2$ and $s = L;H$ and $\sum_x \sigma^s_x = 1\ \forall s$, be the probability used by the market maker to update his beliefs. The market maker anticipates that the informed trades the quantity x when his signal is s with probability σ^s_x. Hence using Bayes' rule, the market maker's updated beliefs are given by

$$
\begin{aligned}
\delta_1\left(\delta;S^1\right) &= \delta\frac{\mu\rho^L_{S^1}+(1-\mu)X^1_S}{\delta\mu\rho^L_{S^1}+(1-\delta)\mu\rho^H_{S^1}+(1-\mu)X^1_S};\\
\delta_1\left(\delta;S^2\right) &= \delta\frac{\mu\rho^L_{S^2}+(1-\mu)X^2_S}{\delta\mu\rho^L_{S^2}+(1-\mu)X^2_S};\\
\delta_1\left(\delta;B^1\right) &= \delta\frac{\mu\rho^L_{B^1}+(1-\mu)X^1_B}{\delta\mu\rho^L_{B^1}+(1-\delta)\mu\rho^H_{B^1}+(1-\mu)X^1_B};\\
\delta_1\left(\delta;B^2\right) &= \delta\frac{(1-\mu)X^2_B}{(1-\delta)\mu\rho^H_{B^2}+(1-\mu)X^2_B}.
\end{aligned}
\tag{8}
$$

Then the second period beliefs are computed using the rule given by (7). As traders can trade one quantity only in the second period, the second period is identical to the second period of the benchmark. Second period updated beliefs have the same form as in the benchmark.

Step 2: Updated beliefs and bid and ask quotes for the particular case of proposition 3.

The equilibrium described in proposition 3 corresponds to the case where $\rho^L_{S^2}=1$ and $\rho^H_{B^2}=1$. Hence the updated beliefs in that particular case can be computed replacing $\rho^L_{S^2}=1$ and $\rho^H_{B^2}=1$ in (8) and using (7). The first period bid and ask prices ($a_1(x)$ and $b_1(x)$) for large quantity ($x=B^2$ and S^2) can be computed using the same steps as in the proof of proposition 1 of the benchmark. They are derived using the zero expected profit condition for those trades.
By definition of the semi-separating equilibrium, small quantities are uninformative. The quotes for small quantities are then given by

$$a_1\left(B^1\right)=b_1\left(S^1\right)=\delta\underline{V}+(1-\delta)\overline{V}=V^*. \tag{9}$$

The second period beliefs and quotes can be computed following the same steps as before.

Step 3: Necessary part.

The first and second period prices computed using step 2 determine an equilibrium such that the informed prefers to trade large and to use non-manipulating strategies if he is better off not bluffing and trading large. The conditional expected profit is defined as follows

$$\Pi\left(x;y;\varepsilon^H_x;\varepsilon^L_x \,\middle|\, s=s\right) \text{ with } s=H;L.$$

The informed's conditional expected profit depends upon the quantity traded in period 1 (x) and in period 2 (y). The informed's conditional expected profits also depends upon the probability by which the market maker thinks the trader with some particular information trades the quantity x. As before ε^s_x denotes the probability by which the market maker thinks the informed with information s trades quantity x.
The informed with signal s, trades large and does not bluff if

$$\pi\left(B^2;B^2;1;0\,\big|\,s=H\right) \geq \pi\left(x;B^2;0;0\,\big|\,s=H\right) \qquad \forall\, x = B^1;S^1; \tag{10}$$

$$\pi\left(S^2;S^2;0;1\,\big|\,s=L\right) \geq \pi\left(x;S^2;0;0\,\big|\,s=L\right) \qquad \forall\, x = S^1;B^1: \tag{11}$$

Substituting for all bid and ask quotes, using (7) and simplifying by $\left(\overline{V} - \underline{V}\right)$, the above conditions are equivalent to

$$\delta_1\left(\delta;B^2\right)\left(1+\frac{((1-\mu)X_B)}{(1-\delta_1(\delta;B^2))\mu+(1-\mu)X_B}B^2\right) \geq \delta\left(1+\frac{(1-\mu)X_B}{(1-\delta)\mu+(1-\mu)X_B}B^2\right); \tag{12}$$

$$\delta_1\left(\delta;B^2\right)\left(1+\frac{((1-\mu)X_B)}{(1-\delta_1(\delta;B^2))\mu+(1-\mu)X_B}B^2\right) \geq \delta\left(-1+\frac{(1-\mu)X_B}{(1-\delta)\mu+(1-\mu)X_B}B^2\right); \tag{13}$$

$$\left(1-\delta_1\left(\delta;S^2\right)\right)\left(1+\frac{(1-\mu)X_S}{\delta_1(\delta;S^2)\mu+(1-\mu)X_S}S^2\right) \geq (1-\delta)\left(1+\frac{(1-\mu)X_S}{\delta\mu+(1-\mu)X_S}S^2\right); \tag{14}$$

$$\left(1-\delta_1\left(\delta;S^2\right)\right)\left(1+\frac{(1-\mu)X_S}{\delta_1(\delta;S^2)\mu+(1-\mu)X_S}S^2\right) \geq (1-\delta)\left(-1+\frac{(1-\mu)X_S}{\delta\mu+(1-\mu)X_S}S^2\right): \tag{15}$$

First, notice that the L.H.S. expressions are always positive. It can be seen that the R.H.S. of (13) is strictly lower than the R.H.S. of (12). The same can be seen for the R.H.S. of (15) with respect to the R.H.S. of (14). Therefore it can be concluded that when conditions (12) and (14) are satisfied the second conditions are trivially verified. Conditions (12) and (14) can be rewritten as conditions (3) and (4). This ends the necessary part.

Step 4: Sufficient part.

In order to prove it, we must show that there does not exist any deviation for the informed leading to higher payoff than trading large when following the information when conditions (3) and (4) are satisfied. Moreover in the following reasoning the market maker's beliefs are held constant. The market maker anticipates that the informed trades the large quantity with probability 1 when following his signal. Under conditions (3) and (4) we have

$$\begin{aligned}\pi\left(S^2;S^2;0;1|s=L\right) &\geq \pi\left(S^1;S^2;0;0|s=L\right)\\ &> \pi\left(B^1;S^2;0;0|s=L\right);\end{aligned} \tag{16}$$

$$\begin{aligned}\pi\left(B^2;B^2;1;0|s=H\right) &\geq \pi\left(B^1;B^2;0;0|s=H\right)\\ &> \pi\left(S^1;B^2;0;0|s=H\right):\end{aligned} \tag{17}$$

From those conditions, it is clear that trading small, bluffing or not, gives lower payoffs than trading large. Moreover, any weighted sum (with non-negative weights whose sum is equal to 1) of $\Pi(S^2; S^2; 0; 1 | s = L)$, $\Pi(S^1; S^2; 0; 0 | s = L)$ and $\Pi(B^1; S^2; 0; 0 | s = L)$ is smaller than $\Pi(S^2; S^2; 0; 1 | s = L)$. Those weighted sums are the payoffs of mixing between S^2, S^1 and B^1 in the first period when the market maker believes that the informed trades the large quantities only. As a consequence all possible deviations lead to lower payoffs. The same reasoning applies for $s = H$.

Step 5: Uniqueness.

First, as pointed out in the text the informed always trades, in equilibrium, the large quantity with a strictly positive probability. Indeed if the market maker anticipates that the informed does not trade the large quantity, the informed is better off trading the large quantity with probability 1. He would trade the large quantity at a price lower than the price of the small quantity.

We now look at the equilibria that cannot be removed trivially. We prove that they cannot exist by contradiction.

Case 1: We look at any symmetric equilibria in which the informed trades a large quantity with probability $\varepsilon^L_{S^2}$ ($\varepsilon^H_{B^2}$) when following his information and bluffs with or trades a small quantity with the converse probability $1 - \varepsilon^L_{S^2}$ ($1 - \varepsilon^H_{B^2}$).

For those equilibria the informed mixes between trading a large quantity when following his information and trading small when following or not his information. Thus the informed must be indifferent between those strategies. Therefore there should exist both $0 < \varepsilon^L_{S^2} < 1$ and $0 < \varepsilon^H_{B^2} < 1$ such that the following two equations are satisfied

$$\Pi\left(S^2; S^2; 0; \varepsilon^L_{S^2} \middle| s = L\right) = \Pi\left(x; S^2; 1 - \varepsilon^H_{B^2}; 1 - \varepsilon^L_{S^2} \middle| s = L\right) \tag{18}$$

$$\Pi\left(B^2; B^2; \varepsilon^H_{B^2}; 0 \middle| s = H\right) = \Pi\left(x; B^2; 1 - \varepsilon^H_{B^2}; 1 - \varepsilon^L_{S^2} \middle| s = H\right) \tag{19}$$

with $x = S^1$ or B^1.
Given the updated beliefs computed using (8) and (7) we can prove that

$$\frac{\partial \Pi\left(S^2; S^2; 0; \varepsilon^L_{S^2} | s = L\right)}{\partial \varepsilon^L_{S^2}} < 0 \text{ and } \frac{\partial \Pi\left(B^2; B^2; \varepsilon^H_{B^2}; 0 | s = H\right)}{\partial \varepsilon^H_{B^2}} < 0. \tag{20}$$

Moreover it can also be checked that $\forall\, x = S^1$ or B^1

$$\frac{\partial \Pi\left(x; S^2; 1 - \varepsilon^H_{B^2}; 1 - \varepsilon^L_{S^2} | s = L\right)}{\partial \varepsilon^L_{S^2}} > 0 \text{ and } \frac{\partial \Pi\left(x; B^2; 1 - \varepsilon^H_{B^2}; 1 - \varepsilon^L_{S^2} | s = H\right)}{\partial \varepsilon^H_{B^2}} > 0; \tag{21}$$

for the informed trading quantity x in equilibrium. We point out that (3) and (4) are equivalent to

$$\Pi(S^2, S^2, 0, 1 | s = L) \geq \Pi(S^1, S^2, 0, 0 | s = L);$$
$$\Pi(B^2, B^2, 1, 0 | s = H) \geq \Pi(B^1, B^2, 0, 0 | s = H).$$

Then applying the previous point and using (20) and (21), the conditions (18) and (19) cannot be verified. The informed is better off following his private information with a large quantity.
The same argument can be used to show that any equilibrium of the following form cannot exist: S-P or P-S without bluffing, S-P or P-S with bluffing on the two sides of the market or bluffing on the side of the market where there exists the semi-separating equilibrium only and S-S or P-P with bluffing on one side only. We then go to the cases where the analysis is not so direct.

Case 2: We now look at the following cases: P-P equilibrium with bluffing on the two sides of the market and S-P or P-S equilibrium with bluffing on the side of the market where there exists a pooling equilibrium only cannot exist.

For those equilibria, it is more complicated to show their non existence. Indeed, the small quantities are traded by the informed with both high and low information on one side of the market at least.
Let us consider a situation where there exists a P-P equilibrium with bluffing on both the sell and buy order sides. In order to have such an equilibrium there must exist $0 < \varepsilon^{H*}_{B^2} < 1$, $0 < \varepsilon^{H*}_{B^1} < 1$, $0 < \varepsilon^{H*}_{S^1} < 1$, with $\varepsilon^{H*}_{B^2} + \varepsilon^{H*}_{B^1} + \varepsilon^{H*}_{S^1} = 1$ and $0 < \varepsilon^{L*}_{S^2} < 1$, $0 < \varepsilon^{L*}_{S^1} < 1$, $0 < \varepsilon^{L*}_{B^1} < 1$, with $\varepsilon^{L*}_{S^2} + \varepsilon^{L*}_{S^1} + \varepsilon^{L*}_{B^1} = 1$, such that the following conditions are simultaneously satisfied

$$\Pi\left(B^2, B^2, \varepsilon^{H*}_{B^2}, 0 \,\middle|\, s = H\right) = \Pi\left(B^1, B^2, \varepsilon^{H*}_{B^1}, \varepsilon^{L*}_{B^1} \,\middle|\, s = H\right); \qquad (22)$$
$$\Pi\left(B^2, B^2, \varepsilon^{H*}_{B^2}, 0 \,\middle|\, s = H\right) = \Pi\left(S^1, B^2, \varepsilon^{H*}_{S^1}, \varepsilon^{L*}_{S^1} \,\middle|\, s = H\right); \qquad (23)$$
$$\Pi\left(S^2, S^2, 0, \varepsilon^{L*}_{S^2} \,\middle|\, s = L\right) = \Pi\left(S^1, S^2, \varepsilon^{L*}_{S^1}, \varepsilon^{H*}_{S^1} \,\middle|\, s = L\right); \qquad (24)$$
$$\Pi\left(S^2, S^2, 0, \varepsilon^{L*}_{S^2} \,\middle|\, s = L\right) = \Pi\left(B^1, S^2, \varepsilon^{L*}_{B^1}, \varepsilon^{H*}_{B^1} \,\middle|\, s = L\right). \qquad (25)$$

If $\varepsilon^{L*}_{B^1} \leq \varepsilon^{H*}_{B^1}$, one can show that $\delta \geq \delta^*_1(\delta; B^1)$ where $\delta^*_1(\delta; B^1)$ is computed using (8) with $\varepsilon^{L}_{B^1}$ and $\varepsilon^{H}_{B^1}$ being replaced by $\varepsilon^{L*}_{B^1}$ and $\varepsilon^{H*}_{B^1}$ respectively. This in turn implies that

$$\Pi\left(B^1, B^2, 0, 0 \,\middle|\, s = H\right) \geq \Pi\left(B^1, B^2, \varepsilon^{H*}_{B^1}, \varepsilon^{L*}_{B^1} \,\middle|\, s = H\right). \qquad (26)$$

Applying (3) and (4) we have

$$\Pi\left(B^2, B^2, \varepsilon^{H*}_{B^2}, 0 \,\middle|\, s = H\right) > \Pi\left(B^1, B^2, \varepsilon^{H*}_{B^1}, \varepsilon^{L*}_{B^1} \,\middle|\, s = H\right).$$

We can conclude that condition (22) cannot be satisfied.
Now let us consider the converse case where $\varepsilon^{L*}_{B^1} > \varepsilon^{H*}_{B^1}$. In that case, we have $\delta < \delta_1^*(\delta, B^1)$ implying the following

$$\Pi\left(B^1, S^2, 0, 0 \,\middle|\, s = L\right) > \Pi\left(B^1, S^2, \varepsilon^{L*}_{B^1}, \varepsilon^{H*}_{B^1} \,\middle|\, s = L\right) > 0.$$

Again applying (3) and (4) we get that

$$\Pi\left(S^2, S^2, 0, \varepsilon^{L*}_{S^2} \,\middle|\, s = L\right) > \Pi\left(B^1, S^2, \varepsilon^{L*}_{B^1}, \varepsilon^{H*}_{B^1} \,\middle|\, s = L\right).$$

Condition (25) is then violated.
It has been proved that under conditions (3) and (4) a P-P equilibrium with bluffing on both the buy and sell order sides cannot exist.
The same argument can be used to prove that a S-P or P-S equilibrium with bluffing on the side where there exists a pooling equilibrium only cannot exist.
This proves the uniqueness of the S-S equilibrium without bluffing under conditions (3) and (4). This point ends the proof of proposition 3.□

Proof of proposition 4

The proof is similar to case 1 of the uniqueness part of proposition 3. Let us look at the more complicated case: S-S equilibrium with bluffing on the two sides of the market. In that case there must exist $0 < \varepsilon^{L*}_{S^2} < 1$ and $0 < \varepsilon^{L*}_{B^1} < 1$ with $\varepsilon^{L*}_{S^2} + \varepsilon^{L*}_{B^1} = 1$ and $0 < \varepsilon^{H*}_{B^2} < 1$ and $0 < \varepsilon^{H*}_{S^1} < 1$ with $\varepsilon^{H*}_{B^2} + \varepsilon^{H*}_{S^1} = 1$ such that the following conditions must be simultaneously satisfied

$$\Pi\left(S^2, S^2, 0, \varepsilon^{L*}_{S^2} \,\middle|\, s = L\right) = \Pi\left(B^1, S^2, 0, 1 - \varepsilon^{L*}_{S^2} \,\middle|\, s = L\right) \geq \Pi\left(S^1, S^2, 1 - \varepsilon^{H*}_{B^2}, 0 \,\middle|\, s = L\right), \tag{27}$$

$$\Pi\left(B^2, B^2, \varepsilon^{H*}_{B^2}, 0 \,\middle|\, s = H\right) = \Pi\left(S^1, B^2, 1 - \varepsilon^{H*}_{B^2}, 0 \,\middle|\, s = H\right) \geq \Pi\left(B^1, B^2, 0, 1 - \varepsilon^{L*}_{S^2} \,\middle|\, s = H\right). \tag{28}$$

Note that $\forall\, \varepsilon^L_{S^2} < 1$ and $\forall\, \varepsilon^H_{B^2} < 1$

$$\Pi\left(S^1, S^2, 1 - \varepsilon^H_{B^2}, 0 \,\middle|\, s = L\right) > \Pi\left(B^1, S^2, 0, 1 - \varepsilon^L_{S^2} \,\middle|\, s = L\right),$$
$$\Pi\left(B^1, B^2, 0, 1 - \varepsilon^L_{S^2} \,\middle|\, s = H\right) > \Pi\left(S^1, B^2, 1 - \varepsilon^H_{B^2}, 0 \,\middle|\, s = H\right).$$

Given the two previous conditions the equilibrium conditions (27) and (28) cannot be satisfied.

The same argument can be used to prove that a S-S equilibrium with bluffing on one side of the market only cannot exist. This ends the proof of proposition 4.□

Sufficient conditions for the existence of a S-P equilibrium without bluffing

The sufficient conditions for the existence of a S-P equilibrium without bluffing are given by:

$$\Pi\left(S^2;S^2;1;0 \mid s=L\right) \geq \Pi\left(S^1;S^2;0;0 \mid s=L\right); \tag{29}$$

$$\Pi\left(S^2;S^2;1;0 \mid s=L\right) \geq \Pi\left(B^1;S^2;1-\epsilon^{H*}_{B^2};0 \mid s=L\right); \tag{30}$$

and

$$\Pi\left(B^1;B^2;0;0 \mid s=H\right) > \Pi\left(B^2;B^2;1;0 \mid s=H\right); \tag{31}$$

$$\Pi\left(B^1;B^2;1-\epsilon^{H*}_{B^2};0 \mid s=H\right) \geq \Pi\left(S^1;B^2;0;0 \mid s=H\right): \tag{32}$$

It can be checked that from (31), there exists $0<\epsilon^{H*}_{B^2}<1$ such that

$$\Pi\left(B^1;B^2;1-\epsilon^{H*}_{B^2};0 \mid s=H\right) = \Pi\left(B^2;B^2;\epsilon^{H*}_{B^2};0 \mid s=H\right):$$

That is the informed with high information mixes between purchasing large and small. The conditions (30) and (32) imply that the informed with a high or a low signal does not bluff. The market maker anticipates the fact that the informed with high information mixes between purchasing both large and small. Condition (29) imply that the informed with low information sells a large quantity.

Proof of proposition 5

In the first step we show that the conditions are sufficient conditions for the existence of a S-P equilibrium with bluffing when the signal is low. The non-existence of equilibrium without bluffing is shown in the second step.

Step 1: Sufficient part.

We must prove that the set of sufficient conditions implies the existence of $0<\hat{\epsilon}^H_{B^2}<1$ and $0<\hat{\epsilon}^L_{S^2}<1$ such that the following set of conditions is satisfied

$$\begin{aligned}\Pi\left(S^2;S^2;0;\hat{\epsilon}^L_{S^2} \mid s=L\right) &= \Pi\left(B^1;S^2;1-\hat{\epsilon}^H_{B^2};1-\hat{\epsilon}^L_{S^2} \mid s=L\right);\\ &\geq \Pi\left(S^1;S^2;0;0 \mid s=L\right);\end{aligned} \tag{33}$$

$$\begin{aligned}\Pi\left(B^2;B^2;\hat{\epsilon}^H_{B^2};0 \mid s=H\right) &= \Pi\left(B^1;B^2;1-\hat{\epsilon}^H_{B^2};1-\hat{\epsilon}^L_{S^2} \mid s=H\right);\\ &\geq \Pi\left(S^1;B^2;0;0 \mid s=H\right):\end{aligned} \tag{34}$$

We show it by proving that there exist two "reaction function" $b^H_{B^2}\left(\epsilon^L_{S^2}\right)$, $b^L_{S^2}\left(\epsilon^H_{B^2}\right)$. The former gives the probability by which the informed with high information mixes between B^2 and B^1 as a function of the probability by which the informed with low signal mixes between S^2 and B^1. The latter gives the probability by which the informed with low information mixes between S^2 and B^1 as a function of the probability by which the informed with high signal mixes between B^2 and B^1. Once the existence of those two reaction functions is established we prove that there exists a crossing point between them. This crossing point gives the equilibrium probability such that bluffing appears. We now go to the proof of the sufficient part.

In the following lemma we establish the existence of a function $\hat{\varepsilon}^H_{B^2}\left(\varepsilon^L_{S^2}\right)$ such that the equality condition of (34) holds.

Lemma 1 $\exists\ 0 < \hat{\varepsilon}^H_{B^2} < 1;\ \forall\ \varepsilon^L_{S^2}$ with $0 < \varepsilon^L_{S^2} < 1$ such that

$$\mu\left(B^2; B^2; \hat{\varepsilon}^H_{B^2}; 0 \,\middle|\, s = H\right) = \mu\left(B^1; B^2; 1 - \hat{\varepsilon}^H_{B^2}; 1 - \varepsilon^L_{S^2} \,\middle|\, s = H\right); \qquad (35)$$

with $\hat{\varepsilon}^H_{B^2} = \varepsilon^{H*}_{B^2}$ when $\varepsilon^L_{S^2} = 1$.

Proof. First, given the form of the updated beliefs $\mu(B^2; B^2; \ldots | s = H)$ and $\mu(B^1; B^2; \ldots | s = H)$ are continuous function of their arguments. Second, it is direct to show that $\mu(B^1; B^2; \ldots | s = H)$ is decreasing with its first argument while increasing with its second. Third we have that

$$\mu\left(B^2; B^2; 0; 0 \,\middle|\, s = H\right) > \mu\left(B^1; B^2; 1; 0 \,\middle|\, s = H\right). \qquad (36)$$

Given the three previous points, the monotonicity of $\mu(B^2; B^2; \ldots | s = H)$ with respect to its first argument (see (20)) and the fact that (4) is violated the existence of $\hat{\varepsilon}^H_{B^2}$ is guaranteed. ∎

In the same way the next lemma establishes the existence of $\hat{\varepsilon}^L_{S^2}\left(\varepsilon^H_{B^2}\right)$ such that the equality condition of (33) holds.

Lemma 2 $\exists\ 0 < \hat{\varepsilon}^L_{S^2} < 1;\ \forall\ 0 \leq \varepsilon^H_{B^2} < \varepsilon^{H*}_{B^2}$ such that

$$\mu\left(S^2; S^2; 0; \hat{\varepsilon}^L_{S^2} \,\middle|\, s = L\right) = \mu\left(B^1; S^2; 1 - \varepsilon^H_{B^2}; 1 - \hat{\varepsilon}^L_{S^2} \,\middle|\, s = L\right). \qquad (37)$$

Proof. First, given the form of the updated beliefs $\mu(B^1; S^2; \ldots | s = L)$ and $\mu(S^2; S^2; \ldots | s = L)$ are continuous function of their arguments.
Second, $\mu(B^1; S^2; \ldots | s = L)$ is increasing with its first argument whereas decreasing with its second. Third, condition (5) being violated is equivalent to

$$\mu\left(S^2; S^2; 0; 1 \,\middle|\, s = L\right) < \mu\left(B^1; S^2; 1 - \varepsilon^{H*}_{B^2}; 0 \,\middle|\, s = L\right). \qquad (38)$$

Fourth, using the updated beliefs, it is direct to prove that

$$\begin{array}{l} \forall\ \varepsilon^H_{B^2} \text{ with } 0 < \varepsilon^H_{B^2} < 1 \\ \mu(S^2; S^2; 0; 0 | s = L) > \mu\left(B^1; S^2; 1 - \varepsilon^H_{B^2}; 1 \,\middle|\, s = L\right). \end{array} \qquad (39)$$

Now using the previous four points, the monotonicity of $\Pi(S^2;S^2;\cdot;\cdot \mid s=L)$ (see (20)) and (39), the existence of $\hat{\varepsilon}^L_{S^2}$ is guaranteed. $\blacksquare$

Lemma 1 and lemma 2 establish the existence of two functions $\hat{\varepsilon}^H_{B^2}\left(\varepsilon^L_{S^2}\right)$ and $\hat{\varepsilon}^L_{S^2}\left(\varepsilon^H_{B^2}\right)$. Those two functions can be understood as reaction functions. The next lemma establishes the existence of a crossing point between those two functions.

Lemma 3 There exist $0 < \tilde{\varepsilon}^H_{B^2} < \varepsilon^{H*}_{B^2}$, $0 < \tilde{\varepsilon}^L_{S^2} < 1$ such that

$$\hat{\varepsilon}^H_{B^2}\left(\tilde{\varepsilon}^L_{S^2}\right) = \hat{\varepsilon}^L_{S^2}\left(\tilde{\varepsilon}^H_{B^2}\right).$$

Proof. First, apply the implicit function theorem on conditions (35) and (37) respectively. We get that

$$\begin{aligned} &\frac{\partial \hat{\varepsilon}^H_{B^2}}{\partial \varepsilon^L_{S^2}} > 0; \qquad \forall\, \varepsilon^L_{S^2}; \\ &\frac{\partial \hat{\varepsilon}^L_{S^2}}{\partial \varepsilon^H_{B^2}} > 0; \qquad \forall\, \varepsilon^H_{B^2} \text{ with } \varepsilon^H_{B^2} < \varepsilon^{H*}_{B^2}. \end{aligned} \tag{40}$$

Second, $\hat{\varepsilon}^H_{B^2}$ is defined for all values of $\varepsilon^L_{S^2}$ (with $\hat{\varepsilon}^H_{B^2} = \varepsilon^{H*}_{B^2}$ for $\varepsilon^L_{S^2} = 1$ and $\hat{\varepsilon}^H_{B^2} > 0$ for $\varepsilon^L_{S^2} = 0$). Third, $\hat{\varepsilon}^L_{S^2}$ is defined for all values of $\varepsilon^H_{B^2} < \varepsilon^{H*}_{B^2}$ (with $\hat{\varepsilon}^L_{S^2} < 1$ for $\hat{\varepsilon}^H_{B^2} = \varepsilon^{H*}_{B^2}$ and $\hat{\varepsilon}^L_{S^2} > 0$ for $\varepsilon^H_{B^2} = 0$). Using the previous three points the existence of a crossing point between those two functions is guaranteed. Figure (1) at the end of this chapter may help to understand the argument of the proof.

This crossing point is such that the equality in both (33) and (34) holds. This means that as $\tilde{\varepsilon}^L_{S^2}$ is strictly smaller than 1, the informed with low information bluffs with probability $1 - \tilde{\varepsilon}^L_{S^2}$. However the uniqueness of this crossing point cannot be established. $\blacksquare$

We still have to show that the informed does not deviate from this equilibrium. Given (20), we have

$$\Pi\left(B^2;B^2;\tilde{\varepsilon}^H_{B^2};0 \mid s=H\right) > \Pi\left(B^2;B^2;\varepsilon^{H*}_{B^2};0 \mid s=H\right) \geq \Pi\left(S^1;B^2;0;0 \mid s=H\right); \tag{41}$$

$$\Pi\left(S^2;S^2;0;\tilde{\varepsilon}^L_{S^2} \mid s=L\right) > \Pi\left(S^2;S^2;0;1 \mid s=L\right) \geq \Pi\left(S^1;S^2;0;0 \mid s=L\right). \tag{42}$$

The second inequality comes from the fact that conditions (3) and (6) are satisfied. From those conditions it is clear that all deviations lead to payoffs lower than doing what the equilibrium specifies.

We showed under the conditions specified in the proposition the existence of $0 < \tilde{\varepsilon}^H_{B^2} < 1$ and $0 < \tilde{\varepsilon}^L_{S^2} < 1$ verifying conditions (33) and (34). This implies the existence of a S-P equilibrium with bluffing when $s = L$. This ends the proof of the sufficient part.

Step 2: Non-existence of equilibrium without bluffing.

We have to check that there do not exist S-S, P-S, S-P nor P-P equilibrium without bluffing.

Given that condition (4) is violated, if the market maker anticipates that the informed with a high signal trades large, the informed is better off trading small. Under the above set of sufficient conditions, an equilibrium without bluffing and a semi-separating equilibrium on the buy order side cannot exist. This rules out the following equilibria: S-S and P-S equilibria without bluffing.

Given that condition (5) is violated, a S-P equilibrium without bluffing cannot exist. Indeed, if the market maker anticipates that the informed does not bluff the informed always bluffs when he receives a low signal. This destroys the S-P equilibrium without bluffing.

The case of the P-P equilibrium without bluffing is not so direct. Again we prove it by contradiction. A P-P equilibrium without bluffing would be such that there would exist $0 < \varepsilon^{H*}_{B^2} < 1$ and $0 < \varepsilon^{L*}_{S^2} < 1$ such that the following conditions should be simultaneously satisfied

$$\Pi\left(S^2,S^2,0,\varepsilon^{L*}_{S^2} \mid s=L\right) = \Pi\left(S^1,S^2,0,1-\varepsilon^{L*}_{S^2} \mid s=L\right) \geq \Pi\left(B^1,S^2,1-\varepsilon^{H*}_{B^2},0 \mid s=L\right), \tag{43}$$

$$\Pi\left(B^2,B^2,\varepsilon^{H*}_{B^2},0 \mid s=H\right) = \Pi\left(B^1,B^2,1-\varepsilon^{H*}_{B^2},0 \mid s=H\right) \geq \Pi\left(S^1,B^2,0,1-\varepsilon^{L*}_{S^2} \mid s=H\right). \tag{44}$$

As we proved that the conditions are sufficient for the existence of a S-P equilibrium with bluffing if the signal is low there exist $0 < \tilde{\varepsilon}^{H}_{B^2} < 1$ and $0 < \tilde{\varepsilon}^{L}_{S^2} < 1$ such that conditions (33) and (34) are satisfied.

First, let us assume that $\varepsilon^{H*}_{B^2} \geq \tilde{\varepsilon}^{H}_{B^2}$. This leads to the fact

$$\Pi\left(B^2,B^2,\tilde{\varepsilon}^{H}_{B^2},0 \mid s=H\right) \geq \Pi\left(B^2,B^2,\varepsilon^{H*}_{B^2},0 \mid s=H\right).$$

On one hand, using the equality conditions for mixing ((34) and (44) we get that

$$\tilde{\delta}\left(\delta,B^1\right) \geq \delta^*\left(\delta,B^1\right), \tag{45}$$

where the tilde and the star represents the equilibrium behavior of the informed. On the other hand, as $\Pi\left(S^1,S^2,.,.|s=L\right)$ is decreasing with its second argument we have

$$\Pi\left(B^1,S^2,1-\tilde{\varepsilon}^{H}_{B^2},1-\tilde{\varepsilon}^{L}_{S^2} \mid s=L\right) \geq \Pi\left(S^1,S^2,0,0 \mid s=L\right) > \Pi\left(S^1,S^2,0,1-\varepsilon^{L*}_{S^2} \mid s=L\right).$$

Moreover, given (43) we have

$$\Pi\left(S^1,S^2,0,1-\varepsilon^{L*}_{S^2} \mid s=L\right) \geq \Pi\left(B^1,S^2,1-\varepsilon^{H*}_{B^2},0 \mid s=L\right).$$

This can be summarized in

$$\Pi\left(B^1; S^2; 1-\hat\lambda^H_{B^2}; 1-\hat\lambda^L_{S^2} \,\middle|\, s=L\right) > \Pi\left(B^1; S^2; 1-\lambda^{H*}_{B^2}; 0 \,\middle|\, s=L\right):$$

This is equivalent to $\underline{\delta}(\delta; B^1) < \delta^*(\delta; B^1)$, contradicting condition (45).

Second, let us assume that $\lambda^{H*}_{B^2} < \hat\lambda^H_{B^2}$. Given that we have

$$\Pi\left(B^1; S^2; 1-\hat\lambda^H_{B^2}; 1-\hat\lambda^L_{S^2} \,\middle|\, s=L\right) > \Pi\left(B^1; S^2; 1-\lambda^{H*}_{B^2}; 0 \,\middle|\, s=L\right): \qquad (46)$$

Using the monotonicity of the function with respect to its two arguments we have

$$\Pi\left(B^1; S^2; 1-\lambda^{H*}_{B^2}; 0 \,\middle|\, s=L\right) > \Pi\left(B^1; S^2; 1-\hat\lambda^H_{B^2}; 0 \,\middle|\, s=L\right)$$
$$> \Pi\left(B^1; S^2; 1-\hat\lambda^H_{B^2}; 1-\hat\lambda^L_{S^2} \,\middle|\, s=L\right):$$

This contradicts (46). This ends the proof of step 2 as well as the proof of proposition 5.

Step 3: Other equilibria.

Let us consider the following parameter configuration: conditions (3) and (5) are satisfied whereas conditions (4) and (6) are not satisfied. It can be checked that we either have a S-P equilibrium with bluffing when $s = H$ or a S-P equilibrium with bluffing for the two values of the signal.
A S-P equilibrium with bluffing when $s = H$ is characterized by the existence of $0 < \lambda^{H*}_{S^1} < 1$, $0 < \lambda^{H*}_{B^2} < 1$ and $0 < \lambda^{H*}_{B^1} < 1$ with $\lambda^{H*}_{S^1} + \lambda^{H*}_{B^2} + \lambda^{H*}_{B^1} = 1$ such that the following equations are satisfied

$$\Pi(S^2; S^2; 0; 1 | s=L) \geq \Pi\left(S^1; S^2; \lambda^{H*}_{S^1}; 0 \,\middle|\, s=L\right);$$
$$\geq \Pi\left(B^1; S^2; 1-\lambda^{H*}_{B^2}-\lambda^{H*}_{S^1}; 0 \,\middle|\, s=L\right);$$
$$\Pi\left(B^2; B^2; \lambda^{H*}_{B^2}; 0 \,\middle|\, s=H\right) = \Pi\left(B^1; B^2; 1-\lambda^{H*}_{B^2}-\lambda^{H*}_{S^1}; 0 \,\middle|\, s=H\right);$$
$$= \Pi\left(S^1; B^2; \lambda^{H*}_{S^1}; 0 \,\middle|\, s=H\right):$$

A S-P equilibrium with bluffing for the two values of the signal is characterized by the existence of $0 < \lambda^{H**}_{S^1} < 1$, $0 < \lambda^{H**}_{B^2} < 1$ and $0 < \lambda^{H**}_{B^1} < 1$ with $\lambda^{H**}_{S^1} + \lambda^{H**}_{B^2} + \lambda^{H**}_{B^1} = 1$ and $0 < \lambda^{L**}_{S^2} < 1$ and $0 < \lambda^{L**}_{B^1} < 1$ with $\lambda^{L**}_{S^2} + \lambda^{L**}_{B^1} = 1$ such that the following equations are satisfied

$$\Pi\left(S^2; S^2; 0; \lambda^{L**}_{S^2} \,\middle|\, s=L\right) = \Pi\left(B^1; S^2; 1-\lambda^{H**}_{B^2}-\lambda^{H**}_{S^1}; 1-\lambda^{L**}_{S^2} \,\middle|\, s=L\right);$$
$$\geq \Pi\left(S^1; S^2; \lambda^{H**}_{S^1}; 0 \,\middle|\, s=L\right);$$
$$\Pi\left(B^2; B^2; \lambda^{H**}_{B^2}; 0 \,\middle|\, s=H\right) = \Pi\left(B^1; B^2; 1-\lambda^{H**}_{B^2}-\lambda^{H**}_{S^1}; 1-\lambda^{L**}_{S^2} \,\middle|\, s=H\right);$$
$$= \Pi\left(S^1; B^2; \lambda^{H**}_{S^1}; 0 \,\middle|\, s=H\right): \qquad \blacksquare$$

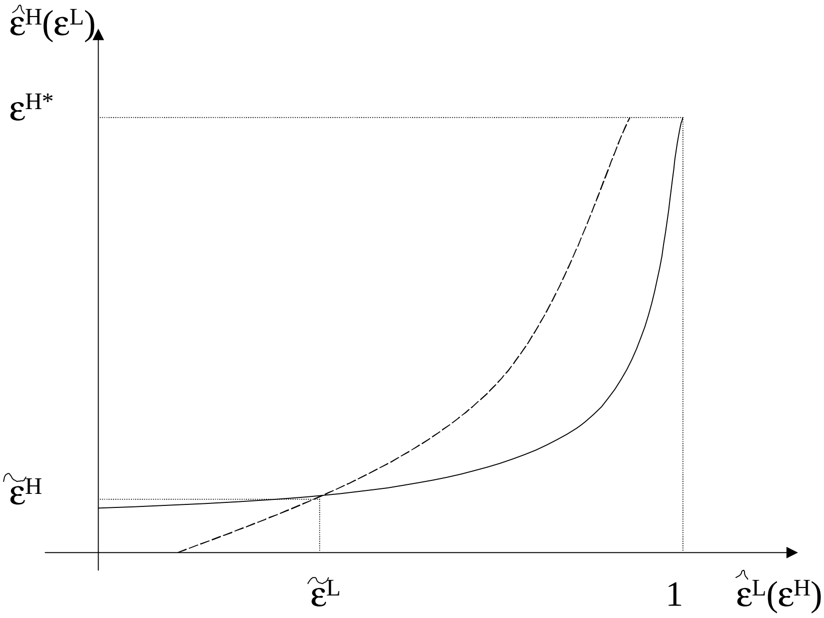

Figure 1: Reaction functions

This figure gives the probabilities of purchasing (selling) large when the signal is high (low) as a function of selling (purchasing) large with a low (high) signal, $\widehat{\varepsilon}^H_{B2}\left(\varepsilon^L_{S2}\right)$ (unbroken line) and $\widehat{\varepsilon}^L_{S2}\left(\varepsilon^H_{B2}\right)$ (broken line) respectively.

Following the signal	Blu±ng
S-S	No
S-P / P-S	Yes (either on one side or on the two sides of the market)
P-P	Yes (either on one side or on the two sides of the market)

Table 1: Types of equilibria.

The ¯rst column gives the form of the equilibrium when the informed follows his private information. On each side of the market, the equilibrium can be one of two forms: separating or pooling. In a separating equilibrium, the informed, when following his signal, trades the large quantity only whereas in a polling equilibrium, he mixes, again when following his signal, between the two quantities. S stands for separating and P for pooling. The ¯rst letter concerns the bid side whereas the second one concerns the ask side. The second column describes whether blu±ng can arise in equilibrium.

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