Irrational Financial Markets

Laurent Germain[*], Fabrice Rousseau[†], and Anne Vanhems[‡]

December 20, 2007

[*]Toulouse Business School, and Europlace Institute of Finance. Contact Address: Toulouse Business School, 20 Boulevard Lascrosses - BP 7010 - 31068 Toulouse Cedex 7 - France, phone: +33 (0)5 61 29 49 49, fax: +33 (0)5 61 29 49 94, e-mail: l.germain@esc-toulouse.fr.

[†]National University of Ireland Maynooth, Maynooth County Kildare, Ireland, phone: + 353 (0)1 7084568, fax: +353 (0)1 7083934, e-mail: fabrice.rousseau@nuim.ie.

[‡]Toulouse Business School, 20 Boulevard Lascrosses - BP 7010 - 31068 Toulouse Cedex 7 - France, phone: +33 (0)5 61 29 49 49, fax: +33 (0)5 61 29 49 94, e-mail: a.vanhems@esc-toulouse.fr.

Irrational Financial Markets

Abstract

We analyze a model where irrational and rational informed traders exchange a risky asset with competitive market makers. Irrational traders misperceive the mean of prior information (optimistic/pessimistic bias), the variance of prior information (better/lower than average effect) and the variance of the noise in their private signal (overconfidence/underconfidence bias). When market makers are rational we obtain results identical to Kyle and Wang (1997). However if market makers are irrational, we obtain that moderately underconfident traders can outperform rational ones and that irrational market makers can fare better than rational ones. Lastly we find that extreme level of confidence implies high trading volume.

1 Introduction

Economic and financial theory have widely used the assumption that agents behave rationally. Such an assumption has failed to explain some properties observed in financial markets such as (*i*) the low responsiveness or sometimes high responsiveness of the price to new information [Ritter (1991) and Womack (1996)], (*ii*) the excessive volume traded [Dow and Gorton (1997)], (*iii*) under-reaction or overreaction of market participants [Debondt and Thaler (1985)], and (*iv*) the excessive volatility observed in financial markets [Shiller (1981, 1989)]. In order to explain these properties, financial economists have assumed that investors may have some psychological traits which would lead them to behave irrationally.

Our paper follows that line of research as it assumes that some traders are irrational. All traders are strategic agents and, moreover, irrational investors have erroneous beliefs about (*i*) the mean of prior information (returns of the risky asset) and, (*ii*) the volatility of the asset returns as well as the variance of the noise in their private information.[1] The former refers to as the optimistic/pessimistic bias. The latter, i.e. (*ii*), refers to as the underconfident/overconfident bias for the misperception of the variance of the noise and

[1] Kandel and Pearson (1995) find that people might have different interpretation of public signal. This motivates the fact that traders may disagree on the interpretation of prior information.

the better/lower-than-average effect for the misperception of the variance of the prior information.[2] We also explore the possibility for the presence of irrational market makers who misperceive both the variance and the mean of prior information. Following such a framework, we find that a moderately underconfident trader can outperform a rational trader when the irrational trader and the market maker have sufficiently different beliefs concerning the expectation of prior information (optimism/pessimism bias).

This surprising result can be explained as follows. Due to her misperception of the mean of prior information, an irrational market maker distorts the price function. She increases (decreases) the overall level of price, through a price drift, if she is optimistic (pessimistic). An irrational trader can outperform a rational trader only if he can successfully commit to trade aggressively. Given the trader's optimism/pessimism bias and the fact that he is privately informed, he has two variables to successfully achieve this commitment: (*i*) his trading on private information and (*ii*) his trading due to his optimism/pessimism bias and due to his response to the price drift. An underconfident trader trades less intensely on private information than a rational trader. However, if he disagrees with the market maker's beliefs of the mean he will trade more aggressively on

---

[2] In our model, the cognitive biases lead to the different priors formed by the market participants. The heterogeneity in the priors is a driving force for the results obtained. The model could alternatively be understood as a model of heterogeneous beliefs that are not a consequence of irrationality. The same results would be obtained.

his second component, i.e. the part of his trading due to his misperception of the mean and to his response to the price drift, than his rational counterpart. The greater the disagreement, the larger the irrational's response to the drift. This can lead to a successful commitment to trade aggressively from a moderately underconfident trader.

This result relies on two important points: (*i*) the market maker's irrationality and (*ii*) the disagreement about the mean of prior information between the irrational traders and the irrational market maker.

The first point is documented in two recent papers. First, Oberlechner and Osler (2007) show that foreign exchange dealers are irrational: they underestimate uncertainty (miscalibration) as well as overestimate their own success (hubris). Moreover, they find that the irrational dealers are not driven out of the market over time. This study is based on a survey sent out to North American currency market professionals. They find that almost 1 in 2 professional displays the miscalibration bias.[3] Second, Greenwood and Nagel (2007) analyze the trading behavior of experienced and inexperienced mutual fund managers during the technology bubble of the late 1990s. They show that irrationality, i.e. optimism, carries over to inexperienced financial market professionals in the performance of their key job functions, with real money at stake.

Krichene (2004) illustrates the second point. His analysis recovers the euro-

[3] The miscalibration effect in expert predictions is a well known fact to psychologists. For a review see Russo and Schoemaker (1991).

dollar rate from option prices for June 2004 as expected by market participants on May 5, 2004. He finds that the market was constituted with two distinct groups of traders. One was expecting an appreciation of the dollar with respect to the euro and one was anticipating a depreciation of the dollar against the euro. Such a situation with the presence of two groups having distinct beliefs can occur during a transition period for the market when some market participants change their beliefs regarding the asset while others keep their beliefs. Our paper can be viewed in that way when we analyze the case where traders and market makers do not have the same beliefs.

As said earlier, we interpret the misperception of the variance of prior information as being the better/lower than average (positive illusions). This misperception together with the overconfident bias were introduced by Odean (1998b), however no psychological interpretation of a known bias was given to that misperception.[4] In addition, we introduce the misperception of the mean of prior information which extends both Kyle and Wang (1997) and Odean (1998b) and enables us to have a more complete parameterization of irrationality. We obtain new results showing the importance of the misperception of the mean onto the level of price, and onto the expected profits of all traders and market makers. We also show that only the ratio of the miperceptions of variances matter for

[4] Hilton (2007) also interprets that misperception as being a representation of the positive illusions bias.

studying irrationality.[5]

Overconfidence or miscalibration has been the focus of an abundant literature in Finance. Most of that literature predicts that overconfident investors trade to their disadvantage. In other words, overconfident investors fare worse than their rational counterpart [Odean (1998b), Gervais and Odean (2001), Caballé and Sákovics (2003), Biais et al. (2004) among others]. However, Kyle and Wang (1997) and Benos (1998) find that moderately overconfident traders may earn larger expected profit than rational ones. Moreover, a common finding to all these papers except Caballé and Sákovics (2003) is that trading volume, and price volatility increase with the level of overconfidence. All these papers differ from ours as none of them consider the possibility of irrational price setters and the misperception of the mean. By considering irrational market makers, we show that trading volume and price volatility may not be monotonic functions of the traders' level of relative confidence. The non-monotonicity of the trading volume is also obtained when traders misperceive the mean of prior information.

Odean (1998b) is the only other paper considering irrational liquidity suppliers, this is done in a Grossman-Stiglitz setting. The risk averse suppliers of liquidity can buy costly information and are all overconfident about private information independently of having acquired or not some information. He finds that overconfident informed liquidity suppliers are outperformed by overconfi-

[5] Both Régner et al. (2004) and Glaser and Weber (2007) show the importance of distinguishing overconfidence from positive illusions.

dent uninformed ones. Being overconfident, too many traders acquire information leading to lower expected utilities for informed traders. Different to Odean (1998b), we show that an irrational market maker can, in expected terms, fare better than a rational one.

There is a large body of evidence in the cognitive psychology literature establishing that people may display, among others, two psychological traits: miscalibration (overconfidence/underconfidence) and "positive illusions" (optimism). Both traits have, independently, received a lot of attention. Miscalibration is defined as the tendency for people to overestimate the precision of their knowledge. Ito (1990) shows its occurrence for participants in the foreign exchange market (for more evidences see Odean (1998b) and Hilton (2001)). The latter bias has been documented in Taylor and Brown (1988, 1994), Sutherland (1992) for the better-than-average effect, Langer (1975), Harris and Middleton (1994), Hoorens (2001) for the illusion of control and Weinstein (1980) for the unrealistic optimism.

Financial practitioners are also well aware of the existence of such psychological traits for investors trading in financial markets. Indeed, they interchangeably use optimistic market for bullish market and pessimistic market for bearish market. The extent of such traits is of great concern for financial institutions as they impact the market through the agents' trading decisions. As a result, the Union des Banques Suisses together with Gallup Organization have launched in

October 1996 the *Index of Investor Optimism*.[6][7]

Our model's implications can be related to known empirical regularities. Indeed, it predicts high trading volume for high level of overconfidence. However our model also predicts the possibility of excessive volume due to extreme underconfidence in the market. Trading volume as a function of the level of confidence can display a U-shape. To the best of our knowledge this is the only model where the volume traded exhibits that property. Excessive volume traded is a well known fact [see Dow and Gorton (1997)] and has been explained by the presence of overconfident traders. We also predict that high level of volatility as it may increase with the traders' level of confidence [see Shiller (1980, 1989)]. However, we show that the volatility increases with the market maker's perceived variance of prior information.

The paper unfolds as follows. In the next section, the general model is presented along with the definition of an equilibrium for our model. In section 3, the model is solved for irrational market makers. In section 4, we derive the

---

6

A detailed methodology used to compute the index can be found at www.ubs.com/investoroptimism. The monthly level of the index is also given from its launch date up to now.

7

Other financial institutions also try to assess the market participants' sentiment. For that reason, in September 2006, a survey denominated "Fund Manager Survey" was conducted on behalf of Merrill Lynch.

equilibrium with irrational market makers. The last section summarizes our results and concludes. All proofs are gathered in the appendix.

2 Model

We study a financial market where a market maker and several traders exchange a risky asset whose future value $\tilde{v}$ follows a Gaussian distribution with zero mean and variance σ_v^2.[8] Traders participating in that market can be either informed or uninformed. The uninformed traders are the so-called noise traders and submit a market order which is the realization of a normally distributed random variable $\tilde{u}$ with zero mean and variance σ_u^2. The informed traders are risk neutral and can be one of two types: rational or irrational. N traders are rational whereas M are irrational. Both types of traders have access to private information, i.e. they observe a noisy signal of the future value of the risky asset

$$\tilde{s}_k = \tilde{v} + \tilde{\varepsilon}_k, \text{ with } \tilde{\varepsilon}_k \sim N\left(0, \sigma_\varepsilon^2\right) \quad \forall k = 1, ..., N + M.$$

These two types of traders differ in the beliefs they hold about both the distribution of the risky asset value (prior information) and the noise in the signal received.

The irrational traders display two psychological traits: an optimism/pessimism

[8] This is also called prior information distribution.

bias as well as an overconfident/underconfident one. We define an optimistic (pessimistic) trader as a trader who has an erroneous belief about the mean of the prior information. An optimistic (pessimistic) trader underestimates (overestimates) this mean and mistakenly believes that it has a value of a (with $a > 0$ for an optimistic trader whereas negative for a pessimistic trader). The second psychological trait concerns the beliefs of the variances of the prior information and of the noise of the private signal. More formally, an overconfident/underconfident trader behaves as if his signal, $\tilde{s}_j = \tilde{v} + \tilde{\varepsilon}_j$ for $j = 1, ..., M$, were drawn according to the following two distributions

$$\begin{aligned} \tilde{v} &\sim N\left(a, \kappa_1 \sigma_v^2\right), \\ \tilde{\varepsilon}_j &\sim N\left(0, \kappa_2 \sigma_\varepsilon^2\right). \end{aligned}$$

The parameter $\frac{1}{\kappa_1}$ embodies the "better than average" effect. When $\kappa_1 < 1$ (> 1) the trader believes that he is "better (lower)-than-average". The parameter κ_2 is the miscalibration parameter: $\kappa_2 < 1$ denotes a purely overconfident trader whereas $\kappa_2 > 1$ denotes a purely underconfident one. Whenever $\kappa_1 = \kappa_2 = 1$, the irrational trader does not misperceive both variances. All irrational traders participating in the market are of the same type, i.e. they misperceive the mean and the two variances in the same way.

The term irrational trader refers to a trader who displays all of the biases. However, that trader rationally anticipates the behavior of both the market maker and the remaining informed traders.

The strategy of each rational trader i is a Lesbegue measurable function, $X_i : \Re \to \Re$, such that $\tilde{x}_i = X_i(\tilde{s}_i)$ for $i = 1, ..., N$. The strategy of each irrational trader j is identically defined: $X_j : \Re \to \Re$ such that $\tilde{x}_j = X_j(\tilde{s}_j)$ for $j = 1, ..., M$.

Finally, the market maker is risk neutral and behaves competitively. She observes the aggregate order flow $\tilde{y} = \sum_{i=1}^{N} \tilde{x}_i + \sum_{j=1}^{M} \tilde{x}_j + \tilde{u}$ before setting the price $\tilde{p}$. Let $P : \Re \to \Re$ denotes a measurable function such that $\tilde{p} = P(\tilde{y})$.

The trading protocol is identical to Kyle (1985).

We now give the definition of an equilibrium for our model.

Definition $\left(X_1^r, ..., X_N^r, X_1^{ir}, ..., X_M^{ir}, P\right)$ *is an equilibrium if the price set by the market marker is such that*

$$\tilde{p} = E\left[\tilde{v} \middle| \tilde{y}\right],$$

and, given that price, the market orders maximize the traders' expected profit conditional on the information received

$$X_i \in \arg\max_{x_i \in \Re} E\left[\left(\tilde{v} - P(\tilde{y})\right) x_i \middle| s = s_i\right] \quad \forall i = 1, ..., N,$$

and

$$X_j \in \arg\max_{x_j \in \Re} E^{ir}\left[\left(\tilde{v} - P(\tilde{y})\right) x_j \middle| s = s_j\right] \quad \forall j = 1, ..., M.$$

The operator E^{ir} denotes the fact that the expectation for the irrational traders is computed given their beliefs about the expectation of the risky asset.

It should be pointed out that all traders know their type. Moreover, all agents know the number of rational and irrational traders as well as the type of the irrational traders. Traders behave strategically meaning that they take into account the impact of their orders onto the price.

3 Rational Market Makers

In this section we derive the equilibrium where the irrational traders have the incorrect beliefs specified above. Let us define $\tau = \frac{\sigma_s^2}{\sigma_v^2}$.

We now characterize the equilibrium.

Proposition 1 *Whenever*

$$M\frac{\kappa_1}{\kappa_2}\left(1+2\tau\right)^2\left[\frac{\kappa_1}{\kappa_2}\left(1-\tau\right)+2\tau\right]+N\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)^2\left(1+\tau\right)\geq 0, \qquad (1)$$

there exists a unique equilibrium of the following form

$$\begin{aligned} x_i^r &= \beta^r s_i, \quad \forall i=1,...,N, \\ x_j^{ir} &= \alpha^{ir}+\beta^{ir}s_j, \quad \forall j=1,...,M, \end{aligned}$$

$$p=\mu+\lambda y=\mu+\lambda\left(\sum_{i=1}^{N}x_i^r+\sum_{j=1}^{M}x_j^{ir}+u\right),$$

where the coefficients are given in the appendix.

The irrational traders outperform the rational traders if and only if

$$\left(\frac{\kappa_1}{\kappa_2}-1\right)\left(\frac{\kappa_1}{\kappa_2}\left(1-2\tau^2\right)+2\tau\left(1+\tau\right)\right)>0.$$

Only moderately overconfident traders ($\frac{\kappa_1}{\kappa_2}>1$) *can outperform rational traders.*

Proof. See Appendix. ■

It should be pointed out that in the previous result only the ratio $\frac{\kappa_1}{\kappa_2}$ matters. A trader who believes that he is less confident (whether under- or over-) than he is "better/lower-than-average", i.e. $\kappa_1<\kappa_2$, behaves as if underconfident. On the contrary, a trader believing that he is more confident than he is "better/lower-than-average" behaves as if overconfident. By taking that into account in the previous proposition, an over (under) confident trader is re-defined as a trader with $\frac{\kappa_1}{\kappa_2}>1$ ($\frac{\kappa_1}{\kappa_2}<1$). The ratio $\frac{\kappa_1}{\kappa_2}$ is then called the relative level of confidence.

This equilibrium is qualitatively the same as in Kyle and Wang (1997). However we extend their model in two directions, we have an oligopoly framework and we introduce the erroneous beliefs regarding the variance and the mean of prior information. The latter implies the drift, μ, of the price function. The presence of optimistic traders induces an inflated order flow which is corrected by the market maker by setting a lower price (negative drift). The converse is true when pessimistic traders are present.

The intuition of the proposition is the same as in Kyle and Wang (1997).

The equilibrium does not exist if condition (1) is not satisfied. This corresponds to the case where the market maker would like to supply infinite liquidity and traders would like to submit unbounded trading volume.

The expected profits are computed under the true distributions of $\tilde{v}$ and $\tilde{\varepsilon}$. We find as in Kyle and Wang (1997) that when an irrational trader is able to credibly commit to trade a large quantity, this trader can outperform the rational trader. This only happens for a moderately overconfident trader. An underconfident investor can never commit to trade large quantities and therefore never outperforms a rational trader. The level of optimism or pessimism, a, does not impact the expected profit. This is due to two reasons. The first one being that a is independent of any relevant information for the market maker. The second one comes from the fact that the market maker perfectly evaluates the part of the order flow coming from the misperception of the mean and therefore rightly correct for it when setting the price.

4 Irrational Market Makers

We now look at the case where the market makers are irrational as well as M traders among the $M + N$ traders.

The irrational traders misperceive the distributions of both $\tilde{v}$ and $\tilde{\varepsilon}_j$ as before. Given the fact that each market maker has no access to any private signal, she misperceives the expectation and variance of the distribution of prior information. Each market maker believes that the distribution of the asset is such that

$$\tilde{v} \to N\left(\bar{a}, K\sigma_v^2\right).$$

We denote $\bar{\kappa} = \frac{1}{K}$ the parameter of irrationality in variance of the market maker. Its effect on the variance is then more comparable to the effect of $\frac{\kappa_1}{\kappa_2}$ for the informed trader. We interpret the parameter $\bar{\kappa}$ as the "better/lower than average" effect. In the remainder of the paper we interchangeably use "variance optimistic" market maker for a "better-than-average" market maker and "variance pessimistic" market maker for a "lower-than-average" market maker.

Let us define a "variance optimistic" or a "better-than-average" market maker as a market maker who believes that the variance of prior information is smaller than it actually is, i.e. $\bar{\kappa} > 1$. A variance pessimistic or a "lower-than-average" market maker believes that the variance is larger, i.e. $\bar{\kappa} < 1$. As before, an optimistic market maker believes that $\bar{a} > 0$. All irrational market makers are assumed to be of the same type. Finally, they behave competitively.

Proposition 2 *Whenever*

$$M\frac{\kappa_1}{\kappa_2}(1+2\tau)^2\left[\frac{\kappa_1}{\kappa_2}\left(\tfrac{1}{\bar{\kappa}}-\tau\right)+2\tau\right]+N\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)^2\left[(2\tau+1)\tfrac{1}{\bar{\kappa}}-\tau\right]\geq 0, \quad (2)$$

there exists a unique linear equilibrium of the following form:

$$\begin{aligned} x_i^r &= \alpha^r + \beta^r s_i, \quad \forall i = 1, ..., N, \\ x_j^{ir} &= \alpha^{ir} + \beta^{ir} s_j, \quad \forall j = 1, ..., M, \end{aligned}$$

$$p = \mu + \lambda y = \mu + \lambda \left(\sum_{i=1}^{N} x_i^r + \sum_{j=1}^{M} x_j^{ir} + u \right).$$

where the coefficients are given in the appendix.

The irrational traders outperform the rational traders if and only if

$$\sigma_v^2 \left(\frac{\kappa_1}{\kappa_2} - 1 \right) \left(\frac{\kappa_1}{\kappa_2} \left(1 - 2\tau^2\right) + 2\tau \left(1 + \tau\right) \right) > \bar{a} a \left(\frac{\kappa_1}{\kappa_2} + 2\tau \right) \left(2\tau + 1\right)^2. \quad (3)$$

Whenever the irrational traders and the market maker hold sufficiently different beliefs about the mean of prior information (either $a > 0$ and $\bar{a} < 0$ or $a < 0$ and $\bar{a} > 0$) an underconfident trader can outperform a rational trader.

Proof. See Appendix. ■

We first comment on condition (2). From this expression, one can see that a variance optimistic market maker exacerbates the occurrence of the non-existence of an equilibrium whereas a pessimistic one alleviates it. This can be explained as follows. A variance optimistic market maker thinks that prior information is more precise than it is and therefore believes that private information is less substantial than it actually is. As a consequence, she adjusts her price less aggressively and increases market depth. Informed traders respond by trading more intensely, implying that the non-existence of equilibrium is more likely to occur. The exact opposite effects take place for a variance pessimistic market maker and therefore explains the fact that the non-existence of equilibrium is less likely to occur in that case.

We now turn to the effect of the market maker's mean misperception onto the level of price. As before the market maker rationally anticipates the presence of irrational traders. She reduces (increases) the overall level of prices due to an inflated (reduced) order flow when optimistic (pessimistic) traders are present. However, this drift of the price function is now affected by her own beliefs of mean of prior information.[9] The combination of the two effects determine the size and the sign of the drift. For expositional clarity we focus on the case where the market maker is optimistic. The pessimistic case is symmetric. An optimistic market maker increases the overall level of price as she wrongly believes that the expectation of the risky asset is higher than it actually is. The traders respond by reducing the size of their market order proportionally to the misperception $\bar{a}$. However, the effect of $\bar{a}$ on the level of prices can be mitigated by the effect of the trader's misperception of the expectation. Whenever the market maker and the irrational traders hold opposite beliefs about the mean of prior information, the trading of the irrational trader has the same effect on the price function as the market maker's mean misperception. This leads to unambiguous shifts of the price function (positive shift with an optimistic market maker). When the market maker and the irrational traders hold the same beliefs, the effect of the irrational trader's trading counteracts the beliefs of the market maker. If the market maker's misperception is high enough, its effect dominates and the price function increases with an optimistic market

[9] When the market maker does not misperceive the mean, the drift is equal to the drift obtained in proposition 1.

maker. If conversely, the irrational traders' misperception is high enough, the price function decreases.

The price sets by the market maker incorporates her irrationality possibly leading to an irrational bubble. Indeed, when the market maker is irrational and due to her misperception of the risky asset's mean, she increases the overall level of prices. This can be understood as the early stage of a price bubble. However, our model being static, we cannot perform a dynamic analysis of that bubble.

The market maker, when rational ($\bar{\kappa} = 1$, $\bar{a} = 0$), obtains zero expected profit. However, when she is irrational, she may obtain an expected profit different from zero. A decrease of $\bar{\kappa}$ has several effects on the market maker's expected profit. A decrease of $\bar{\kappa}$ increases λ which in turn decreases the aggregate order flow. The magnitude of these countervailing effects determine its overall effect on the market maker's expected profit. The effect of the misperception of the mean depends on the sign and magnitude of a and $\bar{a}$. If the market maker and the traders hold opposite beliefs the expected profit is decreased (the second term is negative). If they are both pessimistic or optimistic, the market maker's expected is either increased or decreased.

The following table summarizes, when clear, how an irrational market maker performs overall

$E\left[\Pi^{MM}\right]$	$\bar{\kappa} > 1$		$\bar{\kappa} < 1$	
	$\bar{a} > 0$	$\bar{a} < 0$	$\bar{a} > 0$	$\bar{a} < 0$
$a > 0$	> 0 or < 0	$E\left[\Pi^{MM}\right] < 0$	> 0 or < 0	> 0 or < 0
$a = 0$	$E\left[\Pi^{MM}\right] < 0$	$E\left[\Pi^{MM}\right] < 0$	> 0 or < 0	> 0 or < 0
$a < 0$	$E\left[\Pi^{MM}\right] < 0$	> 0 or < 0	> 0 or < 0	> 0 or < 0

Irrational Market Maker's Expected Profit.

When the market maker is variance pessimistic and for any values of the mean's misperception, the market maker's expected profit can either be positive or negative. This implies that an irrational market maker can fare better than a rational one.

A question of interest is to know whether the irrational traders can outperform the rational traders.

When $\bar{a} = 0$, condition (3) is equivalent to the condition found in proposition 1 implying that the irrational traders earn on average more than the rational traders. Given that the market marker is also irrational, this condition depends now on both the market maker's and the irrational trader's misperception about the mean, and on the irrational trader's beliefs about the variances. It should be pointed out that the condition is independent of $\bar{\kappa}$. Indeed $\bar{\kappa}$ affects the market depth only and has the same effect for the rational and irrational traders.

An irrational trader can outperform a rational trader if he can successfully commit to trade large quantities. Due to the form of the irrational trader's market order, this can be achieved via two variables: α^{ir} and β^{ir}. As already explained in section 3, only a moderately overconfident trader can credibly commit to trade a larger quantity ($\beta^{ir} > \beta^{r}$).[10] In addition, beliefs about the mean of prior information of the market maker and the irrational trader only affect α^{ir}. Contrary beliefs about the mean imply that the irrational trader can commit to trade a larger quantity than his rational counterpart ($\alpha^{ir} > \alpha^{r}$). The greater the disagreement, the larger the quantity.

Given that a moderately overconfident trader can, using only β^{ir}, credibly commit to trade large quantities adding a second variable which increases the quantity traded exacerbates the credibility of the commitment. This leads less moderately overconfident traders to be able to successfully commit to trade large quantities.

An underconfident trader trades less intensely on private information than a rational trader ($\beta^{ir} < \beta^{r}$), implying the result of section 3 that an underconfident trader cannot outperform a rational trader. However, in the current setting, an underconfident trader can increase the quantity traded via his trading due to the misperception of the mean and his response to the price drift (α^{ir}). If the market maker and the irrational trader have sufficiently different beliefs concerning the mean of prior information, the increase in quantity due to α^{ir} can compensate for a lower β^{ir} leading to a credible commitment for an underconfident trader.

[10] In the previous section the irrational trader could not commit to trade larger quantities via α^{ir}. Indeed a rational market maker anticipates the part of the aggregate order flow due to this component and correct for it when setting the price.

Lemma (Price Efficiency and Ex-ante Volatility)

- *The ex-ante volatility is equal to*

$$var\left(p\right)=\frac{\sigma_v^2\left(N\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)+M\frac{\kappa_1}{\kappa_2}(2\tau+1)\right)\left(\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)\left(N+\frac{1}{\bar{\kappa}}(2\tau+1)\right)+M\frac{\kappa_1}{\kappa_2}(2\tau+1)\right)}{\left((N+2\tau+1)\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)+M\frac{\kappa_1}{\kappa_2}(2\tau+1)\right)^2}.$$

  It decreases with $\bar{\kappa}$. It can increase, decrease or be non-monotonic (*initially increasing and then decreasing*) with *the level of relative confidence* ($\frac{\kappa_1}{\kappa_2}$).

- *The price efficiency is given by*

$$var\left(v|\,p\right)=\frac{\sigma_v^2\frac{1}{\bar{\kappa}}\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)(1+2\tau)}{\left(N+\frac{1}{\bar{\kappa}}(2\tau+1)\right)\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)+M\frac{\kappa_1}{\kappa_2}(2\tau+1)}.$$

  It decreases with $\bar{\kappa}$ and decreases with the level of relative confidence ($\frac{\kappa_1}{\kappa_2}$).

Proof. Straightforward. ■

Both the ex-ante volatility and the price efficiency decrease with $\bar{\kappa}$. This implies that both are lower with an optimistic market maker than with a pessimistic one. As described before, an optimistic (pessimistic) market maker sets prices less (more) aggressively by increasing (decreasing) liquidity, traders respond to it by increasing (decreasing) their trading intensity. For the ex-ante volatility, the effect on the market depth dominates the effects on the trading intensity. Regarding now the price efficiency, on the one hand an increase of $\bar{\kappa}$ increases the traders' trading intensity and on the other hand it decreases volatility.

Testable implications and numerical application

The objective of this part is to show testable evidence of irrationality. Our model already provides several implications of irrationality, on the price function for example. We now explore the effect of each of the parameters defining the irrationality on the trading volume.

Result (Trading Volume)

Let us define $\tilde{z} = \sum_{i=1}^{N} |\tilde{x}_i^r| + \sum_{j=1}^{M} \left|\tilde{x}_j^{ir}\right| + |\tilde{u}|$. *The trading volume is given by*

$$
\begin{aligned}
E[\tilde{z}] &= N\alpha^r \operatorname{erf}\left(\frac{\overline{a}(2\tau+1)}{\sigma_v\sqrt{2(1+\tau)}}\right) + M\alpha^{ir} \operatorname{erf}\left(\frac{2\tau a - \overline{a}\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)}{\frac{\kappa_1}{\kappa_2}\sigma_v\sqrt{2(1+\tau)}}\right) \\
&\quad + \sqrt{\frac{2}{\pi}}\left\{\sigma_u + N\sigma_r \exp\left(-\frac{(\overline{a}(2\tau+1))^2}{\sigma_v^2 2(1+\tau)}\right) + M\sigma_{ir}\exp\left(-\frac{\left(2\tau a - \overline{a}\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)\right)^2}{2\sigma_v^2\left(\frac{\kappa_1}{\kappa_2}\right)^2(1+\tau)}\right)\right\},
\end{aligned}
$$

where $\operatorname{erf}(x) = \frac{2}{\pi}\int_0^x \exp\left(-t^2\right) dt$.

Proof. See Appendix. ■

It is a well known fact that overconfidence leads to greater volume traded (see Odean (1998b), Gervais and Odean (2001), Kyle and Wang (1997) and Benos (1998) among others). We also find that result when the market maker is assumed rational ($\bar{a} = 0$, $\bar{\kappa} = 1$) and when the trader does not misperceive the mean ($a = 0$). As a result the trading volume with underconfident traders is lower than that with overconfident traders. This is shown in the following figure.

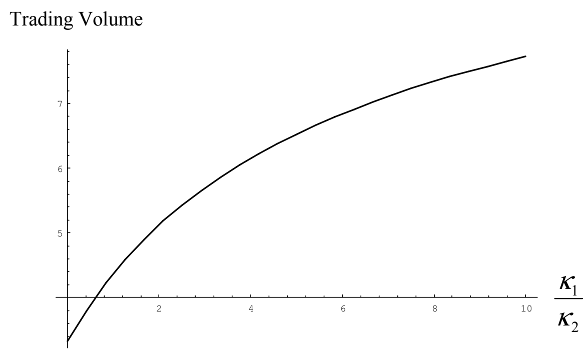

Figure 1: Expected volume as a function of the relative level of overconfidence when the market maker is rational ($\bar{a} = 0$ and $\bar{\kappa} = 1$) and $a = 0$. The simulations are done with $\sigma_v^2 = \sigma_\varepsilon^2 = \sigma_u^2 = 1$ and $M = N = 10$.

However when either $\bar{a} \neq 0$, or $a \neq 0$ or $\bar{\kappa} \neq 1$, this result is dramatically changed. Our model predicts high level of volume traded for extreme level of relative confidence. We obtain that the trading volume can be a non-monotonic function (U-shaped) of the relative level of confidence. As the relative level of confidence decreases the volume can increase. When $\bar{a} \neq 0$, or $a \neq 0$, traders trade against the drift ($\bar{a} \neq 0$), i.e. increase their trading if the drift is negative (lower prices), or trade on the misperception of the mean ($a \neq 0$). The effect of both $\bar{a}$ and a is symmetric. Both situations imply an increase of the volume traded. If this increase is greater than the reduction in volume due to the fact that traders trade less intensely on private information, we obtain the result highlighted in figure 2 below.

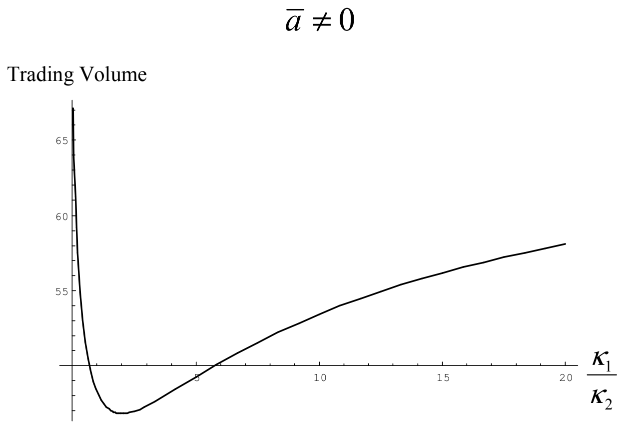

Figure 2: Expected volume as a function of the relative level of overconfidence when $\bar{\kappa} = 1$ and $a = 0$. The simulations are done with $\sigma_v^2 = \sigma_\varepsilon^2 = \sigma_u^2 = 1$ and $M = N = 10$.

As explained before a higher $\bar{\kappa}$ implies more trading, if this overtrading is large enough it can result in large volume traded with underconfident traders (see figure 3 below).

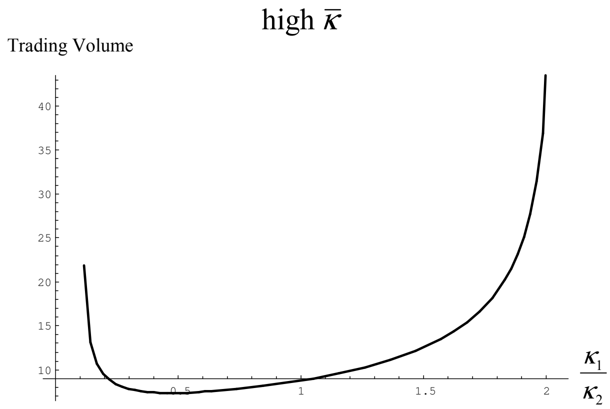

Figure 3: Expected volume as a function of the relative level of overconfidence when $\bar{a} = 0$ and $a = 0$. The simulations are done with $\sigma_v^2 = \sigma_\varepsilon^2 = \sigma_u^2 = 1$ and $M = N = 10$.

To the best of our knowledge, our model is the first one to show that excessive volume can be a result of underconfidence in the market, and that irrationality in mean can impact the trading volume..

The question whether this excessive observed volume is caused by overconfidence or underconfidence in the market could be tested empirically as some indices such as the Index of Investor Optimism or the "Fund Manager Survey" by Merrill Lynch measures the level of confidence in the market. An experimental approach such as the one of Bloomfield et al. (2000) could be also be used to test our model.

5 Conclusion

We develop a model of financial markets where irrational traders along with rational traders trade a risky asset with a market maker. We model irrational traders as traders who, as well as misperceiving the expected returns of the asset, misperceive the variance of both the volatility of the asset returns and the noise in the private signal. As a consequence those traders display different psychological traits: a pessimistic/optimistic one (misperception of the mean), an underconfident/overconfident one (misperception of the variance of the noise in the private information) and the better/lower-than-average effect (misperception of the variance of prior information).

We study two scenarios, in the first one market makers are rational whereas in the second one, they are irrational. In scenario 1, our results are qualitatively the same as in Kyle and Wang (1997). Nevertheless, we extend their setting in two different directions (i) our model deals with an oligopoly framework and, (ii) we allow traders to have erroneous beliefs about the mean and variance of prior information.

When the market maker is irrational, we show that a variance optimistic market maker exacerbates the non-existence of equilibrium whereas a variance pessimistic market maker alleviates it. The introduction of an irrational market maker affects differently the traders. Rational traders have larger expected profits when the market maker is irrational. The impact on the irrational traders' expected profit is not as clear cut. Irrational traders can have lower or greater expected profits due to the introduction of an irrational market maker. We show that a moderately underconfident trader can outperform a rational trader. This is true for an optimistic or pessimistic trader. The necessary condition to obtain that result is that the irrational traders and the market maker must hold opposite beliefs about the mean of prior information. This is a striking and new result. Moreover, we also show that an irrational market maker can in expected terms fare better than a rational one. Odean (1998b) is the other paper considering irrational suppliers of liquidity. He looks at the case where all suppliers are overconfident and shows that informed suppliers are outperformed by uninformed ones. We show that both the volume traded and the volatility might be non-monotonic functions of the traders' relative level of confidence.

Our model predicts high level of volume traded for extreme level of confidence including the case where traders are underconfident. This result raises the question whether the observed high volume is due to overconfidence or to underconfidence. This question could be tested empirically as some indices such as the Index of Investor Optimism or the "Fund Manager Survey" by Merrill Lynch measures the level of confidence in the market. An experimental approach could be also be used to test our model. Our model also predicts high level of volatility.

An interesting extension of the model would be to look at how the results obtained in the present model would be modified in a dynamic setting. A further extension of the present model could include different forms of irrationality. This is left for future research.

6 Appendix

Proof of Proposition 1 (Rational Market Makers)

1. **Equilibrium:** Given the expressions of the market orders submitted by the irrational traders, x^{ir}, and by the rational traders, x^r, the aggregate order flow is equal to

$$y = \sum_{i=1}^{N} x_i^r + \sum_{j=1}^{M} x_j^{ir} + u = \left(N\beta^r + M\beta^{ir}\right) v + \beta^r \sum_{i=1}^{N} \varepsilon_i + \beta^{ir} \sum_{j=1}^{M} \varepsilon_j + N\alpha^r + M\alpha^{ir} + u.$$

The irrational trader maximizes his conditional expected profit

$$\max_{x_j^{ir}} E^{ir}\left(\left(v-p\right) x_j^{ir} \middle| s = s_j\right).$$

Substituting the form of the price as well as the market orders from for the N rational traders, and the $M-1$ irrational traders in the above expression, computing the first order condition and solving it for the market order, we obtain

$$\begin{aligned} x_j^{ir} &= \frac{1}{2\lambda}\left[E^{ir}\left(v \middle| s = s_j\right)\left(1 - (M-1)\lambda\beta^{ir} - N\lambda\beta^r\right)\right. \\ &\quad \left. -\mu - (M-1)\lambda\alpha^{ir} - N\lambda\alpha^r\right]. \end{aligned} \tag{4}$$

We now need to compute $E^{ir}\left(v|\, s=s_j\right)$. On one side and given the normality of the random variables we have that

$$E^{ir}\left(v|\, s=s_j\right)=\gamma\left(s_j-E^{ir}\left(v\right)\right)+E^{ir}\left(v\right)$$

with $\gamma=\frac{cov^{ir}(v,s_j)}{var^{ir}[s_j]}$. Given that $E^{ir}\left(v\right)=a$ and $\tau=\frac{\sigma_\varepsilon^2}{\sigma_v^2}$, we obtain

$$E^{ir}\left(v|\, s=s_j\right)=\frac{\kappa_1}{\kappa_1+\kappa_2\tau}s_j+a\frac{\kappa_2\tau}{\kappa_1+\kappa_2\tau}.$$

Replacing the expression of the conditional expectation into the form of the order (4) and identifying the parameters we have

$$\beta^{ir} = \frac{\kappa_1\left(1-\lambda N\beta^r\right)}{\lambda\left(\left(M+1\right)\kappa_1+2\kappa_2\tau\right)}, \tag{5}$$

$$\alpha^{ir} = \frac{1}{\lambda\left(M+1\right)}\left[a\frac{\kappa_2\tau}{\kappa_1+\kappa_2\tau}\left(1-\left(M-1\right)\lambda\beta^{ir}-N\lambda\beta^r\right)\right. \tag{6}$$
$$\left.-\mu-\lambda N\alpha^r\right].$$

The second order condition is satisfied.

Finally, the rational investor maximizes his conditional expected profit. Given his first order condition and the fact that $E\left(v|\, s=s_i\right)=\frac{s_i}{1+\tau}$, the parameters for the rational investor's market order are such that

$$\beta^r = \frac{\left(1-\lambda M\beta^{ir}\right)}{\lambda\left(N+1+2\tau\right)}, \tag{7}$$

$$\alpha^r = -\frac{1}{\lambda\left(N+1\right)}\left[\mu+\lambda M\alpha^{ir}\right]. \tag{8}$$

The second order condition is satisfied.

The market maker behaves competitively and sets a price such that

$$p = E\left[v|\, y\right] = 0 + \frac{cov\left(v, y\right)}{var\left[y\right]}\left(y - E\left(y\right)\right).$$

Given the expression of the aggregate order flow, the parameters of the price schedule are given by

$$\lambda = \frac{\left(N\beta^{r} + M\beta^{ir}\right)}{\left(N\beta^{r} + M\beta^{ir}\right)^{2} + \left(N\left(\beta^{r}\right)^{2} + M\left(\beta^{ir}\right)^{2}\right)\tau + \frac{\sigma_{u}^{2}}{\sigma_{v}^{2}}}, \tag{9}$$

$$\mu = -\lambda\left(N\alpha^{r} + M\alpha^{ir}\right). \tag{10}$$

Solving the above system of six equations defined by equations (5), (6), (7), (8), (9), and (10) for the six unknowns leads to

for the irrational traders

$$\alpha^{ir} = \frac{2(1+2\tau)\tau\sigma_{u}}{\sigma_{v}\sqrt{M\frac{\kappa_{1}}{\kappa_{2}}(1+2\tau)^{2}\left[\frac{\kappa_{1}}{\kappa_{2}}(1-\tau)+2\tau\right]+N\left(\frac{\kappa_{1}}{\kappa_{2}}+2\tau\right)^{2}(1+\tau)}}a$$

$$\beta^{ir} = \frac{\frac{\kappa_{1}}{\kappa_{2}}(1+2\tau)\sigma_{u}}{\sigma_{v}\sqrt{M\frac{\kappa_{1}}{\kappa_{2}}(1+2\tau)^{2}\left[\frac{\kappa_{1}}{\kappa_{2}}(1-\tau)+2\tau\right]+N\left(\frac{\kappa_{1}}{\kappa_{2}}+2\tau\right)^{2}(1+\tau)}};$$

for the rational traders

$$\beta^{r} = \frac{\left(\frac{\kappa_{1}}{\kappa_{2}}+2\tau\right)\sigma_{u}}{\sigma_{v}\sqrt{M\frac{\kappa_{1}}{\kappa_{2}}(1+2\tau)^{2}\left[\frac{\kappa_{1}}{\kappa_{2}}(1-\tau)+2\tau\right]+N\left(\frac{\kappa_{1}}{\kappa_{2}}+2\tau\right)^{2}(1+\tau)}};$$

for the market maker

$$\begin{aligned}
\mu &= -\frac{2M(1+2\tau)\tau}{(2\tau+N+1)\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)+M\frac{\kappa_1}{\kappa_2}(2\tau+1)}a, \\
\lambda &= \frac{\sigma_v\sqrt{M\frac{\kappa_1}{\kappa_2}(1+2\tau)^2\left[\frac{\kappa_1}{\kappa_2}(1-\tau)+2\tau\right]+N\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)^2(1+\tau)}}{\sigma_u\left[(2\tau+N+1)\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)+M\frac{\kappa_1}{\kappa_2}(2\tau+1)\right]}.
\end{aligned}$$

2. **Expected Profits:** The expected profit of any trader, $h = ir$ or r, can be written as

$$E\left(\Pi^h\right) = E\left((v-p)\,x^h\right) = E\left((v-\mu-\lambda y)\left(\beta^h\left(v+\varepsilon_h\right)+\alpha^h\right)\right).$$

Given the expression of y and after simplification of some of the terms, the expected profit is equal to

$$\begin{aligned}
E\left(\Pi^h\right) &= E\left[\left(v-\lambda\left(\left(N\beta^r+M\beta^{ir}\right)v+\beta^r\sum_{i=1}^{N}\varepsilon_i+\beta^{ir}\sum_{j=1}^{M}\varepsilon_j+u\right)\right)\right. \\
&\qquad \left.\times\left(\beta^h\left(v+\varepsilon_h\right)+\alpha^h\right)\right].
\end{aligned}$$

All random variables are independent and have a zero mean, we therefore get

$$E\left(\Pi^h\right) = E\left(v^2\beta^h\left(1-\lambda\left(N\beta^r+M\beta^{ir}\right)\right)-\lambda\beta^h\varepsilon^h\left(\beta^r\sum_{i=1}^{N}\varepsilon_i+\beta^{ir}\sum_{j=1}^{M}\varepsilon_j\right)\right).$$

This expression simplifies to

for the irrational trader

$$E\left(\Pi^{ir}\right)=\beta^{ir}\left[\sigma_v^2\left(1-\lambda\left(N\beta^r+M\beta^{ir}\right)\right)-\lambda\beta^{ir}\sigma_\varepsilon^2\right],$$

and for the rational trader

$$E\left(\Pi^{h}\right)=\beta^{r}\left[\sigma_v^2\left(1-\lambda\left(N\beta^r+M\beta^{ir}\right)\right)-\lambda\beta^{r}\sigma_\varepsilon^2\right].$$

Using the expressions of β^r, and β^{ir} and after some simplifications we obtain for each type of traders

$$\begin{aligned}
E\left(\Pi^{ir}\right) &= \frac{\sigma_v^2\left(1+2\tau\right)^2\frac{\kappa_1}{\kappa_2}\left[\frac{\kappa_1}{\kappa_2}\left(1-\tau\right)+2\tau\right]}{\lambda\left(\left(2\tau+N+1\right)\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)+M\frac{\kappa_1}{\kappa_2}\left(2\tau+1\right)\right)^2},\\
E\left(\Pi^{r}\right) &= \frac{\sigma_v^2\left(1+\tau\right)\left[\frac{\kappa_1}{\kappa_2}+2\tau\right]^2}{\lambda\left(\left(2\tau+N+1\right)\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)+M\frac{\kappa_1}{\kappa_2}\left(2\tau+1\right)\right)^2}.
\end{aligned}$$

Having the expression of the expected profit for irrational traders and for rational traders, we can compare them. We compute the difference in expected profits. Given the above expressions, finding the sign of $E\left(\Pi^{ir}\right)-E\left(\Pi^{r}\right)$ is equivalent to finding the sign of

$$\left(1+2\tau\right)^2\frac{\kappa_1}{\kappa_2}\left[\frac{\kappa_1}{\kappa_2}\left(1-\tau\right)+2\tau\right]-\left(1+\tau\right)\left[\frac{\kappa_1}{\kappa_2}+2\tau\right]^2.$$

It is straightforward to prove that the previous expression is equal to

$$2\tau\left(\frac{\kappa_1}{\kappa_2}-1\right)\left[\frac{\kappa_1}{\kappa_2}\left(1-2\tau^2\right)+2\left(1+\tau\right)\tau\right]. \tag{11}$$

Whenever $\tau \leq \frac{1}{\sqrt{2}}$, the expression (11) is of the sign of $\kappa_1 - \kappa_2$ and when $\kappa_1 - \kappa_2 > 0$ (< 0), we have $E\left(\Pi^r\right) < E\left(\Pi^{ir}\right)$ ($E\left(\Pi^{ir}\right) < E\left(\Pi^r\right)$). Whenever $\frac{1}{\sqrt{2}} < \tau$, (11) has two positive roots $\kappa_1 = \kappa_2$ and $\frac{\kappa_1}{\kappa_2} = \frac{2(1+\tau)\tau}{2\tau^2-1}$. One can prove that the latter is always greater than the former. For any κ_2 and for κ_1 in the interval $\left[\kappa_2, \frac{2\kappa_2(1+\tau)\tau}{2\tau^2-1}\kappa_2\right]$ we have $E\left(\Pi^r\right) < E\left(\Pi^{ir}\right)$, for any κ_2 and for κ_1 outside the interval we obtain that $E\left(\Pi^{ir}\right) < E\left(\Pi^r\right)$. Given the expression of the irrational's expected profit, one can see that if $\tau > 1$ and $\kappa_1 > \frac{2\tau}{\tau-1}\kappa_2$, irrational traders earn negative expected profits.

Proof of Proposition 2 (Irrational Market Makers)

1. **Equilibrium:** After maximizing the traders expected utility we get for the different parameters

$$\begin{aligned}
\alpha^{ir} &= -\frac{1}{\lambda\left(M+1\right)}\left[\frac{\kappa_2\tau}{\kappa_1+\kappa_2\tau}\left(1-\lambda\left(M-1\right)\beta^{ir}-\lambda N\beta^r\right)\right],\\
\beta^{ir} &= \frac{\kappa_1\left(1-\lambda N\beta^r\right)}{\lambda\left(\left(M+1\right)\kappa_1+2\tau\kappa_2\right)},\\
\alpha^r &= -\frac{1}{\lambda\left(N+1\right)}\left[\mu+\lambda M\alpha^{ir}\right],\\
\beta^r &= \frac{1-\lambda M\beta^{ir}}{\lambda\left(N+1+2\tau\right)}.
\end{aligned}$$

The market maker sets a price, p, such that

$$p = \bar{E}\left[\tilde{v}\right| y\right] = \bar{E}\left[\tilde{v}\right] + \frac{\overline{cov}\left(\tilde{v}, y\right)}{\overline{var}\left(y\right)}\left(y - \bar{E}\left(y\right)\right),$$

where the upper bar denotes that the expectation, covariance and variance are computed given the wrong beliefs of the market maker.

Given the market maker's additive misperception we obtain

$$\lambda = \frac{\left(M\beta^{ir} + N\beta^{r}\right)\frac{1}{\bar{\kappa}}}{\left(M\beta^{ir} + N\beta^{r}\right)^{2}\frac{1}{\bar{\kappa}} + \left(M\beta^{ir^{2}} + N\beta^{r^{2}}\right)\tau + \frac{\sigma_u^2}{\sigma_v^2}}, \tag{12}$$

$$\mu = \left(1 - \lambda M\beta^{ir} - \lambda N\beta^{r}\right)\overline{a} - \lambda M\alpha^{ir} - \lambda N\alpha^{r}. \tag{13}$$

Solving the above system of six equations with six unknowns leads to

for the irrational traders

$$\alpha^{ir} = \frac{(2\tau+1)\left(2\tau a - \bar{a}\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)\right)\sigma_u}{\sigma_v\sqrt{M\frac{\kappa_1}{\kappa_2}(1+2\tau)^2\left[\frac{\kappa_1}{\kappa_2}\left(\frac{1}{\bar{\kappa}}-\tau\right)+2\tau\right]+N\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)^2\left((2\tau+1)\frac{1}{\bar{\kappa}}-\tau\right)}},$$

$$\beta^{ir} = \frac{\frac{\kappa_1}{\kappa_2}(2\tau+1)\sigma_u}{\sigma_v\sqrt{M\frac{\kappa_1}{\kappa_2}(1+2\tau)^2\left[\frac{\kappa_1}{\kappa_2}\left(\frac{1}{\bar{\kappa}}-\tau\right)+2\tau\right]+N\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)^2\left((2\tau+1)\frac{1}{\bar{\kappa}}-\tau\right)}},$$

for the rational traders

$$\alpha^{r} = -\frac{\bar{a}\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)(2\tau+1)\sigma_u}{\sigma_v\sqrt{M\frac{\kappa_1}{\kappa_2}(1+2\tau)^2\left[\frac{\kappa_1}{\kappa_2}\left(\frac{1}{\bar{\kappa}}-\tau\right)+2\tau\right]+N\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)^2\left((2\tau+1)\frac{1}{\bar{\kappa}}-\tau\right)}},$$

$$\beta^{r} = \frac{\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)\sigma_u}{\sigma_v\sqrt{M\frac{\kappa_1}{\kappa_2}(1+2\tau)^2\left[\frac{\kappa_1}{\kappa_2}\left(\frac{1}{\bar{\kappa}}-\tau\right)+2\tau\right]+N\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)^2\left((2\tau+1)\frac{1}{\bar{\kappa}}-\tau\right)}},$$

for the market maker

$$
\begin{aligned}
\mu &= \frac{(2\tau+1)\left[\bar{a}\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)(M+N+1)-2M\tau a\right]}{M\frac{\kappa_1}{\kappa_2}(2\tau+1)+(2\tau+N+1)\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)}, \\
\lambda &= \frac{\sigma_v\sqrt{M\frac{\kappa_1}{\kappa_2}(1+2\tau)^2\left[\frac{\kappa_1}{\kappa_2}\left(\frac{1}{\bar{\kappa}}-\tau\right)+2\tau\right]+N\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)^2\left((2\tau+1)\frac{1}{\bar{\kappa}}-\tau\right)}}{\left[M\frac{\kappa_1}{\kappa_2}(2\tau+1)+N\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)+(2\tau+1)\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)\right]\sigma_u}.
\end{aligned}
$$

2. **Expected Profits** Follow the same steps as in proposition 2 for the expected profits of the traders.

The market maker's expected profit are equal to

$$
E\left[\Pi^{MM}\right] = -NE\left(\Pi^{r}\right) - ME\left(\Pi^{ir}\right) + E\left(\Pi^{Liq}\right).
$$

It is straightforward to show that the expected profit of the liquidity traders, $E\left(\Pi^{Liq}\right)$, are equal to $\lambda\sigma_u^2$. Plug the expressions found for the two types of traders and for the liquidity traders into the expression above and after some manipulations, one can get

for the irrational traders

$$
\begin{aligned}
E\left[\Pi^{ir}\right] &= \frac{(2\tau+1)^2}{\lambda d^2}\left[\sigma_v^2\frac{\kappa_1}{\kappa_2}\left(\frac{\kappa_1}{\kappa_2}(1-\tau)+2\tau\right)\right. \\
&\quad \left. -\bar{a}\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)\times\left(2\tau a-\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)\bar{a}\right)\right],
\end{aligned}
$$

for the rational traders

$$
E\left[\Pi^{r}\right] = \frac{\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)^2}{\lambda d^2}\left[\sigma_v^2(\tau+1)+\bar{a}^2(2\tau+1)^2\right],
$$

for the market maker

$$
\begin{aligned}
E\left[\Pi^{MM}\right] &= \frac{(2\tau+1)\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)}{\lambda d^2}\left[\sigma_v^2\left(\frac{1}{\bar{\kappa}}-1\right)\left(M\frac{\kappa_1}{\kappa_2}(2\tau+1)+N\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)\right)\right. \\
&\left.+\bar{a}(2\tau+1)\left(2M\tau a-\bar{a}\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)(M+N)\right)\right],
\end{aligned}
$$

where $d = (2\tau + N + 1)\left(\frac{\kappa_1}{\kappa_2} + 2\tau\right) + M\frac{\kappa_1}{\kappa_2}(2\tau + 1)$. The comparison of the expected profits is straightforward and follows the same steps as in porposition 1.

Proof of Result:

The volume is given by

$$
y = \sum_{i=1}^{N} x_i^r + \sum_{j=1}^{M} x_j^{ir} + u.
$$

Given the form of both x_i^r and x_j^{ir}, we have that they both follow a normal distribution such that

$$
x_i^r \sim N\left(\alpha^r, \sigma_v^2 (\beta^r)^2 (1+\tau)\right) = N\left(\alpha^r, \sigma_r^2\right)
$$

$$
x_j^{ir} \sim N\left(\alpha^{ir}, \sigma_v^2 \left(\beta^{ir}\right)^2 (1+\tau)\right) = N\left(\alpha^{ir}, \sigma_{ir}^2\right).
$$

From Leone et al. (1961) we obtain

$$
\begin{aligned}
E\left[\left|x_i^r\right|\right] &= \sigma_r \sqrt{\tfrac{2}{\pi}} \exp\left(-\tfrac{(\alpha^r)^2}{2\sigma_r^2}\right) + \alpha^r \operatorname{erf}\left(\tfrac{\alpha^r}{\sqrt{2\sigma_r^2}}\right), \\
E\left[\left|x_j^{ir}\right|\right] &= \sigma_{ir} \sqrt{\tfrac{2}{\pi}} \exp\left(-\tfrac{\left(\alpha^{ir}\right)^2}{2\sigma_{ir}^2}\right) + \alpha^{ir} \operatorname{erf}\left(\tfrac{\alpha^{ir}}{\sqrt{2\sigma_{ir}^2}}\right), \\
E\left[|u|\right] &= \sigma_u \sqrt{\tfrac{2}{\pi}},
\end{aligned}
$$

where the function $\operatorname{erf}(x) = \frac{2}{\pi} \int_0^x e^{-t^2} dt$.

The expected volume or trading volume is then defined as

$$
E\left[\sum_{i=1}^{N} \left|x_i^r\right| + \sum_{j=1}^{M} \left|x_j^{ir}\right| + |u|\right] = E\left[|u|\right] + NE\left[\left|x_i^r\right|\right] + ME\left[\left|x_j^{ir}\right|\right].
$$

This leads to the expected volume equal to

$$
\begin{aligned}
&\sqrt{\tfrac{2}{\pi}} \left\{ \sigma_u + N\sigma_r \exp\left(-\tfrac{(\alpha^r)^2}{2\sigma_r^2}\right) + M\sigma_{ir} \exp\left(-\tfrac{\left(\alpha^{ir}\right)^2}{2\sigma_{ir}^2}\right) \right\} \\
&+ N\alpha^r \operatorname{erf}\left(\tfrac{\alpha^r}{\sqrt{2\sigma_r^2}}\right) + M\alpha^{ir} \operatorname{erf}\left(\tfrac{\alpha^{ir}}{\sqrt{2\sigma_{ir}^2}}\right).
\end{aligned}
$$

Given the expressions of α^r, α^{ir}, σ_r^2, and σ_{ir}^2 we obtain for the expected volume

$$
\begin{aligned}
&\sqrt{\tfrac{2}{\pi}} \left\{ \sigma_u + N\sigma_r \exp\left(-\tfrac{(\overline{a}(2\tau+1))^2}{2\sigma_v^2(1+\tau)}\right) + M\sigma_{ir} \exp\left(-\tfrac{\left(2\tau a - \overline{a}\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)\right)^2}{2\sigma_v^2\left(\frac{\kappa_1}{\kappa_2}\right)^2(1+\tau)}\right) \right\} \\
&+ N\alpha^r \operatorname{erf}\left(\tfrac{\overline{a}(2\tau+1)}{\sigma_v\sqrt{2(1+\tau)}}\right) + M\alpha^{ir} \operatorname{erf}\left(\tfrac{2\tau a - \overline{a}\left(\frac{\kappa_1}{\kappa_2}+2\tau\right)}{\frac{\kappa_1}{\kappa_2}\sigma_v\sqrt{2(1+\tau)}}\right).
\end{aligned}
$$

7 Bibliography

Biais, B., D. Hilton, K. Mazurier, and S. Pouget. "Judgemental Overconfidence, Self-Monitoring and Trading Performance in an Experimental Financial Market," *Review of Economic Studies*, Vol. 72, No. 2 (2005).

Benos, A. "Aggressiveness and Survival of Overconfident Traders," *Journal of Financial Markets*, Vol. 1 (1998), 353-383.

Bloomfield, R., R. Libby and M. W. Nelson. "Underreactions, Overreactions and Moderated Confidence," *Journal of Financial Markets*, Vol. 3 (2000) 113-137.

Caballé, J., and J. Sákovics. "Speculating Against an Overconfident Market," *Journal of Financial Markets*, Vol. 6 (2003) 199-225.

Daniel, K., D. Hirshleifer and A. Subrahmanyam. "Investor Psychology and Security Market Under-and Overreactions," *Journal of Finance*, Vol. 53, No. 6 (1998) 1839-1885.

De Long, J. B., A. Shleifer, L. H. Summers and R. J. Waldmann. "Noise Trader Risk in Financial Markets," *Journal of Political Economy*, Vol. 98, No. 4 (1990) 703-738.

Debondt, W., and R. Thaler. "Does the Stock Market Overreact?" *Journal of Finance*, Vol. 40 (1985) 793-805.

Dow, J., and G. Gorton. "Noise Trading, Delegated Portfolio Management, and Economic Welfare," *Journal of Political Economy*, Vol. 105, No. 5 (1997) 1024-1050.

Gervais, S., and T. Odean. "Learning to be Confident," *Review of Financial*

Studies, Vol. 14, No. 1 (2001) 1-27.

Glaser, M., and M. Weber. "Overconfidence and Trading Volume," *Geneva Risk and Insurance Review*, Forthcoming (2007).

Greenwood, R., and S. Nagel. "Inexperienced Investors and Bubbles," Mimeo (2007).

Harris, and Middleton. "The illusion of control and optimism about health: On being less at risk but no more in control than others," *British Journal of Social Psychology,* 33 (1994) 369-386.

Hilton, D. "The Psychology of Financial Decision-Making: Applications to Trading, Dealing and Investment Analysis," *Journal of Psychology and the Financial Markets*, (2001) 37-53.

Hilton, D. "Overconfidence, Trading and Entrepreneurship: Cognitive and Cultural Processes in Risk-taking," in *Contributions to Economic Analysis*, Vol. 280 (2007).

Hoorens, V. "Why do Controllable Events Elicit Stronger Comparative Optimism than Controllable Events," *International Review of Social Psychology (*2001*).*

Ito, T. "Foreign Exchange Rate Expectations: Micro Survey Data," *American Economic Review*, Vol. 80 (1990) 434-449.

Krichene, N. "Deriving Market Expectations for the Euro-Dollar Exchange Rate from Option Prices," IMF Working Paper, WP/04/196 (2004).

Kyle, A. S. "Continuous Auctions and Insider Trading," *Econometrica*, Vol. 53, No. 6 (1985) 1315-1336.

Kyle, A. S., and F. A. Wang. "Speculation Duopoly with Agreement to Disagree: Can Overconfidence Survive the Market Test," *Journal of Finance*, Vol. 52, No. 5 (1997) 2073-2090.

Langer, E. "The Illusion of Control," *Journal of Personality and Social Psychology*, Vol. 32 (1975) 311-328.

Leone, F., L. Nelson, and R. Nottingham. "The Folded Normal Distribution," *Technometrics*, Vol. 3, No. 4 (1961) 543-550.

Oberlechner, T., and C. Osler. "Overconfidence in Currency Markets," Mimeo (2007).

Odean, T. "Are Investors Reluctant to Realize their Losses?," *Journal of Finance*, Vol. 53, No. 5 (1998a) 1775-1798.

Odean, T. "Volume, Volatility, Price, and Profit When all Traders are Above Average," *Journal of Finance*, Vol. 53, No. 6 (1998b) 1887-1924.

Odean, T. "Do Investors Trade Too Much," *American Economic Review*, Vol. 89, No. 5 (1999) 1279-1298.

Régner, I., D. Hilton, L. Cabantous, and S. Vautier. "Judgmental Overconfidence: One Positive Illusion or Many?," Mimeo Toulouse (2004).

Ritter, J. "The Long-Run Performance of Initial Public Offerings," *Journal of Finance*, Vol. 46, No. 1 (1991) 3-27.

Russo, J., and P. Schoemaker. "Confident Decision-Making: How to Make the Right Decision Every Time," London: Piatkus (1991).

Shiller, R. "Do Stock Prices Move Too Much to Be Justified by Subsequent Changes in Dividends?," *American Economic Review*, Vol. 71 (1981) 421-435.

Shiller, R. "Comovements in Stock Prices and Comovements in Dividends," *Journal of Finance*, Vol. 44, No. 3 (1989) 719-729.

Sutherland, N. S. "*Irrationality: The Enemy Within*," Harmondsworth: Penguin (1992).

Taylor, S., and J. Brow. "Illusion and Well-being: A Social Psychological Perspective on Mental Health," *Psychological Bulletin*, Vol. 103 (1988) 193-210.

Taylor, S., and J. Brown. "Positive Illusion and Well-being Revisited Separating Fact From Fiction," *Psychological Bulletin*, Vol. 116 (1994) 21-27.

Weinstein, N. "Unrealistic Optimism About Future Life Events," *Journal of Personality and Social Psychology*, Vol. 39 (1980) 806-820.

Womack, K. "Do Brokerage Analysts' Recommendations Have Investment Values," *Journal of Finance*, Vol. 51, No. 1 (1996) 137-167.