Tactical Asset Allocation: A Multivariate GARCH Approach

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ABSTRACT

This paper examines how to improve tactical asset allocation by better risk management instead of concentrating on maximising returns. This is achieved by using forecasts of the time-varying conditional covariance matrix of returns obtained from a new specification of the multivariate GARCH process that is particularly well suited to modelling asset returns due to its generality, parameter parsimony and relative ease of estimation. We show that for a portfolio of four UK assets over the period 1976-1997 it would be possible to reduce portfolio risk by on average 5% compared with using the constant sample covariance matrix.

Keywords: Asset allocation, risk management, multivariate GARCH.
JEL Classification: G1

1 Introduction

This paper examines how to improve tactical asset allocation by better risk management instead of concentrating on maximising returns. There are three reasons for this different focus. The first is theoretical. The convention is to use the unconditional covariance matrix of returns. In principle, this is constant and need only be estimated once. Risk management therefore becomes a one time calculation that need not be repeated. For short-term asset allocation, however, one should use the conditional, not the unconditional covariance matrix, and this changes each period - sometimes considerably - with the implication that the mean-variance portfolio frontier is time-varying, not fixed. The second reason is empirical. The conditional covariance matrix is sufficiently serially correlated to be reasonably well predicted, whereas returns have little serial correlation, and hence are largely unpredictable. The third reason is that recent advances in computing power makes this strategy feasible, and hence of potential practical value to fund managers and other large institutional investors. We implement this procedure using a new specification of the multivariate GARCH process that is particularly well suited to modelling asset returns due to its generality, parameter parsimony and relative ease of estimation. This is achieved by separating the conditional second moment matrix of returns into long-run and short-run components, and at the same time ensuring that the time-varying conditional covariance matrix generated by the model is positive definite.

Current practice among institutional investors is normally to construct financial portfolios in two stages, using different personnel for each stage. The first stage is the asset allocation decision which determines the proportions in which the main asset classes - equity, bonds and cash - are held. Typically, this also involves a further breakdown into the class and the national origin (or currency denomination) of the asset. The second stage involves individual asset selection. Most of the effort and resources go into the second stage which is carried out by investment analysts. Relatively few resources go into the first stage and often quite crude methods of analysis are used, such as assuming that the relevant covariance matrix of returns is constant.

The textbook theories of portfolio management focus on the individual assets at the outset. The best known of these are the mean-variance analysis of Markowitz (1952) and the Sharpe-Lintner capital asset pricing model (CAPM). In the former investors are assumed to minimise the riskiness of the total return on the portfolio subject to achieving a target rate of return. For example, a typical requirement of a fund manager is that the portfolio achieve, say, 1% above the market return. In the CAPM the optimal portfolio trades-off expected return and variance to maximise a - typically quadratic -

welfare function in mean and variance over a one period horizon. In practice, because returns - especially equity returns - are not forecastable (they are virtually serially independent), the emphasis is on minimising the variance of the portfolio's return generally by choosing appropriately the proportions in which each asset is held in the portfolio. In making this calculation it is usually assumed that the variance-covariance matrix of returns, and hence portfolio shares, is constant over time. Strictly, CAPM theory requires the use of the conditional covariance matrix of returns, not the unconditional, or long-run, covariance matrix. It can be shown, however, that the conditional covariance matrix will in general be time-varying, and not constant. In the absence of transactions costs, this implies that the optimal portfolio will need to be re-balanced each period. Moreover, unlike returns, the conditional covariance matrix is highly serially correlated and hence its changes are reasonably well predictable. This suggests that the aim of tactical asset allocation should be to exploit the regularities in the covariance structure of returns with the aim of reducing risk. Maximising returns can then be left to the second stage of stock selection.

In this paper we use a multivariate GARCH (M-GARCH) model of returns to forecast their covariances. We then use these forecasts to generate the portfolio frontier period by period and identify the optimal portfolio of risky assets by finding the point of tangency between the portfolio frontier and a line drawn from the risk free rate (the Capital Market Line). Given a target rate of return for the portfolio, the optimal proportions in which each asset should be held can be calculated. Typically this results in going short in at least one asset. Since some investors may be constrained from doing this, we also calculate the optimal proportions when they are constrained to be non-negative.

The illustration in the paper is taken from the viewpoint of a UK investor who each month wishes to form an optimal portfolio allocation consisting of four types of domestic financial asset. Three are risky: UK equities, UK government bonds with more than 15 years to maturity and UK government bonds with less than 5 years to maturity. The fourth is riskless: 30-day Treasury bills. Although this can be seen as simply the first stage decision, the recent successes of index tracker funds compared with managed funds suggests it may be all that is needed to extend the benefits of holding the stock index to a broader portfolio consisting of equity and bonds.

Adopting the two-fund separation theorem, we believe that all investors, regardless of preferences, will hold a combination of only two mutual funds namely the riskless asset and the optimal portfolio of risky assets. Therefore, our aim is not to identify the final investment position of an investor, but rather to identify the proportions in which the risky assets should be held.

Each investor may then choose their preferred combination of these funds based on subjective preference.

The plan of the paper is as follows. In section 2, there is a brief presentation of the mean-variance portfolio theory of Markowitz. Section 3 outlines the econometric techniques used and presents our model. A description of the data is contained in section 4. Section 5 presents and discusses the results of the estimation while section 6 looks at the implications of our results for tactical asset allocation. Finally, section 7 contains our concluding remarks.

2 Mean - Variance Portfolio Analysis

Primarily as a way of introducing notation, in this section we state briefly how we obtain the asset allocation. Since forecasts of equity and bond returns are highly inaccurate due to their near serial independence compared with forecasts of their covariance structure which are highly serially correlated, we focus on choosing a minimum variance portfolio of excess returns over the risk-free rate. In effect this gives us the tangency portfolio of risky assets. The optimal portfolio can then be derived by combining this with the riskless asset by, for example, choosing a target rate of return for the portfolio, or by using the capital asset pricing model of Lintner(1965) and Sharpe(1964) with an explicit trade-off of mean and variance. Either way, the risky assets appear in the optimal portfolio in the same relative proportions as in the tangency portfolio. All of this follows directly from Markowitz's portfolio theory and the two fund separation theorm.[1,2]

Thus the optimal portfolio is obtained using mean-variance analysis as follows.[3] We assume that investors are forming their portfolios for one period only, at the beginning of period t, using the information then available. Let $\mathbf{R}_{t+1} = (R_{1t+1}....R_{nt+1})'$ denote an $nx1$ vector of risky asset returns realised during period t and paid at the beginning of period $t+1$. It is assumed that the conditional distribution of $\mathbf{R}_{t+1}$has mean $E_t\mathbf{R}_{t+1}$,which are not all equal, and non-singular covariance matrix $\mathbf{\Omega}_t = \{\sigma_{ij,t}\}$,for $i, j = 1, 2, ...n$. Also, let $\mathbf{w}_t = (w_{1t}.....w_{nt})'$ be an $nx1$ vector denoting the proportion of an individual's wealth allocated to the ith asset. Since it is assumed that all funds are invested, the sum of the weights must equal one, $\sum_i w_{it} = \mathbf{w}_t'\mathbf{i} = 1$,where $\mathbf{i}$ is an $nx1$ vector of ones. The conditional distribution of the return on the portfolio $R_{p,t+1}$ therefore has expected return

$$E_t R_{p,t+1} = \mathbf{w}_t' E_t \mathbf{R}_{t+1} = \sum_i w_{it} E_t R_{it+1} \tag{2.1}$$

and variance

$$\sigma_{pt}^2 = \mathbf{w}_t'\mathbf{\Omega}_t\mathbf{w}_t = \sum_i\sum_j w_{it}w_{jt}\sigma_{ij,t} \qquad (2.2)$$

It assumed that investors allocate the whole of their wealth by choosing a portfolio of risky assets that minimises the conditional variance of its return subject to a target rate of return for the portfolio of μ_t. Thus, the problem has the standard Markowitz(1952) set-up

$$\begin{array}{cc} \text{Minimise} & \mathbf{w}_t'\mathbf{\Omega}_t\mathbf{w}_t \\ \text{subject to:} & \\ & \mathbf{w}_t'E_t\mathbf{R}_{t+1} = \mu_t \\ & \mathbf{w}_t'\mathbf{i} = 1 \end{array} \qquad (2.3)$$

in which there is no constraint on the sign of the asset weights thereby allowing short sales. Using the method of Lagrange multipliers, where λ_1 and λ_2 are the multipliers for the constraint on the target return and the complete allocation of wealth, the solution to this problem is

$$\begin{aligned} \mathbf{w}_t &= \tfrac{1}{2}\mathbf{\Omega}_t^{-1}\left[\begin{array}{cc} E_t\mathbf{R}_{t+1} & \mathbf{i} \end{array}\right]\left[\begin{array}{c} \lambda_1 \\ \lambda_2 \end{array}\right] \\ &= \mathbf{\Omega}_t^{-1}\left[\begin{array}{cc} E_t\mathbf{R}_{t+1} & \mathbf{i} \end{array}\right]\mathbf{A}_t^{-1}\left[\begin{array}{c} \mu_t \\ 1 \end{array}\right] \end{aligned} \qquad (2.4)$$

$$\begin{aligned} \sigma_{pt}^2 &= \mathbf{w}_t'\mathbf{\Omega}_t\mathbf{w}_t \\ &= \left[\begin{array}{cc} \mu_t & 1 \end{array}\right]\mathbf{A}_t^{-1}\left[\begin{array}{cc} E_t\mathbf{R}_{t+1} & \mathbf{i} \end{array}\right]'\mathbf{\Omega}_t^{-1}\mathbf{\Omega}_t\mathbf{\Omega}_t^{-1}\left[\begin{array}{cc} E_t\mathbf{R}_{t+1} & \mathbf{i} \end{array}\right]\mathbf{A}_t^{-1}\left[\begin{array}{c} \mu_t \\ 1 \end{array}\right] \\ &= \left[\begin{array}{cc} \mu_t & 1 \end{array}\right]\mathbf{A}_t^{-1}\left[\begin{array}{c} \mu_t \\ 1 \end{array}\right] \\ &= \left[\begin{array}{cc} \mu_t & 1 \end{array}\right]\tfrac{1}{\Delta_t}\left[\begin{array}{cc} c_t & -b_t \\ -b_t & a_t \end{array}\right]\left[\begin{array}{c} \mu_t \\ 1 \end{array}\right] \\ &= \tfrac{1}{\Delta_t}(a_t - 2b_t\mu_t + c_t\mu_t^2) \end{aligned} \qquad (2.5)$$

where

$$\mathbf{A}_t = \left[\begin{array}{cc} a_t & b_t \\ b_t & c_t \end{array}\right] = \left[\begin{array}{cc} E_t\mathbf{R}_{t+1}'\mathbf{\Omega}_t^{-1}E_t\mathbf{R}_{t+1} & E_t\mathbf{R}_{t+1}'\mathbf{\Omega}_t^{-1}\mathbf{i} \\ E_t\mathbf{R}_{t+1}'\mathbf{\Omega}_t^{-1}\mathbf{i} & \mathbf{i}'\mathbf{\Omega}_t^{-1}\mathbf{i} \end{array}\right] \qquad (2.6)$$

and $\Delta_t = (a_tc_t - b_t^2) > 0$ by the Cauchy-Schwartz inequality since it is assumed that $\mathbf{\Omega}_t$ is invertible and not all assets have the same mean. Taking the square root of equation (2.5) gives the standard relationship between the standard deviation and the expected return of a portfolio which allows us to draw the portfolio frontier.

3 The Econometric Model

Early work on asset allocation worked within the static framework introduced by Markowitz(1952). In particular, the covariance matrix of returns was assumed to be constant. There is now ample evidence that this assumption is incorrect, and that the covariance matrix of returns is time varying. The first attempts to take this into account assumed that the covariance matrix was sufficiently slowly changing that it could be estimated by the unconditional matrix of past returns and then treated as though it would be constant for a fixed period in the future; see for example, Grubel(1968) and Levy & Sarnat(1970). This is also the assumption implicit in most cross-section tests of CAPM; see, for example, Fama and Macbeth (1973) and Fama and French (1989).

The development of the family of ARCH (Engle(1982)) and GARCH (Bollerslev(1986)) models has made it possible to allow the covariance matrix to be continuously changing. They also help to capture other features of asset returns such as thick tails and volatility clustering. For portfolio analysis it is necessary to use a multivariate ARCH or GARCH framework since the focus is on the covariance of returns. Another reason for using a multivariate model is given by Bollerslev, Engle & Nelson(1994) who observe that

> "Financial market volatility moves together over time across assets and markets. Recognising this commonality through a multivariate modelling framework leads to obvious gains in efficiency. Several interesting issues...also call for an explicit multivariate ARCH approach in order to capture the temporal dependencies in the conditional variances and covariances" (Pp 3002).

Examples of the use of multivariate ARCH models to model asset returns are Frankel(1982), Poterba & Summers(1987), Bollerslev, Engle and Wooldridge(1988), Engle, Frankel, Froot & Rodrigues(1989), Giovannini & Jorion(1990), Thomas & Wickens(1993) and Cumby, Figlewski & Hasbrouck(1994) etc. A survey of these methods is provided by Bollerslev, Chou & Kroner(1992).

A major drawback with this approach is that existing models require a large number of parameters to be estimated, but the more parameters estimated, the more difficult it is to achieve convergence of the likelihood function. For example, even for a multivariate GARCH(1,1) for the most general formulation of the model, termed the vec representation by Engle and Kroner(1993), the number of parameters to be estimated increases at the rate of n^4, where n is the number of variables. Thus, although in principle, for portfolio analysis, one would like a model capable of handling a large number of assets simultaneously, and with a structure flexible enough to capture the dynamic and leptokurtic characteristics of the distribution of asset returns, in practice, this choice is severely limited by numerical problems. In this paper we propose a variant of the multivariate GARCH model that is better suited to portfolio analysis in that it allows a considerable degree of flexibility in the conditional covariance matrix of returns yet is economical in the number of parameters it uses.

Denoting $\mathbf{r}_{t+1}$ as an $nx1$ vector of asset *excess* returns over the risk free rate, a general form of the multivariate GARCH(p,q) model with constant mean can be written as

$$\begin{aligned} \mathbf{r}_{t+1} &= \boldsymbol{\upsilon} + \boldsymbol{\xi}_{t+1} \\ \boldsymbol{\xi}_{t+1} &\mid \Psi_t \sim N(0, \boldsymbol{\Omega}_t) \\ vech(\boldsymbol{\Omega}_t) &= \boldsymbol{\Lambda} + \Sigma_{i=1}^{p} \boldsymbol{\Phi}_i vech(\boldsymbol{\Omega}_{t-i}) + \Sigma_{j=0}^{q-1} \boldsymbol{\Theta}_j vech(\boldsymbol{\xi}_{t-j} \boldsymbol{\xi}'_{t-j}) \end{aligned} \tag{3.1}$$

where $E_t \xi_{t+1} \xi'_{t+1} = \boldsymbol{\Omega}_t$ is the time-varying conditional covariance matrix of excess returns and vech(.) is the vector half-operator which stacks the lower triangle of a square matrix into a column vector. Since $\boldsymbol{\Omega}_t$ is symmetric, $vech(\boldsymbol{\Omega}_t)$ contains all the unique elements of the matrix. $\boldsymbol{\nu}$ is a vector of ones; $\boldsymbol{\xi}_{t+1}$ is a $nx1$ vector of zero mean iid errors. $\boldsymbol{\Phi}$ and $\boldsymbol{\Theta}$ are both square matrices of size $n(n+1)/2$, $\boldsymbol{\Lambda}$ is a size $n(n+1)/2$ vector and n is the number of assets in the problem. $\boldsymbol{\Lambda}, \boldsymbol{\Phi}$ and $\boldsymbol{\Theta}$ are all unrestricted. It follows that in 3.1 there are $n(n+1)/2 + (p+q)n^2(n+1)^2/4$ parameters to be estimated for the covariance matrix and just n for the means. Thus the number of parameters increases at a rate of n^4. Even when $n = 3$ and $p = q = 1$ the conditional second moments require the simultaneous estimation of 78 parameters. This makes equation 3.1 an infeasible model specification for asset allocation, especially if we would like to introduce additonal assets.

In this paper we propose a new specificaction of the multivariate GARCH model that greatly economises on the number parameters required. Our model is a variant of the Berndt, Engle, Kraft & Kroner (BEKK) representation. This ensures that the resulting time-varying covariance matrices of asset excess returns are symmetric and positive definite.[4] Our model also allows for a time-varying conditional mean.

$$\begin{aligned} \mathbf{r}_{t+1} &= \boldsymbol{\upsilon} + \boldsymbol{\Gamma}\mathbf{r}_t + \boldsymbol{\Upsilon} dum87 + \boldsymbol{\xi}_{t+1} \\ \boldsymbol{\xi}_{t+1} &\mid \Psi_t \sim N(0, \boldsymbol{\Omega}_t) \\ \boldsymbol{\Omega}_t &= \mathbf{V}'\mathbf{V} + \boldsymbol{\Phi}'(\boldsymbol{\Omega}_{t-1} - \mathbf{V}'\mathbf{V})\boldsymbol{\Phi} + \boldsymbol{\Theta}'(\boldsymbol{\xi}_t\boldsymbol{\xi}_t' - \mathbf{V}'\mathbf{V})\boldsymbol{\Theta} \end{aligned} \tag{3.2}$$

where the vector of excess returns $r = (uke,\ lbd, sbd)'$, uke is the excess return of UK equities, lbd is the excess return of UK government bonds with more than 15 years to maturity and sbd is the excess return of UK govt bonds with less than 5 years to maturity respectively and $dum87$ is a dummy variable for the October 1987 stock market crash. $\boldsymbol{\Gamma}$ is a $3x3$ matrix of regression parameters and $\boldsymbol{\Upsilon}$ is a $3x1$ vector of parameters. $\mathbf{V}$, $\boldsymbol{\Phi}$ and $\boldsymbol{\Theta}$ are all nxn symmetric matrices. By making the matrices symmetric rather than unrestricted we are able to economise on parameters, since now only $3n(n+1)/2$ parameters are required for the covariance matrix, and the numbers of parameters increases at the rate n^2 instead of n^4. When $n = 3$ the number of parameters to be estimated is reduced from 78 to 18, a substantial saving. It might seem that an equivalent specification would be to make the matrices triangular, but in fact this has the disadvantage of restricting the dynamic structure of the covariance matrix unnecessarily by introducing an additional lag involving cross-effects[5].

It will be noted that in this specification the time-varying covariance matrix is written in error correction form. This has the advantage of separating the long-run from the short-run dynamic structure of the covariance matrix. The first term on the right hand-side of the last equation is the long-run, or unconditional, covariance matrix. The other two terms show the short-run deviation from the long run. By formulating the conditional variance-covariance structure in this way, we can decide more easily if the short-run dynamics have a useful additional contribution to make, and if the increased generality a parameter offers is worth the additional computational burden. As the number of assets increases it may be sensible to further restrict these matrices by, for example, setting some of the coefficients to zero and closing off some of the transmission channels in the long and short run.

4 The Data

This paper uses time series data on broad classes of UK financial assets. In particular, we focus on three risky assets and one riskless asset. The risky assets used in the analysis are equities, represented by the Financial Times All Share Index; long UK government bonds represented by the FT British government stock over 15 years index; and short government bonds represented by the FT British government stock under 5 years index. The data used in this paper are annualised monthly total returns for each asset.[6] The total return data is calculated so as to take account of dividend payments in the case of equities and coupon payments in the case of government bonds. Both dividends and coupon payments are treated as if they were received in equal amounts throughout each working day of the year rather than as a lump sum at one or two distinct points in time. The rate of return on the UK government 30-day Treasury Bill is taken as the risk free rate of interest available to the investor. It is true to say that this asset is riskless at least in the nominal sense. All data is sourced from DATASTREAM.

The data covers a sample period in excess of 20 years beginning in January, 1976 and finishing in February, 1997. This sample yields a total of 251 usable observations. We have chosen to work exclusively with rates of return in excess of the risk free rate. This approach has been adopted to prevent volatility in the risk free rate from incorrectly contributing to the risk of the optimal risky portfolio. Since the risk free rate is perfectly predictable at the start of each period and therefore part of the investor's information set when the allocation decision is made, its inclusion would tend to over-estimate the total risk of the portfolio.

From an econometric point of view, there is a further benefit from working with excess returns, namely that all series are stationary and do not require differencing.[7]

5 Estimation and Results

5.1 Convergence of the Likelihood Function

The model, equation (3.2), was estimated by maximising the log likelihood function

$$LogL = -\frac{nT}{2}\log(2\pi) - \frac{1}{2}\sum_t (\log \mid \mathbf{\Omega}_t \mid - \boldsymbol{\xi}'_{t+1}\mathbf{\Omega}_t^{-1}\xi_{t+1}) \qquad (5.1)$$

recursively using the Berndt, Hall, Hall & Hausmann (BHHH) algorithm. n is the number of assets and T is the number of observations.

Due to the large number of parameters involved, the main practical problem in estimating M-GARCH models is to achieve convergence. By greatly reducing their number, our parameterization significantly improves the speed of convergence.[8] Choosing starting values near the optimum is also very helpful. The error correction structure of (3.2) is useful in this respect as it enables us to use the unconditional error covariance matrix to obtain a consistent estimate of $\mathbf{V}$. Thus, we obtain our initial values of the error term, and hence the unconditional error covariance matrix, from Ordinary Least Squares estimates of the conditional mean equations assuming a homoskedastic error covariance matrix. We then obtain an initial estimate of $\mathbf{V}$ using a choleski factorisation of the resulting estimate of the unconditional covariance matrix. The $\mathbf{\Phi}$ and $\mathbf{\Theta}$ matrices are initialised with an arbitrary small number along the diagonal and zeros elsewhere. In this way we are able to achieve the BHHH convergence criteria.

5.2 Estimates

First we report the estimates for the parameters associated with the conditional mean, and then for the conditional variance. t-statistics are in parentheses.

5.2.1 Conditional Mean

$$\boldsymbol{\upsilon} = \begin{bmatrix} 13.39 \\ (3.75) \\ 4.07 \\ (1.35) \\ 0.93 \\ (0.87) \end{bmatrix}, \mathbf{\Gamma} = \begin{bmatrix} 0.006 & 0.096 & 0.267 \\ (0.09) & (0.95) & (0.83) \\ 0.042 & -0.016 & 0.318 \\ (0.71) & (-0.13) & (1.26) \\ -0.027 & 0.038 & 0.015 \\ (-1.20) & (1.60) & (0.179) \end{bmatrix}, \mathbf{\Upsilon} = \begin{bmatrix} -392.25 \\ (-4.98) \\ 0 \\ \\ 0 \end{bmatrix}$$

5.2.2 Conditional Variance

V, $\mathbf{\Phi}$ and $\mathbf{\Theta}$ are all symmetric matrices.

$$\mathbf{V} = \begin{bmatrix} 57.37 & & \\ (22.58) & & \\ 23.15 & 30.72 & \\ (6.94) & (1.99) & \\ 8.72 & 6.81 & 9.06 \\ (7.02) & (1.77) & (7.33) \end{bmatrix}$$

$$\mathbf{\Phi} = \begin{bmatrix} 0.82 & & \\ (3.44) & & \\ 0.09 & 0.90 & \\ (0.12) & (7.40) & \\ 0.29 & 0.22 & 0.31 \\ (1.37) & (1.11) & (1.5) \end{bmatrix}, \mathbf{\Theta} = \begin{bmatrix} 0.06 & & \\ (1.02) & & \\ 0.03 & 0.08 & \\ (0.60) & (1.20) & \\ 0.11 & 0.02 & 0.16 \\ (4.62) & (0.40) & (1.10) \end{bmatrix}$$

5.3 Discussion of the Results

5.3.1 Conditional Mean

The lack of significance in the estimates of $\mathbf{\Gamma}$ are consistent with the usual finding that total stock and bond returns are serially uncorrelated. Consequently we assume a constant vector of expected asset excess returns to generate the portfolio shares. This has the added advantage that all of the variation in the estimated frontiers, and hence the portfolio shares, can be attributed to variation in the conditional covariance matrix of excess returns.

This is also the assumption made by Cumby, Figlewski and Hasbrouck(1994) who use the historical mean of each asset as its expected value. Jobson & Korkie(1981) advocate the use of global shrinkage based on Stein estimators whereby all assets of the same class have the same expected excess return. This is an extreme case of Stein estimation with the individual asset being assigned a weight of zero and the global mean having a weight of one. Jobson & Korkie show that this approach significantly improved the practical application of the mean-variance framework. Since we are working with financial asset indices as opposed to individual securities, these appoaches reduce to the same thing. Another reason for making this assumption is that the sensitivity of the portfolio shares to small variations in the mean is far greater than that to variations in the covariance matrix, Kallberg and Ziemba (1984). Best and Grauer (1991) show that even small changes in the mean vector can result in dramatic variation in the composition of the estimated optimal portfolio of risky assets.

Continuous re-balancing of the portfolio to changes in the predicted excess return would not only be expensive due to transaction costs, it would also be counter-productive because of the lack of persistence of the deviations of excess returns from their unconditional means. This is not true of rebalancing due to changes in the conditional variance because of their much higher degree of persistence and their lower volatility.

5.3.2 Conditional Variance

The estimates of the elements of $\mathbf{V}$ are all significant at the 10% level and all but one are significant at the 5% level. Although, many of the elements of $\mathbf{\Phi}$ and $\mathbf{\Theta}$ are not significant, even at the 10% level, sufficient are significant to show that there are deviations of the short-run from the long-run covariance matrix. In the main, these are due to autocorrelation in the conditional variances, but there is also a significant effect arising from the interaction between the excess returns on equity and short-term government bonds.

Roughly speaking, and ignoring the other elements, the greater the elements on the leading diagonals of $\mathbf{\Phi}$ and $\mathbf{\Theta}$, the more the conditional covariance matrix deviates from the long-run value. The more significant these elements, the more predictable are these deviations. The estimates suggest that the deviations are both persistent and predictable. Figures **??**-**??** plot the conditional and unconditional variances from the three excess returns. The deviations from the long run are most persistent for the long government bond. It is also clear that equities have predictable and persistent deviations, most notably in 1983-4 and since 1993. These are precisely the conditions in which there is greatest benefit to taking account of a time-varying covariance matrix of excess returns in determining asset allocation.

6 Optimal Asset Allocation

6.1 Frontier Movements

Apart from changes to the target rate of return, variations in the optimal portfolio weights are due entirely to movements in the portfolio frontier brought about by new information on next period's conditional covariance matrix. This new information is the cause of the time variation in the conditional covariance matrix. Some idea of the extent of the movements in the frontier within the sample period can be obtained from Figures 2-6. The position of the frontiers reflect the minimum portfolio standard deviation for a given portfolio return, hence this is just another way of comparing portfolio standard deviations. Figure 2 shows how the frontier has moved over time by displaying the frontier in December at four year intervals from 1976-96. Figure 3 provides information on the distribution of the frontiers since it displays the minimum, maximum, mean and median frontiers for the whole sample. The *global minimum variance portfolio* of the frontier was calculated for each monthly period of the analysis and these portfolios were used to compute the frontiers depicted in this figure. It reveals that the distribution is highly positively skewed with a long tail to the right. The standard deviations of the minimum variance portfolios range from a minimum of approximately 8% in February 1996 to a maximum of 28% in September 1981. The skewness obviously has major implications for the choice of optimal portfolio. Assuming the same frontier for each period by taking the mean or the median frontier would have seriously underestimated the riskiness of the assets in September 1981. Figure 4 examines the last six months of 1981 in more detail. It shows how volatile the frontier can be over a short time horizon.

A very revealing comparison is between the frontiers based on constant covariance matrices computed from both a simple OLS estimate of the unconditional covariance matrix and the long-run matrix ($\mathbf{V}'\mathbf{V}$) of our model and the frontiers obtained from using a time-varying conditional covariance matrix. In Figure 5 we include the frontiers generated by the OLS estimate, the long-run unconditional covariance matrix and the mean and median of the conditional covariance matrices. Both the frontiers associated with the unconditional covariance matrices lie further from the origin than their time-varying counterparts. This shows the considerable reduction in riskiness of the portfolio that can be achieved by using the conditional covariance matrix instead of a constant, unconditional, covariance matrix. The frontier generated by the OLS estimate is to the right of the others, demonstrating that such a simple estimate tends to overestimate the riskiness of the assets and that even in using our model only to estimate the unconditional covariance matrix, there are risk reduction gains to be exploited over using the more simple OLS approach.

Finally, we examine the consequences of allowing the conditional mean to be time varying by not omitting the insignificant terms in the conditional mean in the above calculations. The means and medians of the frontiers computed from including and excluding the lagged dependent variable from the model (and then re-estimating the model) are shown in Figure 6. We find that distribution of the frontiers for the model that includes the lagged excess returns in the model lies to the left of the distribution of the frontiers that restricts the model by excluding these lags. Thus, even if the dynamics in the conditional mean are imprecisely estimated, including them in the model results in a substantial reduction in portfolio risk.

6.2 Optimal Portfolios

6.2.1 Unrestricted weights

The optimal portfolio for each period is constructed by finding the point of tangency of a line drawn through the origin to the implied mean-variance portfolio frontier of excess returns estimated each period from the conditional covariance matrix obtained above. The slope of the tangent in each period (the Sharpe ratio) is given by

$$m_t = \frac{c_t \mu_t - b_t}{\sqrt{(a_t - 2b_t \mu_t + c_t \mu_t^2)(a_t c_t - b_t^2)}} \tag{6.1}$$

At the point of tangency the expected excess return of the optimal portfolio is $E_t R_{p,t+1} = \mu_t = a_t/b_t$. (Further details of the derivation are given in Appendix 1.)

Figure 7 shows how the expected excess return of the optimal portfolio and its standard deviation change over time. It also shows their direct relation, with the standard deviation much more volatile than the excess return. Table 6.1 summarises the key features of the optimal portfolios.

We begin the analysis by computing the optimal asset proportions for a buy and hold (constant proportions) portfolio generated by both our estimates of a constant unconditional covariance matrix. The asset holdings are shown in table 6.2. The two portfolios differ greatly, especially in the importance attached to the government bonds. The portfolio based on the OLS estimate has a large equity holding which is consistent with it producing the most risky portfolio frontier in Figure 5.

Figure 8 shows how the share of total wealth allocated to each asset varies over time in the optimal portfolios and is based on equation (2.4). It also captures the importance of the short-run deviations away from the asset proportions suggested by our long-run unconditional matrix for a buy and hold portfolio. Table 6.3 reports some summary statistics. No restriction on short sales is imposed so the weights can exceed unity and be less than zero. The results show that the optimal portfolio frequently involves taking a short position in the shorter maturity UK government bond, especially in the earlier part of the sample, thereby allowing a larger positive position in the relatively higher return assets. As expected, given the literature on the *equity premium puzzle*, equities are the dominant asset. Their share is on average 70% of the portfolio. It never falls below 38%. On a number of occasions more than 100% of total wealth is invested in equities, the maximum being 160%. The share of the long bond is always less than that of equities and has a mean of 20%. Only once in 251 periods is the long bond shorted. The average holding of the shorter bond is 10% and the variation in its share is the greatest.

Two practical considerations suggest that this may not always be an attractive or viable asset allocation strategy. Firstly, these calculations ignore the transactions costs of continuously rebalancing the portfolio. Given the volatility of the shares, this could be considerable and may act as a deterent to implementing this investment strategy. Secondly, many investors are precluded from going short either by choice or by law. Mutual fund managers in the UK are prohibited by law from holding short positions. We therefore examine optimal asset allocation subject to a non-negativity constraint on asset shares.

6.2.2 Restricted weights

Although it is not possible to provide a closed-form expression for the portfolio shares when a non-negativity constraint is imposed, they can be obtained for each period using Quadratic Programming.[9] Instead of solving for the mean return for the optimal portfolio as above, it is now necessary to specify a target rate of return. We choose the target return to be the average return on the unrestricted optimal portfolio. This implies that, in terms of the mean portfolio return, investors are not penalised by the restriction, and it aids comparisons with the unrestricted case.

The restricted shares are displayed in Figure 9, and summary statistics are reported in Table 6.4. The main change compared with the unrestricted shares is the much lower variation in the shares. Their mean values are hardly altered. For equities the share now ranges between 62% and 72% of the portfolio compared with 38% to 160% previously. The shares of the two types of government bonds are almost a mirror image of each other, and their range of variation is dramatically reduced. This indicates that most of the portfolio rebalancing is between longer-dated and shorter-dated government bonds.

Although this appears to be a much more viable investment strategy than having unrestricted portfolio shares, it should be recalled that we have identified only the relative shares of the risky assets. In practice, it is also necessary to decide the proportion of total wealth to be allocated to the risk-free asset. This will depend on the individual preferences of each investor. If an individual's preference is to bear less risk than that associated with the risky portfolio, then a proportion of wealth should be allocated to the risk-free asset. If an investor is willing to bear more risk than the risky portfolio, then it is necessary to take a short position in the riskless asset and go long only in the risky assets. Whatever the preferences of the individual, total funds can be allocated between these two mutual funds. This paper does not indicate the final investment position of any investor but it identifies the two mutual funds between which resources should be allocated so as to minimise the riskiness of the portfolio in achieving a given target rate of return.

A quick, yet informative, check on the validity of this approach is to compare the actual performances of the constrained time-varying portfolio with a more traditional buy and hold portfolio. The asset proportions of the latter being determined by the long-run unconditional covariance matrix and set out in table 6.2. Since our goal is to minimise risk, we would expect that the variance of the continuously re-balanced portfolio should not be greater than the variance of the other. We would also hope that the returns would not be significantly different. Figures **??** & **??** support our hypothesis. The top panel of the graph shows the ratio of the return on the time-varying portfolio to the return on the buy and hold portfolio. The ratio is usually very close to unity and shows that neither portfolio consistently outperforms the other. The lower panel plots the ratio of the variances. Now we see that our tactical asset allocation strategy systematically delivers lower risk than the more conventional portfolio. The risk reduction is in the order of 5%. In the world of investment where even the slightest advantage can mean massive financial rewards, this reduction is very substantial and highly significant.

7 Conclusion

This paper has examined how to improve tactical asset allocation by better risk management. The main reason for this is that CAPM theory requires the use of the conditional covariance of asset returns and this is time-varying not constant. This implies that portfolios should be regularly rebalanced to reflect this. Moreover, unlike returns, the conditional covariance matrix can be reasonably well predicted. Given recent advances in econometric methodology and computing power this will become a practical strategy for investment managers to use.

A new specification of the multivariate GARCH model is proposed which is particularly well suited to modelling asset returns due to its generality, parameter parsimony and relative ease of estimation. This is achieved by separating the conditional second moment matrix of returns into long-run and short-run components, and at the same time ensuring that the time-varying conditional covariance matrix generated by the model is positive definite.

This strategy was implemented for a representative UK investor holding four domestic assets: equity, two long bonds (a 15-year and a 5-year bond) and cash. Monthly data was used over the period 1976.1-1997.2. Both unconstrained and constrained optimal portfolios that exclude short sales are constructed. It was found that whilst the average portfolio shares for the re-balanced portfolios were similar to the shares based on the unconstrained covariance matrix, there was considerable variation in the short run. The variation in portfolio shares for the unconstrained portfolios were much larger than for the constrianed portfolios. Equities were found to be the dominant asset in every period, accounting for 70% of the portfolio on average. The average share was 20% for the longer bond and 10% for the shorter bond. It was estimated that compared with using constant shares, but maintaining the same average return on the portfolio, a 5% reduction in portfolio risk could be achieved on average through this form of risk management.

These results have encouraged us to extend the analysis to include more assets, both domestic and foreign, are included in the portfolio so that we can undertake global asset allocation, and we can explore how to take account of macroeconomic sources of risk. This will require the development of new theory. The main constraint, however, is the difficulty of achieving convergence of the likelihood function as the number of assets increases.

Acknowledgement

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Notes

[1]For a detailed treatment of portfolio theory, one can refer to Huang and Litzenberger (1988) or Ingersoll (1987).

[2]It may be recalled that when first formulating his theory of portfolio allocation, Markowitz emphasised the importance of minimising risk as opposed to maximising return. The accepted principle of the day was that an investor should choose a portfolio of assets by maximising discounted expected returns. This is not to suggest that these economists totally ignored the concept of risk. Typically, risk was accounted for, e.g. Keynes(1936) or Hicks(1939), by including a risk premium in the expected future asset returns. Markowitz argued that under such an asset allocation strategy, the optimal portfolio would only contain one asset, i.e. the asset with the highest discounted flow of expected future returns. This was clearly inconsistent with the practice of holding diversified portfolios. Markowitz's "mean-variance" rule not only offered an explanation for the practice of diversification in terms of reducing the variance of a portfolio's return, but also showed that investors should diversify over securites with low return covariances. Later, in 1959, Markowitz showed that this type of portfolio selection is firmly grounded as rational choice under uncertainty.

[3]This formulation follows Constantinides and Malliaris (1995) which in turn relies heavily on Roll (1977).

[4]For a discussion of this and a review of alternative specifications, see Bollerslev, Engle & Nelson(1994) and Bera & Higgins(1993).

[5]A simple example will illustrate the timing difference. If we assume that we have a two asset system with returns, $r_{1,t+1}$ and $r_{2,t+1}$, and conditional variances denoted by $H_{11}$and H_{22} and covariance by H_{12}.

Definition 1. **V**, **A** and **B** are defined as symmetric, triangular matrices.

$$H_{11,t} = V_{11}^2 + \Phi_{11}^2 H_{11,t-1} + \Theta_{11}^2 \varepsilon_{1,t}^2$$

and

$$\varepsilon_{1,t+1} = r_{1t} - b_1 - \lambda_1 r_{1,t-1} - \lambda_2 r_{2,t-1}$$

It takes two periods for the second asset to influence the conditional variance of the first.

Definition 2. **V**, **A** and **B** are defined as full, symmetric matrices.

$$\begin{aligned} H_{11,t} &= (V_{11}^2 + V_{12}^2) + \\ & (\Phi_{11}^2 H_{11,t-1} + 2\Phi_{11}\Phi_{12} H_{12,t-1} + \Phi_{12}^2 H_{22,t-1}) + \\ & (\Theta_{11}^2 \varepsilon_{1,t}^2 + 2\Theta_{11}\Theta_{12}\varepsilon_{1,t}\varepsilon_{2,t} + \Theta_{12}^2 \varepsilon_{2,t}^2) \end{aligned}$$

Now the conditional variance of the first asset is influenced by the second with only a one period time lag through the covariance term, $H_{12,t-1}$ and its own variance, $H_{22,t-1}$.

[6]All returns are nominal values. We use nominal returns to be consistent with other studies and using the results of Engle(1984) and Cumby(1988) where it is argued that both the behaviour of both nominal and real returns are substantially the same.

[7]A wide range of Unit root tests, such as (Augmented) Dickey Fuller tests, Stock Watson tests and Phillips Peron tests, were conducted on these series and all results confirm stationarity. Results are available from the authors upon request.

[8]See Clare et al. (1998) for an alternative approach to achieveing convergece based on the use of analytic derivatives.

[9]See Fletcher(1981) for a discussion of Quadratic Programming techniques.

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Appendix 1

Equation of the Portfolio frontier in expected excess return, standard deviation space is

$$sd_t = \sqrt{\frac{a - 2bR_{pt} + cR_{pt}^2}{ac - b^2}} \tag{A.1}$$

Equation of the Capital Market Line

$$sd_t = mR_{pt} \tag{A.2}$$

At the point of tangency, the slopes of these two functions must be equal

$$\begin{aligned} m_t &= \tfrac{1}{2}\left(\tfrac{a-2bR_{pt}+cR_{pt}^2}{ac-b^2}\right)^{\frac{-1}{2}}\left(\tfrac{2cR_{pt}-2b}{ac-b^2}\right) \\ &= \tfrac{cR_{pt-b}}{\left(a-2bR_{pt}+cR_{pt}^2\right)^{\frac{1}{2}}(ac-b^2)^{\frac{1}{2}}} \end{aligned} \tag{A.3}$$

Replace m in A.2 and solve the simultaneous equation system

$$\begin{aligned} \left(\tfrac{a-2bR_{pt}+cR_{pt}^2}{ac-b^2}\right)^{\frac{1}{2}} &= \left(\tfrac{cR_{pt-b}}{\left(a-2bR_{pt}+cR_{pt}^2\right)^{\frac{1}{2}}(ac-b^2)^{\frac{1}{2}}}\right)R_{pt} \\ a - 2bR_{pt} + cR_{pt}^2 &= cR_{pt}^2 - bR_{pt} \\ bR_{pt} &= a \\ R_{pt} &= \tfrac{a}{b} \end{aligned} \tag{A.4}$$

	Mean	Max.	Min.
Excess Return	7.41	16.1	4.34
Standard Deviation	46.0	102.3	26.8

Table 6.1 Key Features of the Optimal Portfolio

	Equity	Long Bond	Short Bond
OLS estimate	71.3%	18.2%	10.5%
Long-run Matrix	69.6%	26.2%	4.2%

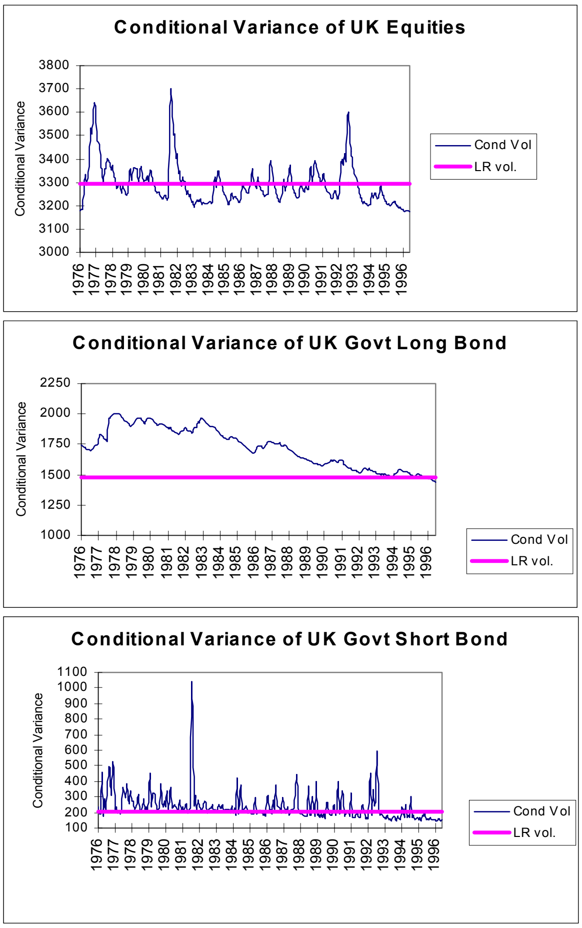
Table 6.2 Optimal Buy and Hold Portfolios

	Mean Weight	Maximum	Minimum.
Equities	69.7%	160%	38.1%
Long Bond	19.8%	84%	-3.6%
Short Bond	10.5%	65%	-144%

Table 6.3 Summary statistics for unrestricted portfolio

	Mean Weight	Maximum	Minimum
Equities	69.4%	72.8%	62.7%
Long Bond	20.3%	37.3%	11.5%
Short Bond	10.3%	15.6%	0%

Table 6.4 Summary statistics for restricted portfolio



Conditional Variance of UK Equities
Conditional Variance
3800
3700
3600
3500
3400
3300
3200
3100
3000
1976 1977 1978 1979 1980 1981 1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996
Cond Vol
LR vol.
Conditional Variance of UK Govt Long Bond
Conditional Variance
2250
2000
1750
1500
1250
1000
1976 1977 1978 1979 1980 1981 1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996
Cond Vol
LR vol.
Conditional Variance of UK Govt Short Bond
Conditional Variance
1100
1000
900
800
700
600
500
400
300
200
100
1976 1977 1978 1979 1980 1981 1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996
Cond Vol
LR vol.

Figure 1:

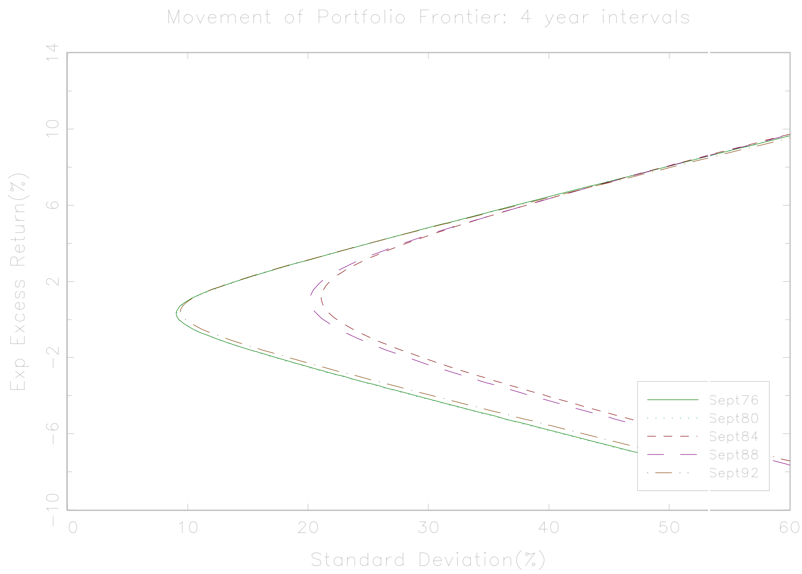
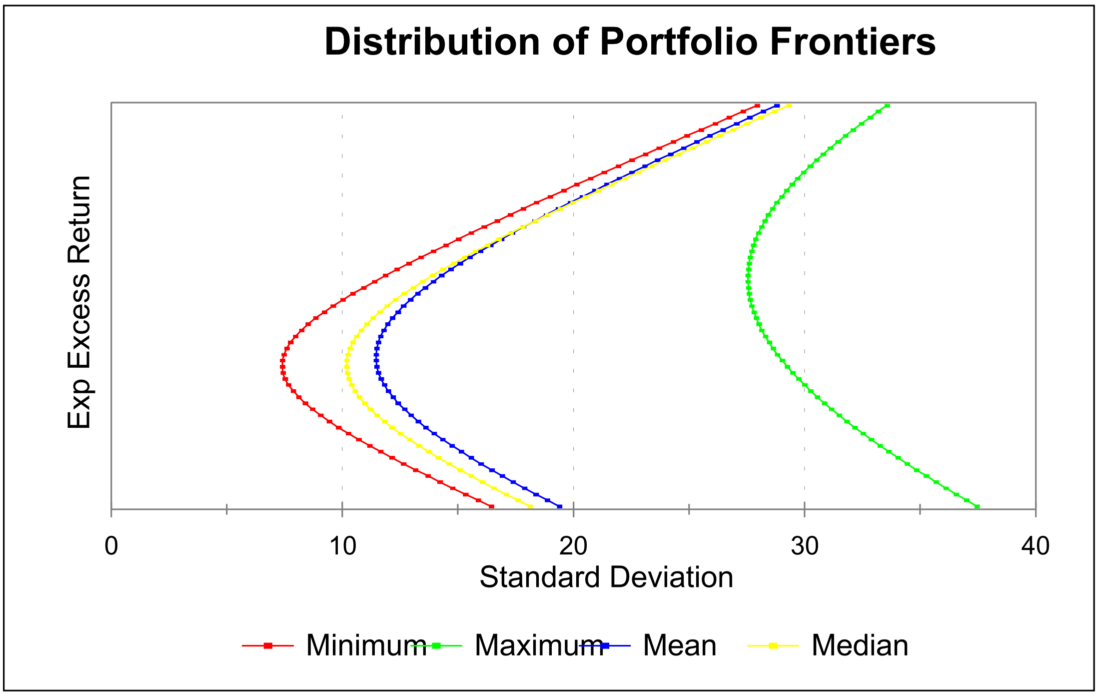
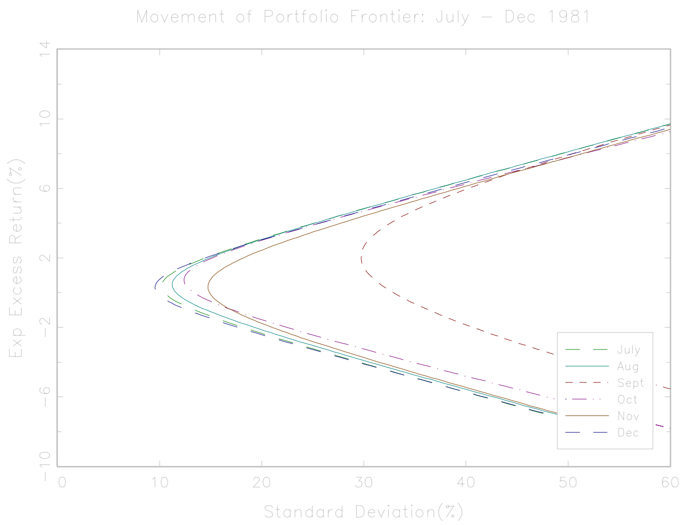

Movement of Portfolio Frontier: 4 year intervals
Exp Excess Return(%)
Standard Deviation(%)
Sept76
Sept80
Sept84
Sept88
Sept92
-10
-6
-2
2
6
10
14
0
10
20
30
40
50
60

Figure 2:



Distribution of Portfolio Frontiers
Exp Excess Return
0
10
20
30
40
Standard Deviation
Minimum
Maximum
Mean
Median

Figure 3:



Movement of Portfolio Frontier: July – Dec 1981
Exp Excess Return(%)
Standard Deviation(%)
July
Aug
Sept
Oct
Nov
Dec

Figure 4:

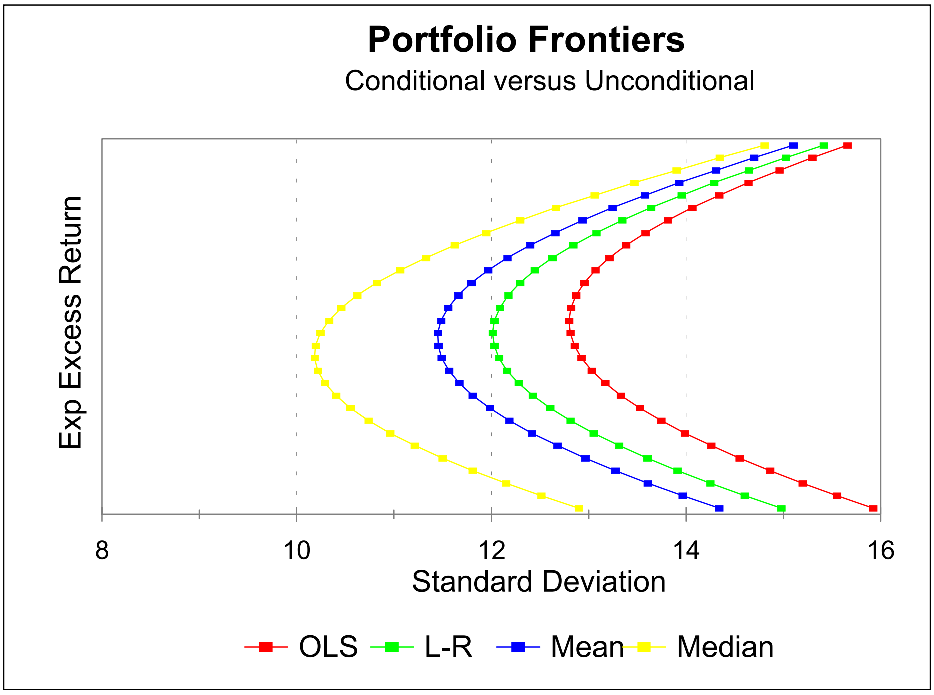


Figure 5:

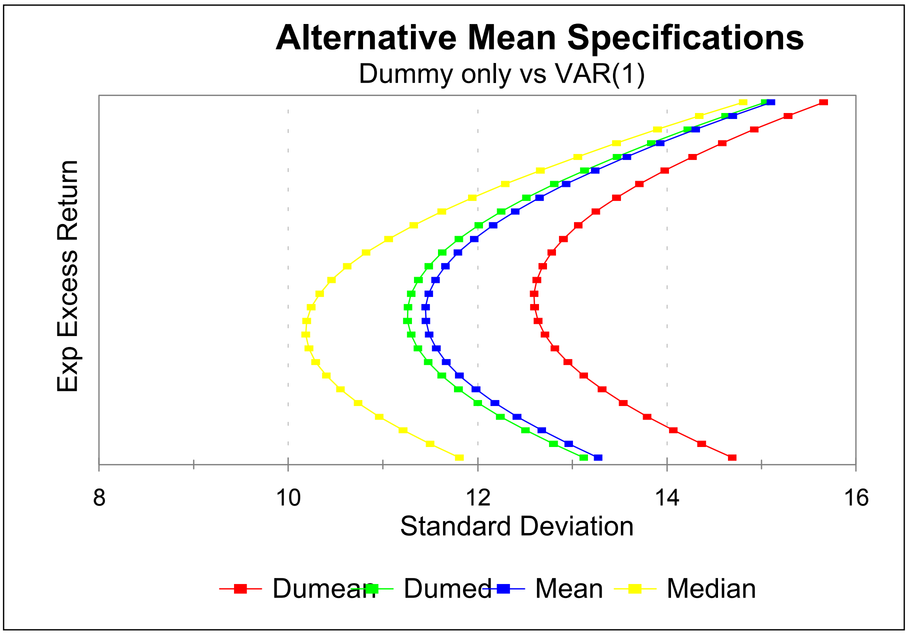


Figure 6:

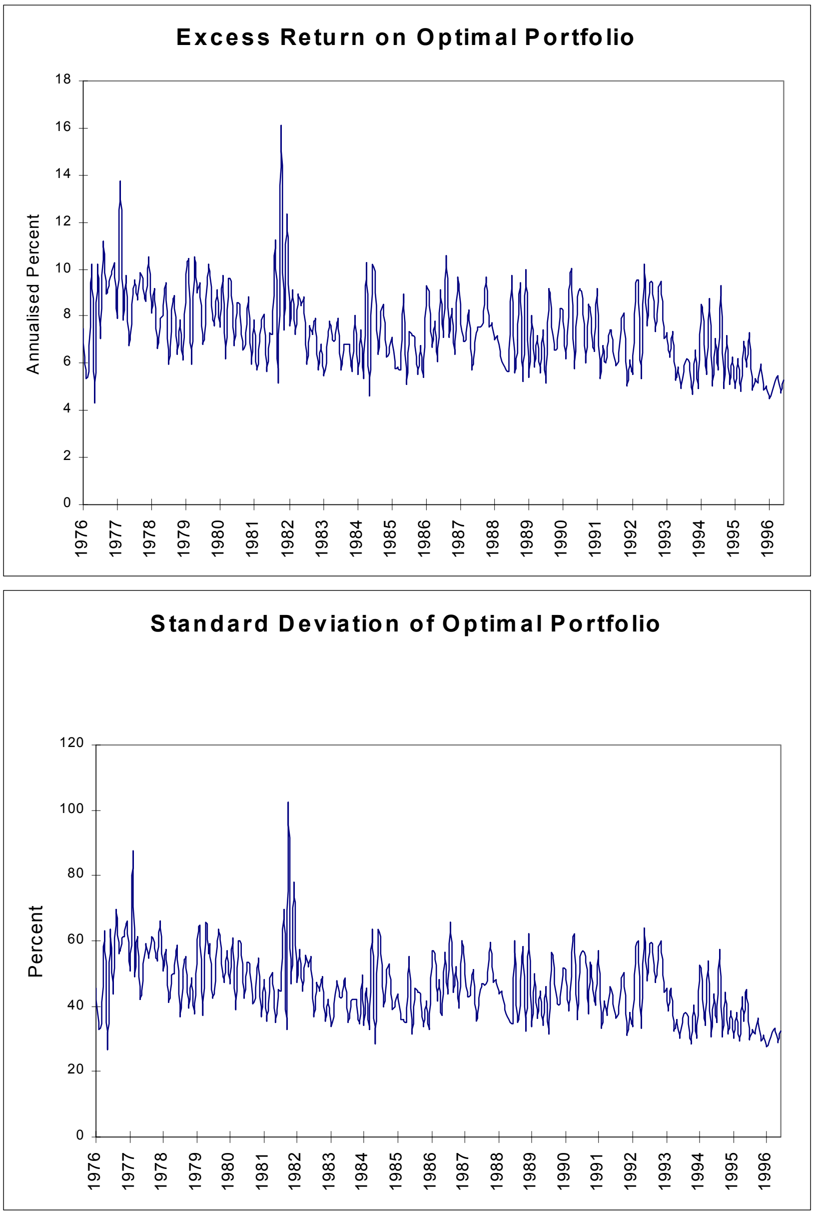
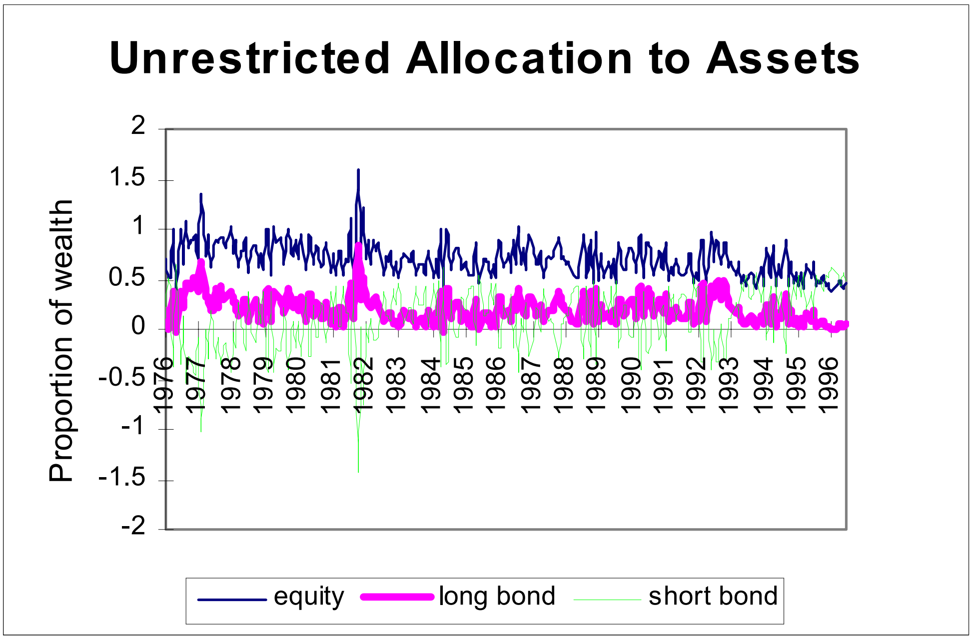

Excess Return on Optimal Portfolio
Annualised Percent
18
16
14
12
10
8
6
4
2
0
1976
1977
1978
1979
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
Standard Deviation of Optimal Portfolio
Percent
120
100
80
60
40
20
0

Figure 7:



Unrestricted Allocation to Assets
Proportion of wealth
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
1976
1977
1978
1979
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
equity
long bond
short bond

Figure 8:

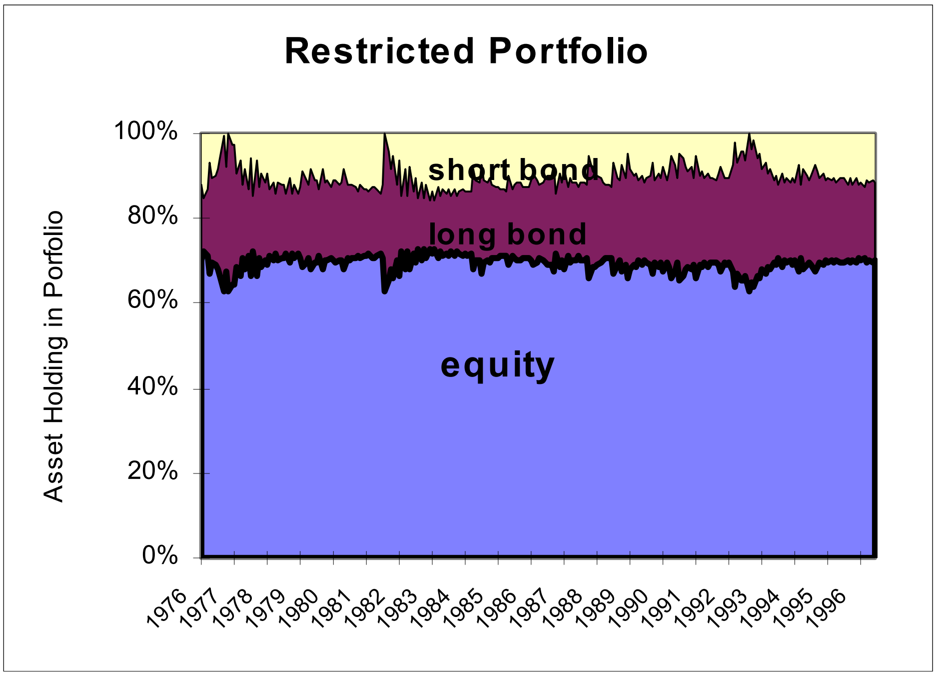

Restricted Portfolio
100%
80%
60%
40%
20%
0%
Asset Holding in Porfolio
short bond
long bond
equity
1976
1977
1978
1979
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996

Figure 9:

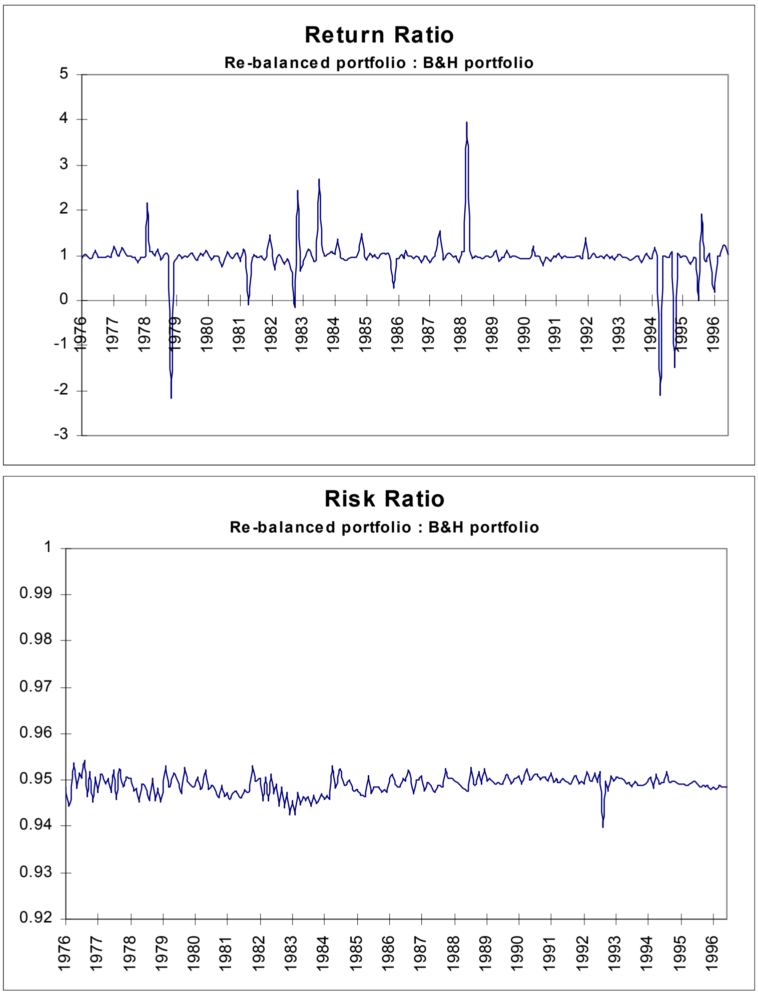

Return Ratio
Re-balanced portfolio : B&H portfolio
5
4
3
2
1
0
-1
-2
-3
1976
1977
1978
1979
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
Risk Ratio
Re-balanced portfolio : B&H portfolio
1
0.99
0.98
0.97
0.96
0.95
0.94
0.93
0.92
1976
1977
1978
1979
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996

Figure 10: