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When Overconfident Traders Meet Feedback Traders

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Abstract

We develop a model in which informed overconfident market participants and informed rational speculators trade against trend-chasers. In line with empirical findings we find a positive relationship between the volatility of prices and the size of the price reversal. We obtain that this result depends on different parameters of the model such as the number of positive feedback traders, how overconfident traders perceive the information of others, as well as the number of informed traders present in the model. In addition we obtain that the presence of positive feedback traders leads to a higher degree of trading activity by both types of informed traders. We also get that overconfidence may lead to lower price volatility and more efficient prices. We obtain that overconfident traders may be better off than their rational counterparts.

Keywords: Overconfidence, Positive Feedback Trading, Excess Volatility, Market Efficiency, Computer Based Trading, Algorithmic Trading.

JEL Classification: D43, D82, G14, G24.

1 Introduction

The analysis of feedback trading has recently seen a revival. Indeed, the Flash Crash of the 6th of May 2010 has been blamed on the extensive use of such strategies together with the use of Computer Based Trading (CBT). On that day, the US equity market dropped by 600 points in 5 minutes and regained almost all the losses in 30 minutes. Positive feedback trading is defined as follows: positive feedback traders buy when prices increase and sell when prices decrease and therefore has been coined trend chasing. In other words, positive feedback traders buy past winners and sell past losers. To the contrary, negative feedback trading leads to asset purchase when prices decrease and sell-off when prices increase. Koutmos (2014) highlights the fact that the use of such feedback strategies may originate for very diverse reasons such as behavioral biases, extrapolative expectations, technical analysis, stop-loss orders and portfolio insurance. When such feedback strategies are coupled with limited human intervention they may have extreme effects. The Introduction of Foresight: The Future of Computer Trading in Financial Markets (2012) and Zygrand et al. (2011) argue that CBT can lead to major share sell-offs leading to further sell-offs triggered by other algorithms.[1] This sequence of sell-offs is very much in the spirit of feedback trading. There are ample evidences of such strategies being used by financial institutions, especially mutual funds (see Baltzer et al., 2019, Frijns et al, 2016, to name but a few). It is also well documented that such strategies are not limited to one asset class but are used in many markets and securities as well as in mature and emerging markets (see for instance recent papers by Menkhoff et al., 2012; Gorton et al., 2013; and Asness et al., 2013).

There are overwhelming evidences that overconfidence and trend chasing are present in financial markets. Both have been established as the two most prominent psychological biases in financial markets (see below for a review). For this reason, the aim of our paper is to answer a multifaceted question of interest, and still unanswered, namely how the interaction between different types of traders including overconfident, feedback and rational traders impacts financial markets. In order to answer that question, we use the dynamic setting of Odean (1998) with price taking overconfident investors and extend it by including both feedback traders à la De Long et al. (1990) and rational investors. For tractability reasons, we keep our theoretical model as simple as possible and therefore include only the three aforementioned types of traders. Introducing liquidity traders would not qualitatively change our results and therefore we omit this extra type of traders for simplicity. The seminal paper by De Long et al. (1990) establishes that rational speculation together with the presence of positive feedback traders can destabilize prices and facilitate the creation of price bubbles. Considering a model where overconfident

[1]However, CBT use as well strategies not mimicking feedback trading. Examples of these strategies are portfolio balancing strategies and pairs trading. Interested readers are referred to Zygrand et al. (2011), Narang (2009), Gomber and Zimmerman (2018), and Farmer and Skouras (2011).

and feedback traders are present enables us to understand how overconfident traders exploit the presence of the latter traders. Our model can be seen as a generalisation of Odean (1998) and De Long (1990) and sits between those two models.

Our model contributes to the overall theoretical literature on behavioral finance and more precisely on understanding the effect of overconfidence in a complex market environment where different types of traders interact. The existing theoretical literature analysing overconfidence and feedback trading focuses separately on each bias and does not consider both biases in conjunction (see Odean, 1998, Kyle et al., 2018, and Germain et al., 2014 for theoretical models of overconfidence and De Long et al., 1990, for feedback trading). To the best of our knowledge, this is the first theoretical model to analyse the interaction of the above three types of traders. The focus of the paper is not to explain feedback trading but to assume its existence and look at how other traders react to its presence. In that vein we assume exogenous feedback trading. Our analysis enhances our comprehension of financial markets. Contrary to papers analysing the impact of overconfidence we find that the volatility of prices can be decrease with overconfidence. We establish a positive relationship between volatility and the size of the price reversals. We predict that the size of the serial correlation and the volatility are linked of the number of feedback traders.

The existence of trend-chasing behavior is a well established fact. Andreassen and Kraus (1990) and Mardyla and Wada (2009) use experiments to show its relevance. In addition, other analyses have empirically found evidences of trend-chasing behavior in financial markets. Frankel and Froot (1988) observe that in the mid-1980's the forecasting services were issuing buy recommendations while maintaining that the dollar was overpriced relative to its fundamental value. Lakonishok et al. (1994) find evidences that individual investors use positive feedback trading strategies and that this behavior can be attributed to an irrational extrapolation of past growth rates. Several studies have focused on the behavior of institutional investors (Shu, 2009, Sias and Starks, 1997, Sias et al., 2001, and Dennis and Strickland, 2002 to name but a few). According to most of these papers, institutional investors use positive feedback strategies and therefore destabilize stock prices. Finally, Bohl and Siklos (2005) find the existence of momentum strategies during episodes of stock market crashes. Evidences of contrarian strategies are also present in financial markets. The short-term portfolio composition strategies suggested by Conrad et al. (1994), and Gervais and Odean (2001) find that US markets showed anomalous, "contrarian" behavior. Moreover, Evans and Lyons (2003) obtain evidence of negative feedback trading, at the daily frequency. Interested readers can consult Koutmos (2014) for a most up to date review of feedback trading. Feedback trading

Overconfidence was first analysed by psychologists. Kahneman and Tversky (1973), and Grether (1980) stress that people overweight salient information. This behavior is well documented in psychology for very diverse situations. Due to the essence of financial markets, overconfidence occurs among market participants. Indeed, the competition between traders implies that the most successful ones survive, leading them to overestimate their own ability. There is ample evidence that this psychological trait is prominent in financial markets. This is true for both experts and professional. For instance see Ben-David et al. (2013), Glaser et al. (2013) and Daniel and Hirshleifer (2015).[2]

Our model features three types of traders in a setting incorporating both the models of Odean (1998) and De Long at al. (1990). We include privately informed traders such as overconfident and rational traders and uninformed myopic feedback traders. Our feedback traders are not sophisticated and we assume that positive (negative) feedback traders buy (sell) past winners and sell (buy) past losers in the spirit of De Long et al. (1990). We further assume that the intensity by which they sell or buy is exogenous such that these traders do not condition their orders on available public information very much in the spirit of stop loss orders where these orders are put in the limit order book and await activation and once activated quantity cannot be controlled by investors. As well as simplifying our analysis, this assumption enables us to directly compare our results with the ones of De Long et al. (1990). A possible extension of the model would be to consider an endogenous β resulting from extracting information from prices. This is left for future research. In our setting, it is clear that feedback traders introduce noise to the model. Overconfident traders misperceive the distribution of their own private information and the one of the other informed traders. They also misperceive public information in the spirit of Odean (1998).

Given such a model we obtain a number of results for the case of positive feedback traders as we mainly focus on that case. First of all we obtain that the equilibrium crucially depends on the amount of feedback activity in the market. If that activity is too large, an equilibrium fails to exist. Both types of informed traders have an incentive to front-run the future trading of positive feedback traders. This triggers even more future trading by feedback traders. This feedback and informed trading may lead to unstable prices and therefore to the nonexistence of an equilibrium. If feedback activity is not too large, an equilibrium exists despite the fact that informed traders take advantage of the feedback traders. Most of the results obtained in the rest of the paper are derived by numerical simulations.

One of the main results of the paper shows that overconfidence can diminish the volatility of prices when positive feedback traders are present in the market. This means that overconfident traders temper the destabilizing role of feedback traders and lead to a more stable market. As explained earlier both types of informed traders front-run the future trading of feedback traders. However, rational traders anticipate the overconfident traders' overreaction to their

[2]An entire volume of the Journal of Economic Perspectives Volume 24 No.4 Fall 2015 is dedicated to Overconfidence. This highlights the importance of that trait and we refer readers to that issue. See, as well, the article from The Guardian by David Shariatmadari on Kahneman entitled *What would I eliminate if I had a magic wand? Overconfidence.*

private information. This leads rational traders to scale down their trading implying a potential decrease of the volatility of prices. This result is at odd with the existing results predicting that if, analysed in isolation, the presence of overconfident traders and feedback traders increases the volatility of prices (see Odean, 1998, and De Long at al., 1990 for instance). However, prices may be more efficient prices due to the fact that the overreaction of overconfident traders may compensate for the lower trading from rational informed traders.

When there is a sufficient large number of trend-chasing speculators, overconfident traders may have higher trading profits than their rational opponents. Kyle and Wang (1997), Benos (1998) and Germain et al. (2014) find a similar result. However some other studies predict the opposite i.e. that overconfident agents trade to their disadvantage (Odean, 1998, Gervais and Odean, 2001, Caballé and Sákovics, 2003, and Biais et al., 2005, among others). Hirshleifer et al. (2006) show that irrational traders can earn positive expected profit in the presence of positive feedback trading if they trade early. Indeed, higher stock prices may attract customers and employees which may reduce the firm's cost of capital and provide a cheap currency for making acquisition. Also, stock prices increase may initially generate cash flow. This simple mechanism does not require irrational traders to be sophisticated enough to think of the positive feedback trading effect and to realize profits. We also find that positive feedback traders earn negative trading profits.

We obtain that price changes can be negatively or positively serially correlated at long horizons. The sign of this serial correlation depends on some of the parameters of the model. The negativity of the serial correlation is a well documented property of prices and is also found in De Long et al. (1990). We obtain that prices can be positively serially correlated when overconfident traders are not too numerous and they believe that the information of the other traders is more precise than it is. This result is also found in Odean (1998) when rational traders trade with overconfident traders who undervalue the signals of other traders. We extend that result to a situation where 3 different types of traders trade with each other. Daniel et al. (1998) consider a situation where investors are overconfident about the precision of their private signals. However, the noisy public information is correctly estimated by all market participants. Price changes exhibit a positive short-lag autocorrelations (called "overreaction phase") and a negative correlation between future returns and long-term past stock market (long-run reversals called "correction phase"). We assume that both rational and overconfident traders are risk averse with the same level of risk aversion. As a side result, we find that increasing the traders' risk aversion increases the negative serial correlation of prices. The literature on feedback trading concludes that the presence of positive (negative) feedback traders leads to negative (positive) serially correlated returns together with an increase (decrease) in volatility (see Shu, 2009, Sias and Starks, 1997, Sias et al., 2001, Bohl and Siklos, 2005, for instance).

Finally, to the extent of trading volume, we obtain that both rational and overconfident traders trade more when feedback traders are present. This result is also present in De Long at al. (1990) for the impact of feedback trading. In addition, it is also found in the literature that the presence of overconfident traders leads to a high degree of trading activity (Odean, 1998, Barber and Odean, 2001, Odean, 1999, Glaser and Weber, 2007, and Statman et al.,2006 among others).

In line with empirical findings we find a positive relationship between the volatility of prices and the size of the price reversal. However our model cannot establish any causality. Nevertheless, we obtain that this result depends on different parameters of the model such as the number of positive feedback traders, how overconfident perceive the information of others, as well as the number of informed traders present in the model.

When we introduce negative feedback traders instead of positive feedback traders, we find that most of the results obtained with the presence of positive feedback are reversed. Price volatility is reduced whereas price efficiency is enhanced due to the presence of contrarian trading. Price volatility and price efficiency increase with both the number of overconfident traders and with their level of overconfidence. The overall volume traded by rational traders increase with the number of negative feedback whereas the volume traded by overconfident is U-shaped with respect to the number of negative feedback. We find the serial correlation of returns to be negative and to decrease with the number of negative feedback traders. Finally the expected profit of the negative feedback traders decrease with the number of negative feedback traders present in the market and is positive for low enough number of negative feedback traders.

If we compare our model to the model of Odean (1998), the main difference is the fact that we introduce feedback traders. We also do not assume that feedback traders have a measure of 1 with the other two types of traders (passive and rational) having a measure of 1 as well. Our set up enables us to analyse the effect of the number of feedback traders.

Due to the number of results we provide a summary table of the results obtained.

Results		
Financial Measures	**Odean (1998)**	**Model with Feedback Traders ($F > 0$) and Rational Traders**
Volatility	Increases with O and κ	Increases or decreases with overconfidence
Quality of Prices	Decreases with O and κ	Increases or decreases with overconfidence
Serial Correlation	Price changes exhibit positive serial correlation	Increases or decreases with O and increases with κ. At long horizon serial correlation can be positive or negative
Trading Volume (Overconfident)	Increases with κ	Increases with F and can be non-monotonic in κ
Trading Volume (Feedback)	NA	Increases with F and κ
Trading Volume (Rational)	NA	Increases with F and decreases with κ
Expected Profits (Overconfident)	Decrease with O and κ. Overconfident earn lower profit than rational traders	Overconfident traders may earn more than rational traders. Increase or decrease with O, κ, F and β
Expected Profits (Feedback)	NA	Always negative and increase with F and β whereas decrease with O and κ
Expected Profits (Rational)	NA	Decrease with F and β, increase with O and increase or decrease with κ

The outline of the paper is as follows. In section 2, we introduce the general model and characterize the different types of traders. In section 3, we derive the trading equilibrium. In section 4, we analyse the effects of overconfidence on some parameters of interest for financial markets when positive feedback traders are present. In section 5, we are interested in understanding the social standpoint. In section 6, we look at how our results are changed when we replace positive feedback traders by negative feedback traders. In section 7, we discuss our model and draw some empirical implications. Finally, we conclude in section 8. All proofs are gathered in the Appendix.

2 The Model

Our model is built on Odean (1998). We extend that model by introducing feedback traders à la De Long et al. (1990). Feedback traders do not base their trading decisions on fundamental values. Instead they react to stock price change. Our aim is then to analyse the impact of feedback traders in a market where another form of behavioral bias is present namely overconfident traders. The overall model is presented now.

We consider four periods where at each of the first three periods ($t = 1, 2, 3$) trade takes place whereas consumption takes place at $t = 4$. Two assets, a riskless and a risky asset, are exchanged during the three trading periods. The riskless interest rate is normalized to zero. The liquidation value of the risky asset, $\tilde{v}$, is assumed to be normally distributed with $\tilde{v} \sim N(\bar{v}, h_v^{-1})$.

We consider a trading system where agents are price takers. At each auction t, the demands for the risky and the riskless assets are x_t and c_t respectively. Let us denote $\bar{x}$ the per capita supply of the risky asset. This supply is the same for each period and known to all traders. Three types of investors trade the assets: informed overconfident traders, informed rational investors and finally uninformed feedback traders. At $t = 0$ both types of informed traders have an endowment c_{0i} of the riskless asset and x_{0i} of the risky asset. Both types also receive information about the liquidation value of the risky asset prior to trading in periods 2 and 3. Overconfident traders believe that their private signals are more accurate than they actually are. Rational investors do not distort their information. There are O overconfident traders and R rational traders. Finally, there are F feedback agents who can be either positive feedback or negative feedback traders.[3] They do not base their trading decisions on fundamental values, instead they react to stock price change. Their order size is proportional to the change in the asset price.

The variables O, R and F denote integer values i.e. actual number of traders.

We denote by P_t the price of the risky asset at time t for $t = 1, 2, 3$. At time $t = 4$, the value of the risky asset is publicly revealed, the price is then equal to the realization of $\tilde{v}$. Trader i's wealth is $W_{ti} = c_{ti} + P_t x_{ti}$ for trading rounds $t = 1, 2, 3$ and $W_{4i} = c_{3i} + \tilde{v} x_{3i}$ for the last trading round.

No information is released before the first trading round. Before each subsequent trading round $t = 2$ and $t = 3$, each rational and overconfident trader receives one of M different signals concerning the liquidation value of the asset. Each trader receives a private signal $\tilde{y}_{ti} = \tilde{v} + \tilde{\varepsilon}_{tm}$, with $\tilde{\varepsilon}_{tm} \sim N(0, h_\varepsilon^{-1})$ and $\tilde{\varepsilon}_{t1}, \ldots, \tilde{\varepsilon}_{tm}$ for $t = 2, 3$ being mutually independent. As in Odean (1998) we assume that $M < O + R$, i.e. there are more traders than signals, and that both O and R are multiple of M, i.e. overconfident traders are, on average, equally informed as their rational counterparts. Let $\bar{Y}_t$ be the average private signal at time t, we have that:

$$\bar{Y}_t = \sum_{m=1}^{M} \frac{\tilde{y_{tm}}}{M} = \sum_{i=1}^{O} \frac{\tilde{y_{ti}}}{O} = \sum_{j=1}^{R} \frac{\tilde{y_{tj}}}{R}.$$

In other words, the informativeness of the private signals is the same for the two groups. This

[3] We mainly analyse the first case in the paper.

setup allows us to exhibit the overconfidence effect without considering informational content bias.

Overconfident market participants believe that the precision of their two signals, the one received at $t = 2$ and the one received at $t = 3$, is equal to κh_ε with $\kappa \geq 1$. They also believe that the $2M - 2$ other signals have a precision equal to γh_ε with $\gamma \leq 1$. Overconfident traders misperceive the distribution of the asset as well. Indeed, they believe that the precision of $\tilde{v}$ equals ηh_v ($\eta \leq 1$). In order to keep the model simple in all the simulations we hold $\eta = 1$. Our framework is consistent with theoretical and empirical findings (see Fabre and François-Heude, 2009, for instance) Indeed, traders tend to overestimate their own signals and to correctly evaluate (or at worst to under-weight) public information.
A rational agent correctly estimates the liquidation value of the risky asset and her private signal. In other words, a rational investor acts as an overconfident trader with $\eta = \kappa = \gamma = 1$.

All informed agents, whether rational or overconfident, are assumed to be risk averse. Their preferences are described by a constant absolute risk aversion (CARA) utility function of the following form

$$u(W) = -e^{-\rho W},$$

where ρ denotes the coefficient of risk-aversion and W final wealth.

Each informed overconfident trader $i = 1, ..., O$ chooses his order at time t, x^o_{ti}, so that

$$x^o_{ti} \in \arg\max E_o[-e^{-\rho W_{ti}} | \Phi_{ti}],$$

and each informed rational trader $j = 1, ..., R$ chooses his order at time t, x^r_{tj}, so that

$$x^r_{tj} \in \arg\max E_r[-e^{-\rho W_{tj}} | \Phi_{tj}],$$

where Φ_{ti} and Φ_{tj} denote the available information to trader i and j at time t.

As in Odean (1998), De Long et al. (1990) and Brown and Jennings (1989), informed traders look one period ahead when solving for their optimal strategy i.e. they are myopic.[4]

Finally, at time $t = 2, 3$, each feedback positive agent $k = 1, ..., F$ submits an order x^f_{tk}, with the following form $x^f_{tk} = \beta(P_{t-1} - P_{t-2})$ where for $t = 2$, $P_{t-2} = \bar{v}$. Feedback traders only participate to the last two rounds of trading.

[4]As mentioned by Odean (1998), assuming myopia when traders conjecture that they do not affect prices leads to the fact the informed traders' demand do not incorporate any hedging demand. This can be seen in Brown and Jennings (1989).

3 The equilibrium

To solve their maximization programs, informed traders whether rational or overconfident assume that prices are linear functions of the average signal(s) such that:

$$P_3 = \alpha_{31} + \alpha_{32}\bar{Y}_2 + \alpha_{33}\bar{Y}_3, \tag{3.1}$$

$$P_2 = \alpha_{21} + \alpha_{22}\bar{Y}_2. \tag{3.2}$$

At each auction, an informed agent determines his demand by considering both his private signal(s) and the price schedule(s). Each informed market participant i has access to the following information $\Phi_{2i} = [y_{2i}, P_2]^T$ and $\Phi_{3i} = [y_{2i}, y_{3i}, P_2, P_3]^T$ for date $t = 2$ and $t = 3$, respectively. Due to the presence of positive feedback trading and when deciding his demand, an informed trader takes into account that his current trade may lead the future price away from fundamentals.

The equilibrium is given below. As in Odean (1998), the equilibrium is not a rational expectation equilibrium. Indeed traders could improve their expected utilities by acting differently.

Proposition 3.1 *If* $\min(\beta F\rho c + d, D) > 0$,[5] *there exists a unique linear equilibrium in the multi-auction market characterized by:*

$$\alpha_{31} = \frac{(O\eta+R)h_v\bar{v} - \rho(O+R+F)\bar{x}}{(O\eta+R)h_v + 2(O(\kappa+\gamma M-\gamma)+RM)h_\varepsilon} + \frac{\rho F\beta(\alpha_{21}-P_1)}{(O\eta+R)h_v + 2(O(\kappa+\gamma M-\gamma)+RM)h_\varepsilon},$$
$$\alpha_{32} = \alpha_{33} + \frac{\rho\beta F\alpha_{22}}{(O\eta+R)h_v + 2(O(\kappa+\gamma M-\gamma)+RM)h_\varepsilon},$$
$$\alpha_{33} = \frac{O(\kappa+\gamma M-\gamma)h_\varepsilon + RMh_\varepsilon}{(O\eta+R)h_v + 2(O(\kappa+\gamma M-\gamma)+RM)h_\varepsilon},$$

where all parameters c, d, D, α_{21}, α_{22} and the different agents' demands over time are given in the Appendix.

Proof: See Appendix.

The existence of the equilibrium crucially depends on the amount of feedback activity in the market and the level of risk aversion of both informed traders. Due to the form of the condition we run some simulations on it and find that if the trading activity of feedback traders is too large, an equilibrium fails to exist. Both types of informed traders have an incentive to front-run the future trading of positive feedback traders. This triggers even more future trading by feedback traders. Large initial feedback trading will lead to unstable prices and therefore the lack of an equilibrium. If feedback activity is not too large (in other words if βF is low enough), an equilibrium exists despite the fact that informed traders take advantage of the feedback traders.

[5] This condition is showed in the Appendix.

By comparison, De Long et al. (1990) also obtain a condition for the existence of a stable solution. In their article the intensity of feedback trading is bounded above by the product of the risk aversion coefficient and the variance of a shock. It should be pointed out that the number of feedback traders is equal to the number of informed investors. It can be seen in the graphs displayed later that, for the given configuration of parameters, the maximum number of feedback traders that allows the existence of the equilibrium is 35 for 20 informed traders (10 rational traders and 10 overconfident traders) close to 2/3 the number of traders in the market. Moreover, the increase in informed traders' risk aversion leads to a decrease in the maximum feedback intensity allowing an equilibrium to exist. For instance, all other things being equal, if the risk aversion coefficient goes from 2 to 5 then the maximum possible number of feedback traders drops from 35 to 11 and ends up at only 3 when the risk aversion coefficient goes up to 10.

The number of positive feedback traders has an impact on the different parameters α except on α_{33}. Indeed, at the last auction, informed agents cannot trigger feedback trading on the basis of their new information. Nevertheless, all prices are influenced and connected by the presence of trend-chasing traders. More precisely, the link between P_3 and $\bar{Y}_2$ (captured by α_{32}) depends on the link between P_2 and $\bar{Y}_2$ (i.e. α_{22}). The greater feedback trading (β and F) the stronger this link is. Similarly, the informed traders' risk aversion, ρ, strengthens this link.

As both types of informed traders are aware of the presence of positive feedback traders, they take that into account when trading. Indeed, upon, for instance, receiving good news before both auctions, informed traders take larger position based on that information at $t = 2$ in order to drive prices up. This triggers even more buying later on from feedback traders which enables them to unload their position at an inflated price resulting in positive expected profit.

In Hirshleifer et al. (2006), irrational traders do not anticipate the feedback effect and the rise in price causes stakeholders (for instance workers) to make greater firm-specific investments when they anticipate the growth of the firm. This in turn increases the final payoff of the risky asset. Nevertheless, risk averse traders trade less aggressively and dampen the feedback effect.

The prices, P_2 and P_3, are not fully revealing. Only the average signal can be inferred from prices. Prices are prevented to be fully revealing due to different reasons: private information is noisy, and both overconfident traders and feedback traders add up some noise to prices due to their behaviour. We refer interested readers to the rational expectations literature.

4 Volatility, Quality of Prices, Serial Correlation of prices and Trading Volume

In this section, we focus on how overconfident trading, due to the presence of feedback traders, impacts on the volatility of prices, measured as the variance of prices, on the quality of prices at t (the variance of the difference between the price and the liquidation value, $var(P_t - \tilde{v})$), the serial correlation of prices and on the different market participants' trading volume. It should be pointed out that all numerical analyses are done when the equilibrium existence condition is satisfied.

4.1 Volatility

The following result analyses, among other things, the effect the feedback traders have on the overconfident traders' behaviour. This result is obtained using numerical procedures. In our simulations when we do not precise the parameter values we set $\gamma = 1$, $\beta = 1$, $\kappa = 1$, $O = 10$, $R = 10$, $F = 10$, $\beta = 2$, $\rho = 2$, $x = 1$, $h_v = h_\epsilon = 1$.

Result 4.1 (Volatility)

1. *Overconfidence, measured by κ or the number of overconfident traders O, can diminish the volatility of prices. It crucially depends on the number of feedback traders. For a large number of feedback traders, the price volatility decreases with overconfidence. When $F = 0$, the volatility of prices increases with κ and O.*

2. *The volatility of prices can increase or decrease with γ.*

We find that the effect of overconfident traders on the excess volatility depends critically on the number of feedback traders in the market. When there are no positive feedback traders, we obtain the result of Odean (1998) and Caballé and Sákovics (2000). Indeed in that case our model is very close to the two previous models. However, when positive feedback traders are present this result can be reversed. This result also contradicts the finding of Benos (1998). We find that the main source of excess volatility is due to feedback trading rather than the trading from overconfident traders. Hence, the presence of overconfident traders can alleviate the effect of the feedback traders and lead to a more stable market. This can be explained as follows, when increasing the number of overconfident traders keeping constant the number of rational traders, the following forces are at work. On the one hand, increasing the number of overconfident traders stabilizes prices as it increases the risk bearing capacity of the market. On the other hand it destabilizes prices as more traders anticipate the trend-chasing behavior. However, the reaction of both rational and overconfident traders is not identical. Indeed rational investors also anticipate the impact of the presence of overconfident traders on the future price and scale down their contemporary trading as they anticipate that overconfident traders trade “too much”. The overall effect is such that for small values of F, the volatility of prices is increased by the presence of overconfident traders whereas for large values of F, but still satisfying the equilibrium existence condition, the volatility of prices decreases with overconfidence. The more positive feedback traders in the market, the larger the latter effect. In other words, overconfident traders commit to trade more on their information the greater F and introducing more information counterbalances the effect of positive feedback trading. The effect of trend chasing, alone, is as described in De Long et al. (1990), namely that it increases volatility in the market. This works through two channels. Increasing F increases the “amount of feedback trading” which in turn also impacts how both types of informed traders trade.

It should be highlighted that when analysing the effect of the number of feedback traders on price volatility we find the same result as De Long et al. (1990) i.e. that price volatility increases with the number of feedback traders. This is proved in the Appendix.

The following two figures (Figure 1 and Figure 2) illustrate the effect of overconfidence on the volatility of prices as described in the previous result.

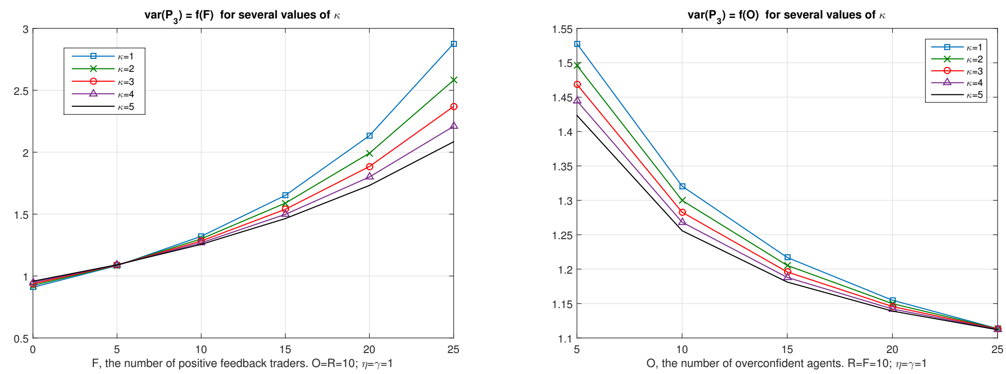

Figure 1: *The variance of prices at time* $t = 3$ *as a function of the number of feedback traders and of the number of overconfident traders.*

The effect of γ on the volatility of prices also depends on the number of feedback traders. Figure 3 illustrates this relationship: when F is large enough the volatility decreases with the underestimation of the precision of the other signals. This result is consistent with Odean (1998) as he obtains that the smaller the parameter γ, the greater is the volatility of prices. In our model, the feedback trading enhances always the volatility of prices. However, since the overconfident agents’ orders convey information, these orders can counterbalance the effect of feedback trading provided that the number of feedback traders is low. In this case, the larger the parameter γ the greater the volatility of prices.

We now turn to the quality of prices.

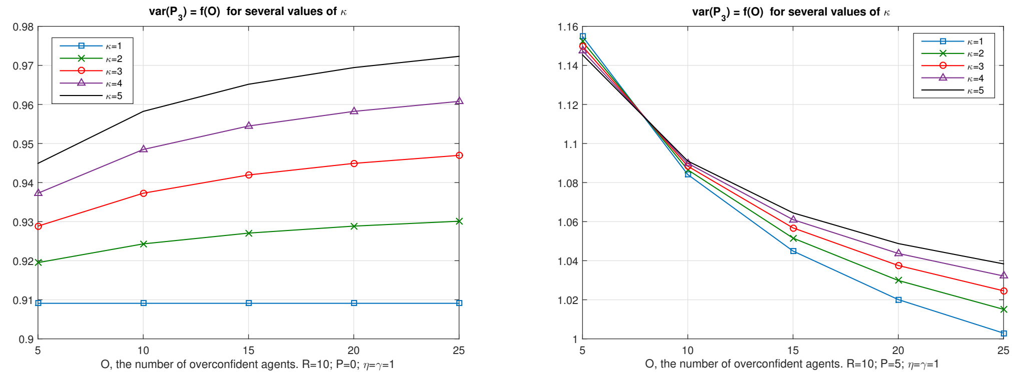

Figure 2: *The variance of prices at time $t = 3$ as a function of the number of feedback traders and of the number of overconfident traders.*

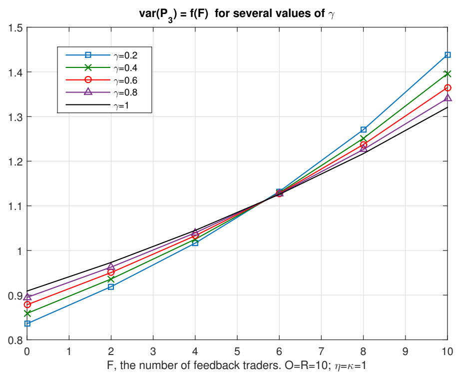

Figure 3: *The variance of prices at time $t = 3$ as a function of the number of feedback traders and of the parameter γ for $O = 10$ and $R = 10$.*

4.2 Quality of Prices

We now examine the behavior of the quality of prices. We use as in Odean (1998) the following measure for the quality of prices at time t: $var(P_t - \tilde{v})$. Kyle (1985) uses a slightly different measure defined as the variance of the liquidation value conditional on past and contemporary order flow. To compare our results to Odean (1998) we take the same definition.

Result 4.2 (Quality of Prices)

1. *Depending on the number of feedback traders, the quality of prices can either increase (for large F) or decrease (for small F or even $F = 0$) with O.*

2. *It increases with κ (for large F), decreases with it (for small F or even $F = 0$) or is non-monotonic with κ for intermediate values of F.*

As explained earlier both types of informed traders front-run the future trading by feedback traders. In addition, overconfident traders alter the quality of prices due to their irrationality.[6] The impact of the overconfidence on the quality of prices depends on whether there are feedback traders present and on their number. If there is no trend-chasing behavior, the presence of overconfident traders moves prices away from fundamentals and diminishes market efficiency. However, when the number of feedback traders is large (but still such that the equilibrium exists), the quality of prices improves with both the number of overconfident traders and their level of overconfidence as measured by the parameter κ. Rational traders anticipating the reaction of overconfident reduce their trading leading to less efficient prices. However, as overconfident traders are trading more intensely on their private information this has a beneficial effect on price informativeness. When the number of feedback traders is large the latter effect dominates leading to overall more informative prices. This result is in contrast with the result obtained by Odean (1998) but is in accordance with Benos (1998).

As anticipated and in accordance with De Long et al. (1990), we obtain that more feedback traders lead to a lower quality of prices. Indeed, these traders add some noise to prices directly and indirectly through the front-running of informed traders. We provide the proof in the Appendix. Figure 4 and Figure 5 illustrate the impact of both overconfident trading and feedbak trading on the quality of prices by considering different scenarios from the absence of trend-chasers to a large number of feedback traders.

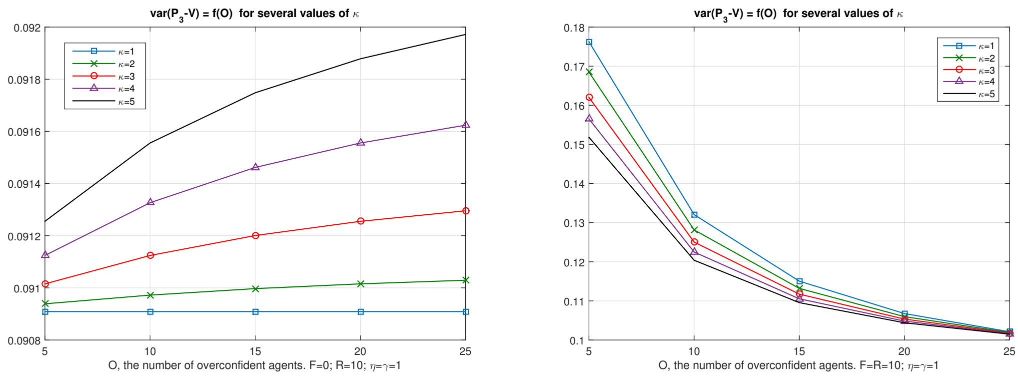

Figure 4: *The quality of prices at time $t = 3$ as a function of the number of overconfident agents and of the parameter κ for $F = 0$ and $F = 10$.*

[6]Ko and Huang (2007) show that arrogance can be a virtue. Indeed, overconfident investors believe that they can earn extraordinary returns and will consequently invest resources in acquiring information pertaining to financial assets. In our model there is no information-seeking activity which could permit to obtain such a positive externality.

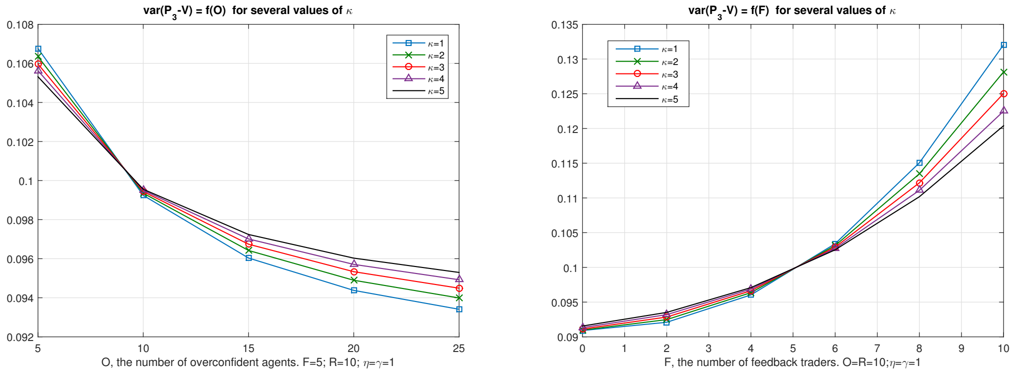

Figure 5: *The quality of prices at time $t = 3$ as a function of the number of overconfident agents and of the parameter κ for $F = 5$ and the quality of prices at time $t = 3$ as a function of the number of feedback traders and of the parameter κ for $O = 10$ $R = 10$.*

4.3 Serial Correlation of Prices

As the behavior of the serial correlation of prices cannot be established formally, the result below establishes how the serial correlation changes in the model without overconfident traders. The result with overconfident traders is established with simulations.

Result 4.3 (Serial Correlation of Prices)

1. *When only rational traders are present, the serial correlation of prices is negative and close to zero.*
2. *The serial correlation of prices in absolute value increases with the number of overconfident traders (for low F or even $F = 0$) and is non-monotonic (initially increasing) with O for large F. It also increases with the level of overconfidence κ.*
3. *The serial correlation of prices decreases with γ. In other words, the more precise the other traders' information is believed to be the smaller the price changes.*

The serial correlation of prices depends critically on the overconfidence level and on the number of positive feedback traders. In the presence of positive feedback trading, the serial correlation is generally negative. It implies that positive feedback trading destabilizes the price schedule. Informed traders cannot keep price at fundamentals or reduce the fluctuation of prices.

The term $cov(P_3 - P_2, P_2 - P_1)$ describes the correction phase in Daniel et al. (1998). They show that overconfident traders begin by overreacting to their private signals. In the second phase, irrational market participants correct their beliefs and their order as new public information arrives, this is defined as the "correction phase". In our model, agents update their beliefs concerning their private information. The informed market participants know that their earlier trades move prices away from fundamentals as they try to exploit the presence of feedback trading. The size of the departure of prices from fundamentals increases with the number of feedback traders. At date 3, informed participants correct their demands after observing their last signal which leads to price reversal. Odean (1998) finds such negative correlation by considering overconfident agents only. When rational investors are introduced to the model, he shows that the serial correlation of prices may be positive provided overconfident agents sufficiently undervalue the signals of others. We extend his former result, as we show that informed rational traders cannot prevent feedback as well as overconfident traders to destabilize prices.

We also can formally establish that the number of feedback traders increase the serial correlation when only rational and feedback traders are present. See the proof provided in Appendix.

Figure 6 illustrates the last point of result 4.3. It shows that the price change is more important when each overconfident trader underestimates the precision of the other traders' private information. However, there is a positive momentum when each overconfident trader underestimates the other specific market participants' signals.

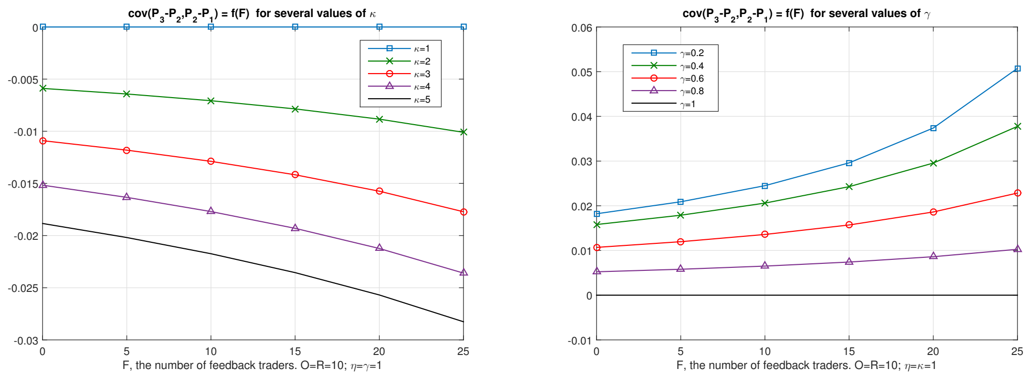

Figure 6: *The price changes as a function of the number of feedback traders for different values of* γ.

The empirical literature on feedback trading concludes that the presence of positive (negative) feedback traders leads to negative (positive) serially correlated returns together with an increase (decrease) in volatility (see Shu, 2009, Sias and Starks, 1997 , Sias et al., 2001, Bohl and Siklos, 2005, Conrad et al., 1994, to name a few). However, we find that when the market is made of positive feedback traders and rational traders the effect of the number of feedback traders is very small and the serial correlation of prices is close to zero. The effect of F increases with the level of overconfidence. It can be seen that the combination of rational investors, positive feedback traders and overconfident traders can lead to positive serial correlation of prices despite the presence of positive feedback traders. This result is in sharp contrast with the literature such as Bohl and Reitz (2006) and Daniel et al. (1998). Bohl and Reitz (2006) find a possible link between positive feedback trading and negative return auto correlation during period of high volatility in the German Neuer Market. Daniel et al. (1998) find that overconfidence and long-run reversals of returns may be linked. They also find that the momentum effect is stronger for high volume stocks.

4.4 Trading Volume

Result 4.4 (Trading Volume)

1. *The trading volume by both rational and overconfident traders increases with the number of positive feedback traders.*
2. *The trading volume by overconfident traders can be a non-monotonic function of κ and this comparative static depends on the number of feedback traders whereas the trading volume by rational traders decreases with κ.*

When positive feedback traders are present, both types of informed agents trade more aggressively. They anticipate that the initial price increase will stimulate buying by feedback traders at the subsequent auctions. In doing so, they drive prices up higher than fundamentals. Consequently, positive feedback traders respond by trading even more. Feedback trading and overconfidence enhance trading. (De Long et al., 1990, find that effect for the impact of feedback trading and Odean, 1998, for the impact of overconfidence). Our result is consistent with empirical findings. Glaser and Weber (2007) show that investors who think that they are above average in terms of investment skills or past performance trade more. Statman et al. (2006) use U.S. market level data and argue that after high returns subsequent trading volume will be higher as investment success increases the degree of overconfidence. This is also confirmed by Kim and Nofsinger (2007) for Japanese traders on the Tokyo Stock Exchange. Odean (1999) analyses the trading of 10,000 investors and find the aforementioned relationship between volume and overconfidence.

In Figure 7, we compare the volume from overconfident traders and from rational traders with no feedback traders. As expected, we see that overconfident traders trade more aggressively than their rational counterparts. The volume from overconfident investors increases with κ, whereas, as explained before, the volume from rational investor's order decreases with κ. However, both expected volumes decrease with the number of overconfident traders.

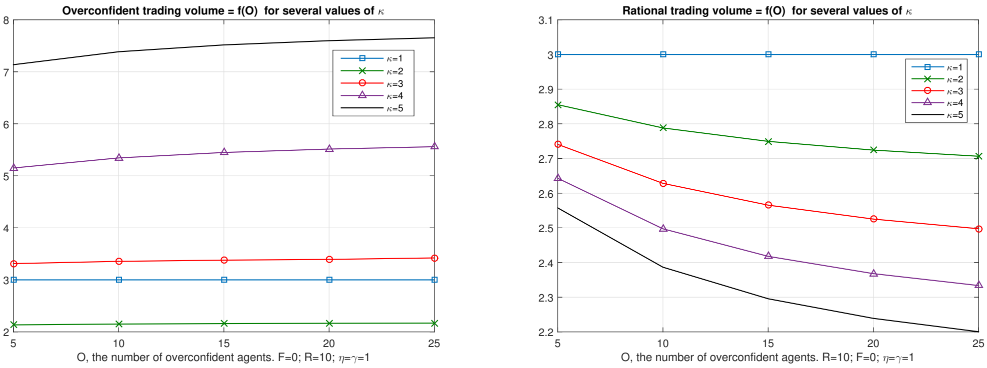

Figure 7: *The total individual overconfident trading volume as a function of the number of overconfident agents, for different values of the parameter* κ.

5 Trading Performance

We now look at trading profits. For the sake of simplification, we compare the expected profits of each investor. Of course this doesn't consider the risk aversion or the level of overconfidence. However, it is an indicator of the trading profits of the market participants.

Result 5.1 (Trading Profits)

1. *If both* κ *and the number of feedback traders are large overconfident traders may earn positive trading profits. In that case overconfident traders are better off than their rational counterparts. For a moderate level of overconfidence (for instance* $\kappa = 2$*) and for a relatively large number of feedback traders, we also find the latter result.*

2. *The trading profits of overconfident traders can be increasing, decreasing or non-monotonic with positive feedback activity, measured by the number of positive feedback traders and* β*. The effect of* O *and* κ *on the profit of the overconfident traders is ambiguous: For large* F*, it is increasing with* κ *whereas decreasing with* κ *for small* F*, it decreases with* O *for low* β *whereas it increases with* O *for large* β*.*

3. *The trading profits earned by rational traders decrease with the feedback trading (*F *and* β*). They increase with the number of overconfident traders and can increase with* κ *for large* F *and low* O *and decrease with* κ *for low* F *and large* O*.*

Proof: See Appendix for the derivation of the expressions of the trading profits for the different traders.

Due to the fact that informed investors anticipate the presence of feedback trading, feedback traders either buy shares at an inflated price or sell shares at a too low a price. Such trading behavior leads these agents to lose money.

When there are only rational and overconfident traders in the market, we find that overconfident traders earn less than their rational counterparts. This result is consistent, among others, with Odean (1998), Gervais and Odean (2001). When the 3 types of traders are present (overconfident, rational and positive feedback agents) overconfident agents may outperform rational traders. This result confirms Benos (1998), Kyle and Wang (1997) and Germain et al. (2014). In these papers pure liquidity traders are present. Hirshleifer et al. (2006) show that irrational traders can earn positive expected profit in the presence of positive feedback trading if they trade early.

The last part of the first point in the above result stems from the fact that the trend-chasers' trading volume decreases with κ and increases with γ.

We dedicate the next section to contrarian trading, i.e. feedback traders selling when prices increase.

6 On Contrarian Trading

In this section we look at the case of contrarian trading. Given the setup of the model, we can investigate the effect of negative feedback trading by considering a negative β (negative feedback trading intensity). Negative feedback traders or contrarian traders sell securities when prices rise.

First of all, it should be noticed that due to their trading behavior, negative feedback traders limit the movement of prices. Informed traders upon receiving some information will always follow their information and trade larger quantities as they know that due to negative feedback trading subsequent prices will not fully reflect their information. This alters some of the comparative statics we found earlier for the case of positive feedback trading.

We find the obvious result that the volatility of prices decreases with the number of negative feedback and increases with the number of overconfident.

The quality of prices in both periods decreases with the number of negative feedback whereas the quality of prices in the third period increases with the number of overconfident and with κ.

The serial correlation of prices is negative and decreases with the number of negative feedback traders.

The overall volume traded by rational investors increases with the number of negative feedback and decreases with the level of overconfidence in the market, κ. As for the case of positive feedback traders, the overall volume traded by overconfident traders is non-monotonic in the number of negative feedback traders. For a low number of feedback traders, the volume traded by overconfident decreases with F whereas for a large number of feedback traders it increases with F. The range for which it decreases with F increases as the level of overconfidence, κ, increases.

Finally, we find that the expected profit of the negative feedback traders decreases with the number of negative feedback present in the market and that they can derive positive trading profits for a low number of negative feedback traders. In that case the second round gains compensate the third round losses. However, as overconfident traders become more overconfident, measured by an increase of κ, the expected profit decreases. When feedback traders are negative, they can earn positive trading profits as they sell (buy) at high (low) price due to the informed traders behavior.

7 Discussion and Empirical Implications

In this section, we are interested in understanding how prices change in a financial market. We have shown that positive feedback trading induces an increase in price volatility and worsens market efficiency more than overconfident trading. When positive feedback agents are introduced to the market, overconfident investors may obtain greater expected profits than rational speculators. Positive feedback traders can suffer important losses if there are too numerous.

In order to investigate the level of prices, we have numerically simulated the price pattern over time as a function of the number of positive feedback traders, the number of overconfident traders and the risk aversion coefficient ρ.

Figure 8 shows the price pattern over time as a function of the number of feedback traders and the number of overconfident traders. Feedback trading is based on the previous prices movement and appears as lagged trading. It can be seen that the number of both overconfident traders and feedback traders impact prices. The more feedback traders, the more the price initially increases and the more the prices decrease between period 2 and 3. In the second graph we can see that increasing the number of overconfident do not increase the variation of prices for a relatively large number of feedback traders (here equal to 10).

We observe, in figure 9, that both the volatility of price and the price change increase with the risk aversion coefficient. Informed investors react abruptly to new information and lead prices to be more volatile. This result emphasizes that the risk aversion of the market participants can explain part of the excess volatility observed in the market.

Figure 10 shows that the price changes are less extreme when the market participants are less risk averse. We also show how the difference between the overconfident investors' demand and the rational one depends on F and on the speculators' risk aversion. As the risk aversion

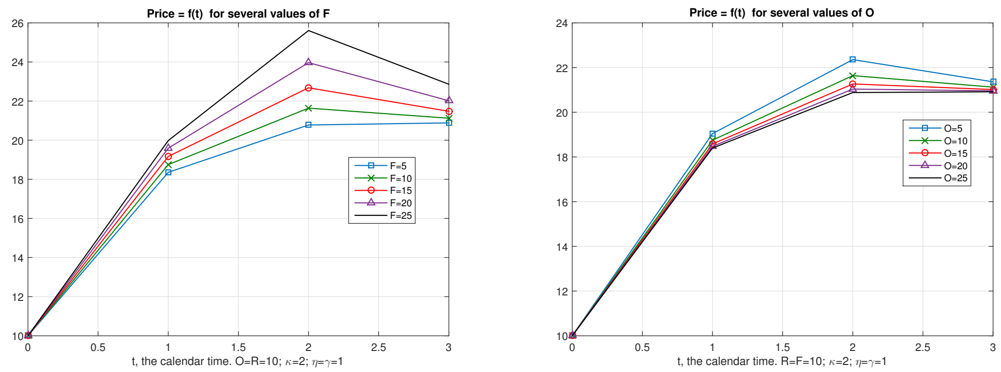

Figure 8: *Prices over time for different values of F (the number of positive feedback traders)and for different values of O (the number of overconfident agents).*

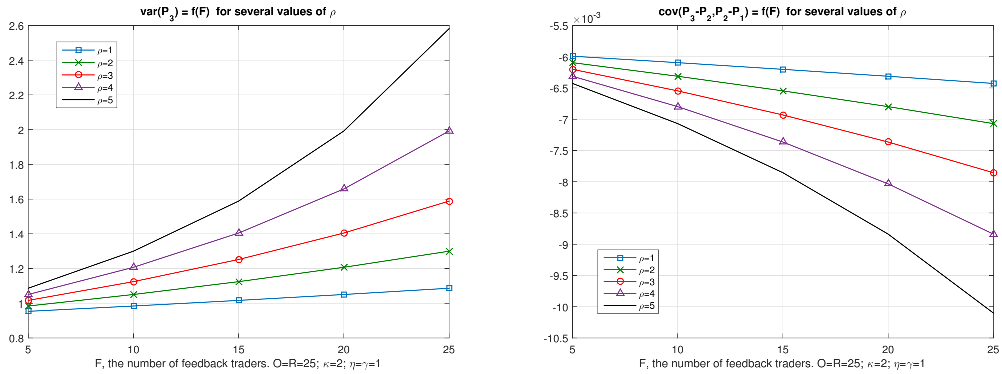

Figure 9: *The volatility of prices at date $t = 3$ and the $cov(P_3 - P_2, P_2 - P_1)$, for different values of the parameter ρ.*

increases and the number of feedback traders increases, rational and overconfident traders tend to trade in the same manner. The difference is extreme when there are a large number of feedback traders with a low level of risk aversion.

In our model, we have considered overconfident agents dealing with rational traders. One might have assumed two types of privately informed rational investors that differed from one another in their risk-aversion level. From the literature we know that less (more) risk averse investors trade more (less) on their private information. In the presence of feedback traders, less risk averse trader will react more than more risk averse traders, potentially leading to the even lower reaction to private information of more risk averse traders. This is a similar effect to the one described in the paper potentially leading to the result of volatility. It may also happen that the effect of the trading of less risk averse traders is not large enough to compensate of the trading of more risk averse traders leading to the decrease of volatility. Whether the effect of an overconfident trader is comparable to a less risk rational trader is an open question and would need to be analysed in a fully developed model.

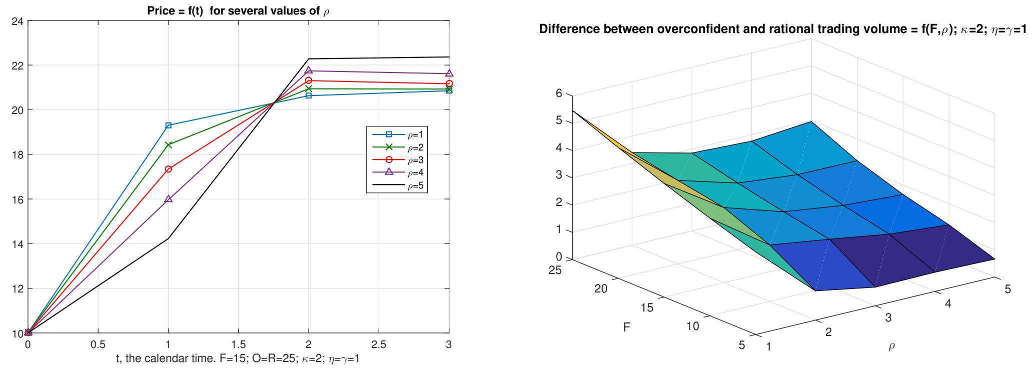

Figure 10: *Price over time for different values of ρ and difference demands between overconfident and rational investors.*

Empirically we find some testable implications.

Our model finds a positive relationship between the volatility of prices and the size of the price reversal. Indeed, when the price volatility is low (high), the serial correlation of prices is low (high). The causality is unclear in our model. This would have to be tested. The literature on feedback trading concludes that the presence of positive (negative) feedback traders leads to negative (positive) serially correlated returns together with an increase (decrease) in volatility. For instance Bohl and Siklos(2005) puts forward that relationship between negative serially correlated returns and high volatility when positive feedback traders are present. For further evidences, see Shu (2009), Sias and Starks (1997), and Sias et al. (2001), for instance.

Our model predicts that the size of the serial correlation and the volatility are linked to the number of positive feedback traders. An increase in the number of positive feedback traders implies more volatility and more negative serial correlation. If the two phenomena are observed in unison this could give an indication of the presence of feedback traders.

De Long et al. (1990) find that prices exhibit a positive correlation at short horizons whereas at long horizons price changes are negatively serially correlated. We show that this property depends on different parameters of the model such as the number of positive feedback traders, how overconfident perceive the information of others, as well as the number of informed traders present in the model.

However, this property can be reversed when there are no rational traders and overconfident traders are not too numerous and undervalue the information of others. In that case we find that prices exhibit a negative correlation at short horizons whereas at long horizons price changes are negatively serially correlated.

If it is believed that CBT exacerbates the volume of feedback trading and from a policy point of view, our paper, as others, would recommend the limitation of Computer Based Trading (CBT) or algorithmic trading (AH) which would lead to feedback loop destabilizing markets. Our specification of feedback traders resembles the actual behavior of computer based trading or algorithmic trading in the sense that both CBT and AH are very mechanical. Our feedback traders chase the trend in this mechanical aspect and this aspect is one major factor in the creation of asset price bubbles.[7] In our model, increasing CBT, as measured by an increase of F or β, implies more volatility, greater price changes that both can possibly lead to bubbles and then crashes.

8 Conclusion

Our paper analyses the interaction between different type of traders in a financial market. We shed some light on the result of the competition between feedback traders and two types of informed traders some being rational and others being overconfident. This enables us to revisit some results found in the price taking model by Odean (1998), in his price taking model, whereby overconfidence increases volatility of prices and deceases the quality of prices in the light of the result found in De Long et al. (1990) where rational speculation together with the presence of positive feedback traders can destabilize prices and facilitate the creation of price bubbles.

It is shown in De Long et al. (1990) that positive feedback traders increase price volatility. Their trade is based on past prices which are determined by the informed trading in earlier stages. This causes a temporary miscoordination between traders. In our model, both overconfident and rational traders anticipate the behavior of feedback-positive agents and front-run the future trading by feedback traders. The model finds some striking results. Due to the competition between overconfident traders and rational traders, we find that overconfidence may decrease the volatility of prices and improve price quality. The presence of overconfident traders leads rational traders to scale down their trading implying a decrease of the volatility of prices caused by positive feedback traders. Overconfident traders commit to react more to their private information. Introducing more information counterbalances the effect of positive feedback.

We also find that the presence of feedback traders leads to an increase of the volume of both rational and overconfident traders. It can also be the case that the expected profits from trading for overconfident traders are superior to the ones obtained by rational traders.

In line with empirical findings we find a positive relationship between the volatility of prices and the size of the price reversal. However our model cannot establish any causality. Nevertheless, we show that this result depends on different parameters of the model such as the number of positive feedback traders, how overconfident perceive the information of others, as well as the

[7] Feedback trading is often cited as the reason of the bubble (see Zhou and Sornette, 2006, 2009, Cajueiro et al. 2009, and Abreu and Brunnermeier, 2003.)

number of informed traders present in the model. In addition we obtain that the presence of positive feedback traders leads to more volume being traded by both types of informed traders.

A possible and interesting extension of the above model would be to consider more sophisticated feedback traders. We could indeed think of feedback traders having an endogenous sensitivity to the changes of price and may try to extract information from public information. Such an assumption would enrich our model at the expense of losing further tractability. This line of research is left for future research.

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10 Appendix

Notation

Notation	
Terminal (liquidation) value of risky asset	$\tilde{v} \sim N(\bar{v}, h_v^{-1})$
Number of informed overconfident traders	O
Number of informed rational traders	R
Number of feedback traders	F
Time	$t = 0, ..., 4$
Overconfidence parameter	$\kappa \geq 1$
Parameter of underweighting signals from others	$\gamma \leq 1$
Parameter of underweighting priors	$\eta \leq 1$
Private signals at time t for informed trader i	$\tilde{y}_{ti} = \tilde{v} + \tilde{\varepsilon}_{tm}$
Error terms in signals	$\tilde{\varepsilon}_{tm} \sim N(0, h_\varepsilon^{-1})$
Number of signals	$m = 1, ..., M$
Average private signal at time t	$\bar{Y}_t$
Informed traders' risk absolute risk aversion coefficient	ρ
Per capita supply of risky asset	$\bar{x}$
Price of risky asset at time t	P_t
Information available to trader i at time t	Φ_{ti}
Feedback trading intensity	β
Endowment of the risky and riskless assets at time $t = 0$ for an informed trader i	x_{0i} and c_{0i}
Demands for the risky and riskless assets at time $t = 1, 2, 3$ from an overconfident trader i	x_{ti}^o and c_{ti}^o
Demands for the risky and riskless assets at time $t = 1, 2, 3$ from a rational trader j	x_{tj}^r and c_{tj}^r
Demand for the risky asset at time $t = 2, 3$ from a feedback trader k	x_{tk}^f
Informed trader's i wealth at time $t = 0, ..., 3$	W_{ti}
Expected Profits of an overconfident trader i at time t	Π_{ti}^o
Expected Profits of a rational trader j at time t	Π_{tj}^r
Expected Profits of a feedback trader k at time t	Π_{tk}^f
Quality of Prices at time t	$var(P_t - \tilde{v})$
Serial Correlation of Prices	$cov(P_3 - P_2, P_2 - P_1)$
Volatility of Prices at time t	$var(P_t)$

Proof of Proposition 3.1: Equilibrium

This proposition is proved by backward induction, we then start with the last period, i.e. $t = 3$. Let us introduce the following notations: a lower or upper script o stands for the overconfident trader, r stands for the rational trader and finally f stands for the feedback trader.

Third round $t = 3$

At time $t = 3$, each trader, noted i with $i = 1, ..., O$ when overconfident or j with $j = 1, ..., R$ when rational, has private information Φ_{3i} or Φ_{3j} respectively which have a multivariate distribution. The information available for trader i and j is $\Phi_{3i} = [y_{2i}, y_{3i}, P_2, P_3]^T$ and $\Phi_{3j} = [y_{2j}, y_{3j}, P_2, P_3]^T$ respectively.

An overconfident trader infers the mean of this distribution, $E_o(\Phi_{3i})$, and the variance-covariance matrix, Ψ_o, as follows:

$$E_o(\Phi_{3i}) = [\bar{v}, \bar{v}, \alpha_{21} + \alpha_{22}\bar{v}, \alpha_{31} + (\alpha_{32} + \alpha_{33})\bar{v}]^T,$$

$$\Psi_o = \begin{bmatrix} \frac{1}{\eta h_v} + \frac{1}{\kappa h_\varepsilon} & \frac{1}{\eta h_v} & \frac{\alpha_{22}}{\eta h_v} + \frac{\alpha_{22}}{M\kappa h_\varepsilon} & \frac{\alpha_{32}+\alpha_{33}}{\eta h_v} + \frac{\alpha_{32}}{M\kappa h_\varepsilon} \\ \frac{1}{\eta h_v} & \frac{1}{\eta h_v} + \frac{1}{\kappa h_\varepsilon} & \frac{\alpha_{22}}{\eta h_v} & \frac{\alpha_{32}+\alpha_{33}}{\eta h_v} + \frac{\alpha_{33}}{M\kappa h_\varepsilon} \\ \frac{\alpha_{22}}{\eta h_v} + \frac{\alpha_{22}}{M\kappa h_\varepsilon} & \frac{\alpha_{22}}{\eta h_v} & \frac{\alpha_{22}^2}{\eta h_v} + \frac{\alpha_{22}^2(\gamma+M\kappa-\kappa)}{M^2\kappa\gamma h_\varepsilon} & C_1 \\ \frac{\alpha_{32}+\alpha_{33}}{\eta h_v} + \frac{\alpha_{32}}{M\kappa h_\varepsilon} & \frac{\alpha_{32}+\alpha_{33}}{\eta h_v} + \frac{\alpha_{33}}{M\kappa h_\varepsilon} & C_1 & C_2 \end{bmatrix},$$

with $C_1 = \frac{\alpha_{22}\alpha_{33}}{\eta h_v} + \alpha_{22}\alpha_{32}\left(\frac{1}{\eta h_v} + \frac{\gamma+M\kappa-\kappa}{M^2\kappa\kappa\gamma h_\varepsilon}\right)$, and $C_2 = \frac{(\alpha_{32}+\alpha_{33})^2}{\eta h_v} + (\alpha_{32} + \alpha_{33})^2\left(\frac{\gamma+M\kappa-\kappa}{M^2\kappa\gamma h_\varepsilon}\right)$.

The mean, $E_r(\Phi_{3j})$, and the variance-covariance matrix Ψ_r for the rational agent are obtained by setting $\eta = \kappa = \gamma = 1$ in $E_o(\Phi_{3i})$ and Ψ_o.

By solving the mean-variance problem, we obtain the ith insider's orders:

$$x_{3i}^o = \frac{E_o(\tilde{v}|\Phi_{3i}) - P_3}{\rho var_o(\tilde{v}|\Phi_{3i})},$$

$$x_{3j}^r = \frac{E_r(\tilde{v}|\Phi_{3j}) - P_3}{\rho var_r(\tilde{v}|\Phi_{3j})}.$$

On the other hand, we know that each feedback trader $k = 1, ..., F$ determines her order by considering the trend of prices as follows :

$$x_{3k}^f = \beta(P_2 - P_1).$$

Using the projection theorem, we obtain the following

$$E_o(\tilde{v}|\Phi_{3i}) = \frac{(y_{2i} + y_{3i})(\kappa - \gamma)h_\varepsilon + (\bar{Y}_2 + \bar{Y}_3)\gamma h_\varepsilon M + \eta h_v \bar{v}}{\eta h_v + 2(\kappa + \gamma M - \gamma)h_\varepsilon},$$

$$var_o(\tilde{v}|\Phi_{3i}) = \frac{1}{\eta h_v + 2(\kappa + \gamma M - \gamma)h_\varepsilon},$$

$$E_r(\tilde{v}|\Phi_{3j}) = \frac{(\bar{Y}_2+\bar{Y}_3)h_\varepsilon M + h_v\bar{v}}{h_v+2Mh_\varepsilon},$$

$$var_r(\tilde{v}|\Phi_{3j}) = \frac{1}{h_v+2Mh_\varepsilon}.$$

Replacing the above into the expressions of the different orders for the different types of traders we obtain

$$\begin{array}{rcl} x^o_{3i} &=& \frac{1}{\rho}[(y_{2i}+y_{3i})(\kappa-\gamma)h_\varepsilon + (\bar{Y}_2+\bar{Y}_3)\gamma h_\varepsilon M + \eta h_v\bar{v} - P_3(\eta h_v + 2(\kappa+\gamma M-\gamma)h_\varepsilon)], \\ x^r_{3j} &=& \frac{1}{\rho}[(\bar{Y}_2+\bar{Y}_3)h_\varepsilon M + h_v\bar{v} - P_3(h_v+2Mh_\varepsilon)], \\ x^f_{3k} &=& \beta(P_2-P_1). \end{array}$$

In equilibrium the total demand must be equal to the exogenous total supply, this is given by

$$\sum_{i=1}^{O} x^o_{3i} + \sum_{j=1}^{R} x^r_{3j} + \sum_{k=1}^{F} x^f_{3k} = (O+R+F)\bar{x}. \tag{10.3}$$

From (10.3), the price P_3 can be obtained as a function of $\bar{Y}_2$, $\bar{Y}_3$, P_2 and P_1

$$\begin{aligned} P_3 = \frac{1}{\Lambda}[&((O\gamma+R)Mh_\varepsilon + (\kappa-\gamma)Oh_\varepsilon)(\bar{Y}_2+\bar{Y}_3) + (O\eta+R)h_v\bar{v} \\ &- \rho(O+R+F)\bar{x} + \rho F\beta(P_2-P_1)], \end{aligned}$$

with $\Lambda = (O\eta+R)h_v + 2(O(\kappa+\gamma M-\gamma)+RM)h_\varepsilon$, $\bar{Y}_2 = \tilde{v} + \frac{\varepsilon_{2q}}{M} + \frac{1}{M}\sum_{l\neq q}\varepsilon_{2l}$ and $\bar{Y}_3 = \tilde{v} + \frac{\varepsilon_{3q}}{M} + \frac{1}{M}\sum_{l\neq q}\varepsilon_{3l}$ and with q and l being arbitrary informed traders (rational or overconfident).

Using equation (3.1) from the text and identifying the parameters αs we obtain:

$$\begin{aligned} \alpha_{31} &= \tfrac{(O\eta+R)h_v\bar{v}-\rho(O+R+F)\bar{x}}{\Lambda} + \tfrac{\rho F\beta}{\Lambda}(\alpha_{21}-P_1), \\ \alpha_{32} &= \tfrac{O(\kappa+\gamma M-\gamma)h_\varepsilon+RMh_\varepsilon}{\Lambda} + \tfrac{\rho\beta F}{\Lambda}\alpha_{22}, \\ \alpha_{33} &= \tfrac{O(\kappa+\gamma M-\gamma)h_\varepsilon+RMh_\varepsilon}{\Lambda}. \end{aligned}$$

Second round $t=2$

Using the third round's results, we can obtain the second round's parameters.

Let us introduce the notations:

$$B_o^T = cov_b(P_3, \Phi_{2i}) = [cov_b(y_{2i}, P_3), cov_b(P_2, P_3)],$$

$$B_r^T = cov_r(P_3, \Phi_{2j}) = [cov_r(y_{2j}, P_3), cov_r(P_2, P_3)].$$

Using the projection theorem, we get

$$E_l(P_3|\Phi_{2q}) = E_l(P_3) + B_l^T var_l(\Phi_{2q})^{-1}(\Phi_{2q} - E_l(\Phi_{2q})),$$

$$var_l(\Phi_{2q}) = \begin{pmatrix} var_l(y_{2q}) & \alpha_{22}cov_l(y_{2q}, \bar{Y}_2) \\ \alpha_{22}cov_l(y_{2q}, \bar{Y}_2) & \alpha_{22}^2 var_l(\bar{Y}_2) \end{pmatrix},$$

where $l = o, r$ and E_l and var_l denote the fact that they are computed following trader l's beliefs.

We obtain

$$\begin{aligned} E_l(P_3|\Phi_{2q}) &= (\alpha_{32} + \alpha_{33})\bar{v} + \alpha_{31} + \tfrac{1}{L_l}((cov_l(y_{2q}, P_3)D_1^l + cov_l(P_2, P_3)\tfrac{D_3^l}{\alpha_{22}})(y_{2q} - \bar{v}) \\ &\quad + cov_l(y_{2q}, P_3)D_3^l + cov_l(P_2, P_3)\tfrac{D_2^l}{\alpha_{22}}(\bar{Y}_2 - \bar{v})), \end{aligned}$$

with $l = o, r$ and with

$$\begin{aligned} 0cm D_1^o = var_o(\bar{Y}_2) &= \tfrac{1}{\eta h_v} + \tfrac{(\gamma + M\kappa - \kappa)}{M^2\gamma\kappa h_\varepsilon}, \\ D_2^o = var_o(y_{2i}) &= \tfrac{1}{\eta h_v} + \tfrac{1}{\kappa h_\varepsilon}, \\ D_3^o = -cov_o(y_{2i}, \bar{Y}_2) &= -\left(\tfrac{1}{\eta h_v} + \tfrac{1}{M\kappa h_\varepsilon}\right), \\ L^o = var_o(y_{2i})var_o(\bar{Y}_2) - cov_o(y_{2i}, \bar{Y}_2)^2 &= \tfrac{(M-1)[((M-1)\gamma+\kappa)h_\varepsilon + \eta h_v]}{M^2\eta\kappa\gamma h_v h_\varepsilon^2}. \end{aligned}$$

The expressions D_1^r, D_1^r, D_3^r, and L^r can be obtained by setting $\kappa = \gamma = \eta = 1$ in D_1^o, D_1^o, D_3^o, and L^o.

At the second round, informed trader i's order is:

$$x_{2i}^o = \frac{E_o(P_3|\Phi_{2i}) - P_2}{\rho var_o(P_3|\Phi_{2i})},$$

$$x_{2j}^r = \frac{E_r(P_3|\Phi_{2j}) - P_2}{\rho var_r(P_3|\Phi_{2j})}.$$

The kth feedback agent's order is:

$$x_{2k}^f = \beta(P_1 - P_0).$$

By equating exogenous supply and demand, we have:

$$(O + R + F)\bar{x} = \sum_{i=1}^{O} \frac{E_o(P_3|\Phi_{2i}) - P_2}{\rho var_o(P_3|\Phi_{2i})} + \sum_{j=1}^{R} \frac{E_r(P_3|\Phi_{2j}) - P_2}{\rho var_r(P_3|\Phi_{2j})} + F\beta(P_1 - P_0).$$

First round $t = 1$

In the first round, none of the traders participating to the market are informed. The different agents' orders are:

$$x_{1i}^o = \frac{E_o(P_2) - P_1}{\rho var_o(P_2)} = \frac{\alpha_{21} + \alpha_{22}(\bar{v} + b) - P_1}{\rho \alpha_{22}^2 var_o(\bar{Y}_2)}.$$

$$x_{1j}^r = \frac{E_r(P_2) - P_1}{\rho var_r(P_2)} = \frac{\alpha_{21} + \alpha_{22}\bar{v} - P_1}{\rho \alpha_{22}^2 var_r(\bar{Y}_2)},$$

There are no feedback traders in the first round. However, the agents who will become informed subsequently anticipate the presence of such behavior for the next two rounds.

Again, the no-excess supply equation leads to

$$(O + R)\bar{x} = \sum_{i=1}^{O} x_{1i}^o + \sum_{j=1}^{R} x_{1j}^r.$$

The price P_1 can then be derived:

$$P_1 = \alpha_{21} + \alpha_{22}\bar{v} + \frac{-\rho\alpha_{22}^2\bar{x}(O + R)var_r(\bar{Y}_2)var_o(\bar{Y}_2) + O\alpha_{22}bvar_r(\bar{Y}_2)}{Ovar_r(\bar{Y}_2) + Rvar_o(\bar{Y}_2)}.$$

From the above expression the parameters α_{21} and α_{22} can be identified.

After some computations, one can show that $\alpha_{22} = \frac{N}{c\rho\beta F + d}$ where N is independent of F, with

$$\begin{aligned}
c &= -(M-1)[Ovar_r(P_3|\Phi_{2j})L^r s_1 + Rvar_o(P_3|\Phi_{2i})L^o s_2],\\
d &= \Lambda\eta h_v M^2 \kappa\gamma {h_\varepsilon}^2 L^o L^r (Ovar_r(P_3|\Phi_{2j}) + Rvar_o(P_3|\Phi_{2i})),\\
N &= (M-1)[O(\kappa + \gamma(M-1)) + MR](Ovar_r(P_3|\Phi_{2j})L^r z_1 + Rvar_o(P_3|\Phi_{2i})L^o z_2),
\end{aligned}$$

with

$$\begin{aligned}
s_1 &= [\kappa + \gamma(M-1)]h_\varepsilon + \eta h_v,\\
s_2 &= \eta\kappa\gamma(Mh_\varepsilon + h_v),\\
z_1 &= 2(\kappa + \gamma(M-1))h_\varepsilon + \eta h_v,\\
z_2 &= 2\eta\kappa\gamma M h_\varepsilon + \eta\kappa\gamma h_v.
\end{aligned}$$

Thus, we note that $c < 0$, $d > 0$ and $N > 0$.

After some computations we obtain the expression of α_{21} :

$$\alpha_{21} = \frac{A_1/B_1 + B_2}{D},$$

with

$$D = 1 - \frac{\rho var_o(P_3|\Phi_{2i})(2Mh_\varepsilon + h_v)\alpha_{33}^2 \beta F}{O(2Mh_\varepsilon + h_v)\alpha_{33}^2 + RMh_\varepsilon(Mh_\varepsilon + h_v)var_o(P_3|\Phi_{2i})},$$

$$B_1 = \frac{O(2Mh_\varepsilon + h_v)\alpha_{33}^2}{Mh_\varepsilon(Mh_\varepsilon + h_v)} + Rvar_o(P_3|\Phi_{2i}),$$

$$B_2 = \frac{\rho var_o(P_3|\Phi_{2i})(2Mh_\varepsilon + h_v)\alpha_{33}^2 \beta F \alpha_{22}}{O(2Mh_\varepsilon + h_v)\alpha_{33}^2 + Rvar_o(P_3|\Phi_{2i})Mh_\varepsilon(Mh_\varepsilon + h_v)}(v + E),$$

and

$$E = \frac{-\alpha_{22}(O + R)\left(\frac{1}{h_v} + \frac{1}{Mh_\varepsilon}\right)\left(\frac{1}{\eta h_v} + \frac{\gamma + M\kappa - \kappa}{M^2\kappa\gamma h_\varepsilon}\right)}{O\left(\frac{1}{h_v} + \frac{1}{Mh_\varepsilon}\right) + R\left(\frac{1}{\eta h_v} + \frac{1}{M^2\kappa\gamma h_\varepsilon}\right)},$$

$$A_1 = O\frac{(2Mh_\varepsilon + h_v)}{Mh_\varepsilon(Mh_\varepsilon + h_v)}\alpha_{33}^2(\alpha_{31} + Jv) + Rvar_o(P_3|\Phi_{2i})G + H,$$

$$J = \left((\alpha_{32} + \alpha_{33}) - \frac{\left(\frac{\alpha_{32}+\alpha_{33}}{\eta h_v} + \frac{\alpha_{32}}{M\kappa h_\varepsilon}\right)\left(\frac{\gamma + M\kappa - \kappa}{M^2\kappa\gamma h_\varepsilon} - \frac{1}{M\kappa h_\varepsilon}\right)\left(\frac{\alpha_{33}}{\eta h_v} + \alpha_{33}\left(\frac{1}{\eta h_v} + \frac{\gamma + M\kappa - \kappa}{M^2\kappa\gamma h_\varepsilon}\right)\right)\left(\frac{1}{\kappa h_\varepsilon} - \frac{1}{M\kappa h_\varepsilon}\right)}{\left(\frac{1}{\eta h_v} + \frac{1}{\kappa h_\varepsilon}\right)\left(\frac{1}{\eta h_v} + \frac{\gamma + M\kappa - \kappa}{M^2\kappa\gamma h_\varepsilon}\right) - \left(\frac{1}{\eta h_v} + \frac{1}{M\kappa h_\varepsilon}\right)^2}\right),$$

$$G = \alpha_{31} + (\alpha_{32} + \alpha_{31})v - \frac{\left(\frac{\alpha_{33}}{h_v} + \alpha_{32}\left(\frac{1}{h_v} + \frac{1}{Mh_\varepsilon}\right)\right)\left(\frac{1}{h_\varepsilon} - \frac{1}{Mh_\varepsilon}\right)v}{\left(\frac{1}{h_v} + \frac{1}{h_\varepsilon}\right)\left(\frac{1}{h_v} + \frac{1}{Mh_\varepsilon}\right) - \left(\frac{1}{h_v} + \frac{1}{Mh_\varepsilon}\right)^2},$$

$$H = \frac{-\rho(2Mh_\varepsilon + h_v)\alpha_{33}^2 var_o(P_3|\Phi_{2i})((O + R + F)x + \beta F P_0)}{Mh_\varepsilon(Mh_\varepsilon + h_v)}.$$

We obtain the condition of equilibrium by considering that the parameter before the mean of the signals α_{22} must be strictly positive.

In other words, the existence condition relates to prices being positive. In other words a sufficient condition ensuring that prices are positive is that both α_{21} and α_{22} are positive. Given that $\alpha_{22} = \frac{N}{c\rho\beta F + d}$ and that $N > 0$ for α_{22} to be positive we need $\beta F \rho c + d > 0$. The parameter α_{21} is defined as being equal to $\frac{A_1/B_1 + B_2}{D}$. It can be easily established that the numerator is positive. As a consequence for α_{21} to be positive, we need $D > 0$. In other words to ensure that both conditions are positive we need that $\min(\beta F \rho c + d, D) > 0$. We then run simulations for that equation and find that it defines a non-empty set.

Expressions for $t = 2$ and $t = 3$ volatility (Result 4.1) and their derivatives with respect to F

The variance for $t = 2$ is given by $var(P_2) = \alpha_{22}^2 var(\bar{Y}_2)$ with $\alpha_{22} = \frac{N}{c\rho\beta F + d}$.

The derivative of $var(P_2)$ with respect to F is then equal to

$$\frac{\partial var(P_2)}{\partial F} = \frac{\partial \alpha_{22}^2}{\partial F} var(\bar{Y}_2) = 2\alpha_{22}\frac{\partial \alpha_{22}}{\partial F} var(\bar{Y}_2) = 2\alpha_{22}\left(-c\rho\beta\frac{N}{(c\rho\beta F + d)^2}\right) var(\bar{Y}_2).$$

Given that $var(\bar{Y}_2) > 0$, $\alpha_{22} > 0$ and $-c\rho\beta\frac{N}{(c\rho\beta F+d)^2} > 0$ it can be established that $\frac{\partial var(P_2)}{\partial F} > 0$.

- **t = 3**

The variance for $t = 3$ prices is given by

$$\begin{aligned} var(P_3) &= \alpha_{32}^2 var(\bar{Y}_2) + \alpha_{33}^2 var(\bar{Y}_3) + 2\alpha_{32}\alpha_{33} cov\left(\bar{Y}_2, \bar{Y}_3\right) \\ &= (\alpha_{32}^2 + \alpha_{33}^2)(\frac{1}{h_v} + \frac{1}{Mh_\varepsilon}) + \frac{2\alpha_{32}\alpha_{33}}{h_v}. \end{aligned}$$

Using the fact that $\alpha_{32} = \alpha_{33} + \frac{\rho\beta F}{\Lambda}\alpha_{22}$, we can rewrite the variance of prices as follows

$$var(P_3) = \left(\alpha_{33} + \frac{\rho F\beta}{\Lambda}\alpha_{22}\right)\left[\left(\alpha_{33} + \frac{\rho F\beta}{\Lambda}\alpha_{22}\right)\left(\frac{1}{h_v} + \frac{1}{Mh_\varepsilon}\right) + \frac{2\alpha_{33}}{h_v}\right] + \alpha_{33}^2\left(\frac{1}{h_v} + \frac{1}{Mh_\varepsilon}\right).$$

The derivative is then given by the following expression

$$\frac{\partial var(P_3)}{\partial F} = 2\left(\frac{\rho\beta F}{\Lambda}\frac{\partial \alpha_{22}}{\partial F} + \frac{\rho\beta F\alpha_{22}}{\Lambda}\right)\left[\left(\alpha_{33} + \frac{\rho\beta F\alpha_{22}}{\Lambda}\right)\left(\frac{1}{h_v} + \frac{1}{Mh_\varepsilon}\right) + \frac{\alpha_{33}}{h_v}\right].$$

Knowing that the condition of the equilibrium gives that $\alpha_{22} > 0$ and $\frac{\partial \alpha_{22}}{\partial F} > 0$ we deduce can that $\frac{\partial var(P_3)}{\partial F} > 0$.

Expressions for $t = 2$ and $t = 3$ quality of prices (Result 4.2) and their derivatives with respect to F

We now look at the quality of prices. We first start with $t = 2$ and then turn to $t = 3$.

The quality of prices for $t = 2$ is given by $var(P_2 - \tilde{v})$ which is given by the following expression

$$var(P_2 - \tilde{v}) = \frac{\alpha_{22}^2}{h_v} + \frac{\alpha_{22}^2}{Mh_\varepsilon} + \frac{1}{h_v} - 2\frac{\alpha_{22}}{h_v} = \frac{(\alpha_{22} - 1)^2}{h_v} + \frac{\alpha_{22}^2}{Mh_\varepsilon}.$$

The derivative with respect to F is positive and given by

$$\frac{\partial var(P_2-\tilde{v})}{\partial F}=\frac{2(\alpha_{22}-1)}{h_v}\frac{\partial\alpha_{22}}{\partial F}+\frac{2\alpha_{22}}{Mh_\varepsilon}\frac{\partial\alpha_{22}}{\partial F}=2\frac{\partial\alpha_{22}}{\partial F}[\frac{\alpha_{22}-1}{h_v}+\frac{\alpha_{22}}{Mh_\varepsilon}].$$

In order to find the quality of prices for $t=3$, we proceed as for $t=2$. The quality of prices is then equal to

$$var(P_3-\tilde{v})=\frac{(\alpha_{32}+\alpha_{33}-1)^2}{h_v}+\frac{\alpha_{32}^2+\alpha_{33}^2}{Mh_\varepsilon}.$$

The derivative is then

$$\frac{\partial var(P_3-\tilde{v})}{\partial F}=\frac{\partial\alpha_{32}}{\partial F}\left(\frac{2(\alpha_{32}+\alpha_{33}-1)}{h_v}+\frac{2\alpha_{32}}{Mh_\varepsilon}\right).$$

Since $\alpha_{22}>\frac{N}{d}>1$, we conclude that $\frac{\partial var(P_2-\tilde{v})}{\partial F}>0$ and $\frac{\partial var(P_3-\tilde{v})}{\partial F}>0$.

Expression for the serial correlation of prices (Result 4.3) and its derivative with respect to F

We now look at the serial correlation of prices. It is given by $cov(P_3-P_2,P_2-P_1)$.

We have

$$cov(P_3-P_2,P_2-P_1)=cov(P_3-P_2,P_2)=cov(P_3,P_2)-cov(P_2,P_2).$$

After some manipulations, it can be rewritten as

$$cov(P_3-P_2,P_2-P_1)=\alpha_{22}\left(var(\bar{Y_2})\left(\alpha_{32}-\alpha_{22}\right)+\alpha_{32}cov\left(\bar{Y_2},\bar{Y_3}\right)\right).$$

Using the fact that $\alpha_{32}=\alpha_{33}+\frac{\rho\beta F}{\Lambda}\alpha_{22}$, $cov\left(\bar{Y_2},\bar{Y_3}\right)=\frac{1}{h_v}$ and that $var(\bar{Y_2})=\frac{1}{h_v+Mh_\varepsilon}$, we obtain

$$cov(P_3-P_2,P_2-P_1)=\alpha_{22}\left[\left(\alpha_{33}+\alpha_{22}\left(\frac{\rho\beta F}{\Lambda}-1\right)\right)\frac{1}{h_v+Mh_\varepsilon}+\frac{\alpha_{33}}{h_v}\right].$$

The derivative with respect to F is then given by

$$\frac{\partial cov(P_3-P_2,P_2-P_1)}{\partial F}=\left[\frac{\rho\beta}{\Lambda}\alpha_{22}^2+\frac{\partial\alpha_{22}}{\partial F}\left(\alpha_{33}+2\alpha_{22}\left(\frac{\rho\beta F}{\Lambda}-1\right)\right)\right]\left(\frac{1}{h_v}+\frac{1}{Mh_\varepsilon}\right)+\alpha_{33}\frac{\partial\alpha_{22}}{\partial F}\frac{1}{h_v}.$$

We now consider the case where $O=0$, after developing the different coefficients, we find:

$$\frac{\partial cov(P_3-P_2,P_2-P_1)}{\partial F}\frac{(c\rho\beta F+d)^3}{-c\rho\beta NR(M-1)var_b(P_3|\Phi_{2i})}=\left(-\rho\beta F+R(2Mh_\varepsilon+h_v)\right)Z$$

with

$$Z = \left[1 + \left(\frac{Mh_\varepsilon + h_v}{2Mh_\varepsilon + h_v} h_\varepsilon\right)\left(\frac{2}{h_v} + \frac{1}{Mh_\varepsilon}\right) - \left(\frac{1}{h_v} + \frac{1}{Mh_\varepsilon}\right)\right]$$

Expressions for Result 5.1: Trading profits

We now compute the expected profits for all type of traders for each trading round.

Third Round Expected Profits

Trader's $i = 1, ..., O$ overconfident expected profits, Π^o_{3i}, is given by

$$\Pi^o_{3i} = E(x^o_{3i}(\tilde{v} - P_3)) = E(x^o_{3i}\tilde{v}) - E(x^o_{3i}P_3),$$

where the demand, x^o_{3i}, is

$$x^o_{3i} = \alpha^*(y_{2i} + y_{3i}) + \beta^*(Y_2 + Y_3) - \gamma^* P_3 + \delta^*,$$

with $\alpha^* = \frac{(\kappa-\gamma)h_\varepsilon}{\rho}$, $\beta^* = \frac{\gamma M h_\varepsilon}{\rho}$, $\gamma^* = \frac{\eta h_v + 2(\kappa+\gamma(M-1))h_\varepsilon}{\rho}$ and $\delta^* = \frac{\eta h_v \bar{v}}{\rho}$.

After some simplifications and plugging in all the above expressions we obtain

$$\begin{aligned}\Pi^o_{3i} &= (\alpha^* + \beta^*)[\tfrac{2}{h_v} + 2\bar{v}^2 - 2\alpha_{31}\bar{v} - (\alpha_{32} + \alpha_{33})(\tfrac{2}{h_v} + 2\bar{v}^2 + \tfrac{1}{Mh_\varepsilon})] \\ &- \gamma^*[\alpha_{31}(1 - 2(\alpha_{32} + \alpha_{33}))\bar{v} - \alpha^2_{31} + (\alpha_{32} + \alpha_{33})(1 - (\alpha_{32} + \alpha_{33}))(\tfrac{1}{h_v} + \bar{v}^2) \\ &- \tfrac{(\alpha^2_{32} + \alpha^2_{33})}{Mh_\varepsilon} - \alpha^2_{31}] + \delta^*[\bar{v}(1 - (\alpha_{32} + \alpha_{33})) - \alpha_{31}].\end{aligned}$$

We now focus on the rational trader's $j = 1, ..., R$ expected profits, Π^r_{3j}. In order to compute Π^r_{3j} the same steps as for calculating the expected profit of the overconfident traders can be followed. The demand for the rational trader, x^r_{3j}, has the same linear form as the demand for the overconfident trader. The coefficients are given by $\alpha^* = 0$, $\beta^* = \frac{Mh_\varepsilon}{\rho}$, $\gamma^* = \frac{h_v + 2Mh_\varepsilon}{\rho}$ and $\delta^* = \frac{h_v \bar{v}}{\rho}$.

A rational trader's expected profits, Π^r_{3j}, can be obtained from (10.4) by replacing $\kappa = \gamma = \eta = 1$.

The feedback trader's $k = 1, ..., F$ expected profits Π^f_{3k} depends on the first and second round prices P_1 and P_2 through their demand $x^f_{3k} = \beta(P_2 - P_1)$. It can be calculated by computing the following expression

$$\Pi^f_{3k} = E(x^f_{3k}(\tilde{v} - P_3)) = E(x^f_{3k}\tilde{v}) - E(x^f_{3k}P_3).$$

After some computations and simplifications, it is given

$$\begin{aligned}\Pi^f_{3k} &= \beta\left[(\alpha_{21} - P_1)(\alpha_{31} + (1 - (\alpha_{32} + \alpha_{33}))\bar{v}) - \alpha_{22}\alpha_{31}\bar{v}\right] \\ &+ \beta\left[(\tfrac{1}{h_v} + \bar{v}^2)[\alpha_{22}(1 - \alpha_{32} - \alpha_{33})] - \alpha_{22}\alpha_{32}\tfrac{1}{Mh_\varepsilon}\right].\end{aligned}$$

Second Round Expected Profits

We now determine the overconfident trader's expected profits Π^o_{2i}. As before the expected profits are given by

$$\Pi^o_{2i} = E(x^o_{2i}(\tilde{v} - P_2)) = E(x^o_{2i}\tilde{v}) - E(x^o_{2i}P_2),$$

where $x^o_{2i} = \frac{E_o[P_3/\Phi_{2i}]-P_2}{\rho var_o[P_3/\Phi_{2i}]}$.

After some computations, we obtain

$$\Pi^o_{2i} = \frac{1}{\rho var_o[P_3/\Phi_{2i}]}[(A_o - \alpha_{21})(-\alpha_{21} + (1-\alpha_{22})\bar{v})(S_o + T_o - \alpha_{22})(\frac{1}{h_v} + \bar{v}^2 - \alpha_{21}\bar{v} - \alpha_{22}(\frac{1}{h_v} + \bar{v}^2 + \frac{1}{Mh_\varepsilon}))],$$

with

$$A_o = \alpha_{31} + (\alpha_{33} + \alpha_{22})\bar{v} - (cov_o(\tilde{y_{2i}}, P_3))(D^o_1 + D^o_3) + (\frac{cov_o(P_2,P_3)(D^o_2+D^o_3)}{\alpha_{22}})\frac{\bar{v}}{L^o},$$

$$S_o = \frac{cov_o(\tilde{y_{2i}},P_3)}{L^o}D^o_1 + \frac{cov_o(P_2,P_3)}{\alpha_{22}L^o}D^o_3,$$

$$T_o = \frac{cov_o(\tilde{y_{2i}},P_3)}{L^o}D^o_3 + \frac{cov_o(P_2,P_3)}{\alpha_{22}L^o}D^o_2.$$

The expected profits of a rational trader can be computed following the same steps, we then obtain that

$$\Pi^r_{2j} = \frac{1}{\rho var_r[P_3/\Phi_{2j}]}[(A_r - \alpha_{21})(-\alpha_{21} + (1-\alpha_{22})\bar{v})(S_r + T_r - \alpha_{22})(\frac{1}{h_v} + \bar{v}^2 - \alpha_{21}\bar{v} - \alpha_{22}(\frac{1}{h_v} + \bar{v}^2 + \frac{1}{Mh_\varepsilon}))],$$

with

$$A_r = \alpha_{31} + (\alpha_{33} + \alpha_{22})(\bar{v}) - (cov_r(\tilde{y_{2j}}, P_3))(D^r_1 + D^r_3) + (\frac{cov_r(P_2,P_3)(D^r_2+D^r_3)}{\alpha_{22}})\frac{\bar{v}}{L^r},$$

$$S_r = \frac{cov_r(\tilde{y_{2j}},P_3)}{L^r}D^r_1 + \frac{cov_r(P_2,P_3)}{\alpha_{22}L^r}D^r_3,$$

$$T_r = \frac{cov_r(\tilde{y_{2j}},P_3)}{L^r}D^r_3 + \frac{cov_b(P_2,P_3)}{\alpha_{22}L^r}D^r_2.$$

The expected profits of a feedback trader is given by:

$$\Pi^f_{2k} = E(x^f_{2k}(\tilde{v} - P_2)) = E(x^f_{2i}\tilde{v}) - E(x^f_{2k}P_2),$$

with $x^f_{2k} = \beta(P_1 - P_0)$.

The prices P_1 and P_0 are known before trading, the expected profit can be written as

$$\Pi^f_{2k} = \beta(P_1 - P_0)(\bar{v}(1-\alpha_{22}) - \alpha_{21}).$$

First Round Expected Profits

At the first round only the rational and the overconfident agents are trading. However, they are not informed at $t = 1$. As a consequence, the price at the first round P_1 is known before any trade is done.

The expected profits of an overconfident trader are given by

$$\Pi_{1i}^o = E[x_{1i}^o(\tilde{v} - P_1)],$$

with $x_{1i}^o = \frac{\alpha_{21}+\alpha_{22}\bar{v}-P_1}{\rho\alpha_{22}^2 var_o(\bar{Y}_2)}$. It is then straightforward to find that

$$\Pi_{1i}^o = \frac{\alpha_{21} + \alpha_{22}\bar{v} - P_1}{\rho\alpha_{22}^2 var_o(\bar{Y}_2)}(\bar{v} - P_1).$$

The expected profits of a rational trader are given by

$$\Pi_{1j}^r = \frac{\alpha_{21} + \alpha_{22}\bar{v} - P_1}{\rho\alpha_{22}^2 var_r(\bar{Y}_2)}(\bar{v} - P_1).$$